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mekanika.

2.1) Find the Velocity $\dot{x}$ and the Position x as Functions of the time t for a Particle of massa m, which starts from rest at $x=0$ and $t=0$, subject to the following force functions:

a. $F_x = F_0 + Ct$.
b. $F_x = F_0 \sin Ct$
c. $F_x = F_0 e^{Ct}$.

where F_0 and C are Positive Constants.

2.2) Find the Velocity $\dot{x}$ as a Function of the displacement x for a Particle of mass m, which Starts from rest at $x=0$, subject to the following force functions:

a. $F_x = F_0 + Cx$.
b. $F_x = F_0 e^{-Cx}$
c. $F_x = F_0 \cos Cx$.

where F_0 and C are Positive constants.

2.3) Find the Potential energy function $V(x)$ for each of the Forces in Problem 2.2.

2.4). A Particle of mass m is constrained to Lie along a Frictionless, horizontal Plane subject to a force given by the expression $F(x) = -kx$. it is Projected from $x=0$ to the right along the Positive x direction with initial kinetic energy $T_0 = \frac{1}{2}kA^2$. k and A are Positive Constants. Find (a). the Potential.

energy function V<x> for this force. <b> the kinetic energy, and <C> the total energy of the particle as a function of its position. <d> find the turning points of the motion. <e> sketch the potential, kinetic, and total energy functions. <optional : use. mathcad OR mathematica to plot these to plot these functions set k and A each equal to I>.

2.8} given that the velocity of a particle in rectilinear motion varies with the displacement x according to the equation.

$$\dot{x} = bx^{-3}$$

where b is a positive constant, find the force acting on the particle as a function of x. <hint : $F = m\ddot{x} = m\dot{x}\, d\dot{x}/dx$>.

2.11}. A metal block of mass m slides on a horizontal surface that has been lubricated with a heavy oil so that the block suffers a viscous that varies as the $\frac{3}{2}$ power of the speed.

$$F\langle v\rangle = -cv^{3/2}.$$

if the initial speed of the block is v_0 at $x=0$, show that the block cannot travel farther than $2mv_0^{\frac{1}{2}}/c$

2.R} A gun is fired straight up. assuming that the air drag on the bullet varies quadratically with speed. Show that the speed varies with height acording to the equations.

$$v^2 = Ae^{-2kx} - \frac{g}{k} \quad \langle\text{upward motion}\rangle.$$

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$V^2 = \frac{g}{k} - Be^{2kx}$ <downward motion>.

in which A and B are constants of integration, g is acceleration of gravity, and $k = C_2/m$ where C_2 is the drag constant and m is the mass of the bullet. <note: x is measured possitive upward, and the gravitational force is assumed to be constant)

2.13} use the above result to show that, when the bullet hits the ground on its return, the speed is equal to the expression.

$$\frac{V_0 V_t}{\langle V_0^2 + V_t^2 \rangle^{\frac{1}{2}}}$$

in which V_0 is the initial upward speed and

$$V_t = \langle mg/C_2 \rangle^{\frac{1}{2}} = \text{terminal speed}$$
$$= \langle g/k \rangle^{\frac{1}{2}}.$$

<this result allows one to find the fraction of the initial kinetic energy lost through air friction).

2.14}. A particle of mass m is released from rest a distance b from a fixed origin of force that attracts the particle according to the inverse square law:

show that the time required for the particle to reach the origin is.

$$\pi \left\langle \frac{mb^3}{8k} \right\rangle^{\frac{1}{2}}.$$

2.15}. Show that the terminal speed of a falling spherical object is given by

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$V_t = [\langle mg/C_2 \rangle + \langle C_1/2C_2 \rangle^2]^{\frac{1}{2}} - \langle C_1/2C_2 \rangle.$

when both the liniear and the quadratic terms in the drag Force are taken into account.

Jawaban.

(2.1) a.) $\dot{x} = \frac{d\dot{x}}{dt} = \frac{d\dot{x}}{dx} \cdot \frac{dx}{dt} = x' \frac{d\dot{x}}{dt}.$

$\dot{x} \frac{d\dot{x}}{dt} = \frac{1}{m} \langle F + Ct \rangle$

$\dot{x} d\dot{x} = \frac{1}{m} \langle F + Ct \rangle dt.$

$\int \dot{x}\, d\dot{x} = \int \frac{1}{m} \langle F + Ct \rangle dt.$

$\frac{1}{2} \dot{x} = \frac{1}{m} \langle F \cdot x + \frac{Ct}{2} \rangle$

$\frac{x}{m} = \langle 2F + Ct \rangle^{\frac{1}{2}}.$

b). $\dot{x} = \frac{d\dot{x}}{dt} = \frac{d\dot{x}}{dx} \frac{dx}{dt} = \frac{\dot{x}\, d\dot{x}}{dt}.$

$\dot{x} \frac{d\dot{x}}{dt} = \frac{1}{m} \langle F \sin Ct \rangle.$

$\dot{x}\, d\dot{x} = \frac{F}{m} \sin Ct\, dt.$

$\int \dot{x}\, d\dot{x} = \int \frac{F}{m} \sin Ct\, dt.$

$\frac{1}{2} \dot{x} = \frac{F}{Cm} \cos Ct.$

$\dot{x} = \langle -\frac{2F}{Cm} \cos Ct \rangle^{\frac{1}{2}}.$

c) $\dot{x} = \frac{d\dot{x}}{dt} = \frac{d\dot{x}}{dx} \cdot \frac{dx}{dt} = \dot{x} \frac{d\dot{x}}{dt}.$

$\dot{x} \frac{d\dot{x}}{dt} = \frac{1}{m} Fe^{Ct}$

$\dot{x}\, d\dot{x} = \frac{F}{m} e^{Ct}\, dt$

$\int \dot{x}\, d\dot{x} = \int \frac{F}{m} e^{Ct}\, dt$

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$\frac{1}{2}\dot{x}^2 = \frac{F}{cm}(e^{ct}-1)$

$\dot{x} = (\frac{2F}{cm})(e^{-ct}-1))^{\frac{1}{2}}$

2.2 a) $V(x) = -\int_x^x (F+Cx)\,dx.$

$= [(Fx + \frac{Cx^2}{2})] + c.$

$= -Fx - \frac{Cx^2}{2} + c.$

b). $V(x) = -\int_x^x F\, e^{-cx}\, dx.$

$= -(-\frac{F}{c} e^{-cx}) + c.$

$= \frac{F}{c} e^{-cx} + c.$

2.8 $\dot{x} = bx^{-3}$, $F = m\ddot{x} = m\dot{x}\frac{d\dot{x}}{dx}$

$F = m\dot{x}\frac{d\dot{x}}{dx}$

$= m(bx^{-3})\, d(bx^{-3})/dx$

$= m(bx^{-3})(-3bx^{-4})$

$= -3mb^2x^{-7}.$

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2.11 Block sliding under Forre

$F(v) = -cv^{3/2}$

$= m\frac{dv}{dT} = m\frac{dv}{dx}\frac{dx}{dt} = mv\frac{dv}{dx}.$

The distance trovelled is.

$-cv^{3/2} = mv\frac{dv}{dx}$

$\int_0^x dx = \int_{v_0}^v -\frac{mv}{cv^{3/2}}\, dv$

$x = -\frac{m}{c}\int_{v_0}^v \frac{dv}{\sqrt{v}} = -\frac{m}{c}\int_{v_0}^v v^{-\frac{1}{2}}\, dv$

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$$= -\frac{m}{c} \frac{V^{\frac{1}{2}}}{\frac{1}{2}} \Big|_{V_0}^{V}$$

$$= -\frac{2m}{c} \langle \sqrt{V_0} - \sqrt{V} \rangle$$

The particle is stopped when $V=0$, so the maximum distance trowelled is

$$X_{max} = \frac{2m\sqrt{V_0}}{c}.$$

2.3 a) $F_x = -\frac{dV}{dx} = F_0 + cV$

$$V\langle x \rangle = -[F_0 x + \frac{c}{2}x^2] + c$$

b). $F_x = -\frac{dV}{dx} = F_0 e^{-cx}$

$$\int_{V\langle x_0 \rangle}^{V\langle x \rangle} dV = -F_0 \int_{x_0}^{x} e^{-cx} \, dV.$$

$$V\langle x \rangle - V\langle x_0 \rangle = \frac{F_0}{c} [e^{-cx} - e^{-cx_0}]$$

c). $F_x = -\frac{dV}{dx} = F_0 \cos cx$

$$\int_{V\langle x_0 \rangle}^{V\langle x \rangle} dV = -F_0 \int_{F_0}^{x} \cos\langle cx \rangle \, dV$$

$$V\langle x \rangle - V\langle x_0 \rangle = -\frac{F_0}{c} [\sin cx - \sin cx_0]$$

2.4 Penyelesaian:

a). $V\langle x \rangle = -\int F\langle x \rangle \, dx$

$$= -\int \langle -kx \rangle \, dx$$

$$= \frac{kx^2}{2}.$$

b). $E = T_0 = \frac{1}{2} kA^2$

$$E = T\langle x \rangle - 1\,V\langle x \rangle$$

$$T\langle x \rangle = E - V\langle x \rangle$$

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$$T\langle x\rangle = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$$

$$T\langle x\rangle = \frac{k}{2}\langle A^2 - x^2\rangle.$$

c). Energi kinetik total.

$$E = T_0 = \frac{1}{2}kA^2.$$

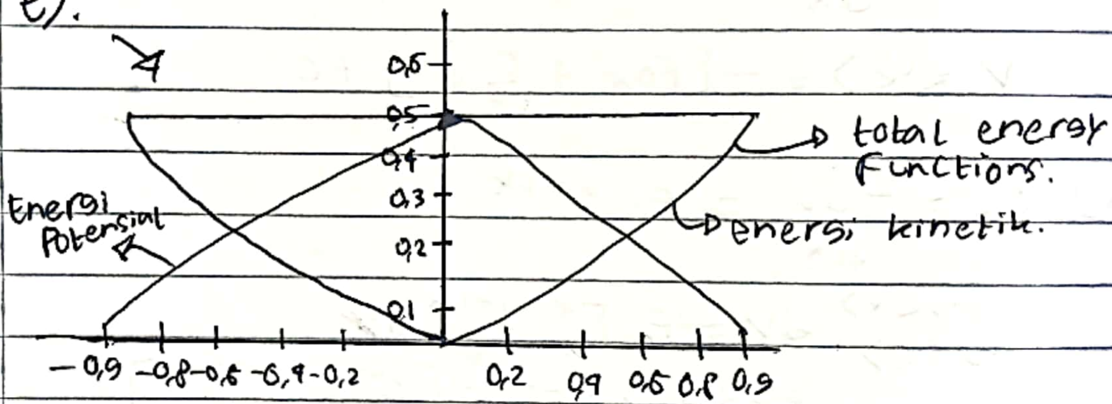
d). $E = V\langle x\rangle$

$$E = V\langle x\rangle$$

$$\frac{1}{2}kA^2 = \frac{1}{2}kx^2$$

$$x = \pm A.$$

e).



(2.12) * Upwark motion.

$$ma_x = \sum F_x$$

$$ma_x = -mg - c_2 v^2$$

$$a_x = \frac{dv}{dt} = \frac{dv}{dx}\cdot\frac{dx}{dt}$$

$$mv\frac{dv}{dx} = -mg - c_2 v^2$$

$$v\frac{dv}{dx} = -g - c_2 v^2$$

$$v\frac{dv}{dx} = -g - \frac{c_2}{m}v^2$$

$$k = \frac{c_2}{m}$$

$$v\frac{dv}{dx} = -g - kv^2$$

$$\int_{v_0}^{v} \frac{v\,dv}{-g-kv^2} = \int_0^x dx.$$

$$y = -g - kv^2$$

$$dy = -2kv\,dv$$

$$v\frac{dv}{dx} = -g \cdot kv^2$$

$$y = -g - kv^2$$

$$dy = -2kv\,dv$$

$$v\,dy = -\frac{1}{2k}\,dy.$$

$$\int_{v_0}^{v} -\frac{1}{2k} \cdot \frac{dy}{y} = \int_0^x dv.$$

$$v\,dv = -\frac{1}{2k}\,dy.$$

$$\int_{v_0}^{v} \frac{-1}{2k} \cdot \frac{dy}{y} = \int_0^x dv.$$

$$-\frac{1}{2k}\ln(y)\Big|_{v_0}^{v} = \int_0^x dv.$$

$$= \frac{1}{2k}\ln\left(\frac{-g-kv^2}{-g-kv_0^2}\right) = x.$$

$$\frac{-g-kv^2}{-g-kv^2} = (-g-kv_0^2)e^{-2kx}.$$

$$v^2 = \left(\frac{g}{k} + v_1^2\right)e^{-2kx}.$$

$$A = \frac{g}{k} + v_0^2$$

$$v^2 = Ae^{-2kx} - \frac{g}{k}$$

* downwark motion.

$$ma_x = \sum F_x.$$

$$ma_x = -mg + c_2v^2$$

$$a_x = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = \frac{dv}{dx} \cdot v$$

$$mv\frac{dv}{dx} = -mg + C_2V^2$$

$$V\frac{dv}{dx} = -g + \frac{C_2}{m}v^2$$

$$k = \frac{C^2}{m}$$

$$V\frac{dv}{dx} = -g + kv^2$$

$$\int_0^v \frac{v\,dv}{-g+kv^2} = \int_{x_0}^{x} dx.$$

$$y = -g + kv^2$$

$$dy = 2k\,v\,dv$$

$$v\,dv = \frac{1}{2k}\,dy.$$

$$\int_0^v \frac{1}{2k}\frac{dy}{y} = \int_{x_0}^{x} dx.$$

$$\frac{1}{2k}\ln\langle y\rangle\Big|_0^v = x\Big|_{x_0}^{x}$$

$$\frac{1}{2k}\ln\left[-g + kv^2\right]\Big|_0^v = x - x_0$$

$$\frac{1}{2k}\ln\left[\frac{-g+kv^2}{-g}\right] = x - x_0$$

$$\frac{-g+kv^2}{-g} = e^{2k\langle x-x_0\rangle}$$

$$-g + kv^2 = -ge^{2k\langle x-x_0\rangle}$$

$$kv^2 = -ge^{2k\langle x-x_0\rangle} + g$$

$$V^2 = -\frac{g}{k}e^{-2kx_0}e^{2kx} +$$

$$B = \frac{g}{k}e^{-2kx_0}$$

$$V^2 = \frac{g}{k} - Be^{2kx}$$

(2.13). $v^2 = \left(\frac{g}{k} + v_0^2\right) e^{-2ky} - \frac{g}{k}$ (upward motion)

$v^2 = -\frac{g}{k} e^{-2ky_0} e^{2ky} + \frac{g}{k}$ (downward motion

$0 = \left(\frac{g}{k} + v_0^2\right) e^{-2kh} - \frac{g}{k}$

$e^{-2kh} = \dfrac{\frac{g}{k}}{\frac{g}{k} + v_0^2}$

$v^2 = -\frac{g}{k} e^{-2kh} e^{2k0} + \frac{g}{k}$

$= -\frac{g}{k} \cdot \dfrac{\frac{g}{k}}{\frac{g}{k} + v_0^2} \cdot 1 + \frac{g}{k}$

$v_t^2 = \frac{g}{k}$

$v^2 = -v_t^2 \cdot \dfrac{v_t^2}{v_t^2 + v_0^2} + v_t^2$

$= v_t^2 \left(\dfrac{v_t^2 + v_0^2 - v_t^2}{v_t^2 + v_0^2}\right) = \dfrac{v_t^2 v_0^2}{v_t^2 + v_0^2}$

$v = \dfrac{v_t^2 \cdot v_0^2}{\sqrt{v_t^2 + v_0^2}}$

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(3.14) $F(x) = -kx^{-2}$

$m \frac{d\dot{x}}{dt} = -kx^{-2}$

$\frac{d\dot{x}}{dt} = \frac{dx}{dt} \cdot \frac{d\dot{x}}{dx}$

$= \dot{x} \frac{d\dot{x}}{dx}$

$m\dot{x} \frac{d\dot{x}}{dx} = -kx^{-2}$

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$$\int_0^{\dot{x}} \dot{x}\, dx = -\frac{k}{m}\int_b^{x} \frac{dx}{x^2}$$

$$\frac{1}{2}\dot{x}^3 = \frac{k}{m}\left\langle \frac{1}{x} - \frac{1}{b} \right\rangle$$

$$\dot{x} = \sqrt{\frac{2k}{mb}\left\langle \frac{b-x}{x}\right.}$$

$$\frac{dx}{dt} = \sqrt{\frac{2k}{mb}\left\langle \frac{bx}{x} \right\rangle}$$

$$dt = \sqrt{\frac{mb}{2k}\left\langle \frac{x}{bx} \right\rangle}\, dx$$

$$\int_0^t dt = \sqrt{\frac{mb}{2k}}\int_b^0 \left\langle \frac{x}{b-x} \right\rangle^{\frac{1}{2}} dx$$

$$t = \sqrt{\frac{mb}{2k}} \left\langle b\int_b^0 \frac{dx}{b-x} - \int_b^0 dx \right\rangle.$$

$$\int_0^t dt = \sqrt{\frac{mb^3}{2k}}\int_b^0 \left\langle \frac{\frac{x}{b}}{1-\frac{x}{b}} \right\rangle^{\frac{1}{2}} d\left\langle \frac{x}{b} \right\rangle$$

$$t = \sqrt{\frac{mb^3}{2k}}\int_{\frac{\pi}{2}}^{0} \frac{\sin\theta \left\langle 2\sin\theta\cos\theta \right\rangle}{\cos\theta}\, d\theta.$$

$$t = \sqrt{\frac{2mb^3}{k}}\int_{\frac{\pi}{2}}^{0} \sin^2\theta\, d\theta$$

$$t = \pi\sqrt{\frac{mb^3}{8k}}.$$