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Tugas Mekanika (kerjakan soal no. 2.1, 2.2, 2.3, 2.4, 2.8, 2.11, 2.12, 2.13, 2.14 dan 2.15

2.1 Find the velocity $\dot{x}$ and the position $\ddot{x}$ as functions of the time t for ~~a~~ particle of mass m, which starts from rest at $x = 0$ and $t = 0$, subject to the following force functions :

(a) $F_x : F_0 + ct$
(b) $F_x : F_0 \sin ct$
(c) $F_x : F_0 e^{ct}$, where F_0 and c are positive contants

2.2 Find the velocity $\dot{x}$ as a function of the displacement x for a particle of mass m, which starts from rest at $x = 0$, subject to the following force functions :

(a) $F_x = F_0 + cx$
(b) $F_x = F_0 e^{-cx}$
(c) $F_x = F_0 \cos cx$

2.3 Find the potential energy function $V(x)$ for each of the forces in Problem 2.2.

2.4 A particle of mass m is constrained to lie along a frictionless, horizontal plane subject to a force given by the expression $F(x) = -kx$. It is projected from $x = 0$ to the ~~point~~ right along the positive x direction with initial kinetic energy $T_0 = 1/2\, kA^2$. k and A are positive constants. ~~...~~ Find (a) the potential energy function $V(x)$ for this force ; (b) the kinetic energy ; and (c) the total energy of the particle as a function of its position, (d) find the turning points of the motion, (e) sketch the potential, kinetic, and total energy functions. (Optinal : Use mathkad or Mathematica to plot these functions. Let k and A each equal to 1)

2.8 Given that the velocity of a particle in rectilinear motion varies with the displacement x according to the equation

$$\dot{x} = bx^{-3}$$

Where b is a positive constant, find the force acting on the particle as a function of x. (Hint $F = m\ddot{x} = m\dot{x}\, d\dot{x}/dx$)

2.11 A metal block of mass m slides on a horizontal surface that has been lubricated with a heavy oil so that the block suffers a viscous resistance that varies as the 3/2 power of the speed $F(v) = -cv^{3/2}$

PAPERLINE

2.12 A gun is fired straight up. Assuming that the air drag on the bullet varies quadratically with speed. Show that the speed varies with height according to the equations

$$V^2 = Ae^{-2kx} - \frac{g}{k} \quad \text{(upward motion)}$$

$$V^2 = \frac{g}{k} - Be^{2kx} \quad \text{(downward motion)}$$

In which A and B are constants of integration. g is the acceleration of gravity, and $k = C_2/m$ where C_2 is the drag constant and m is the mass of the Bullet. (note : x is measured positive upward, and the gravitational force is assumed to be constant).

2.13 Use the above result to show that, when the bullet hits the ground on its return, the speed will be equal to the expression

$$\frac{V_0 V_t}{(V_0^2 + V_t^2)^{1/2}}$$

In which V_0 is the initial upward speed and

$$V_t = (mg/C_2)^{1/2} = \text{terminal speed} = (g/k)^{1/2}$$

(this result allows one to find the fraction of the initial kinetic energy lost through air friction)

2.14 A particle of mass m is released from rest a distance b from a fixed origin of force that attracts the particle according to the invers square law :

$$F(x) = -kx^{-2}$$

Show that the time required for the particle to reach the origin is

$$T = \left(\frac{mb^3}{8k}\right)^{1/2}$$

2.15 Show that the terminal speed of a falling spherical object is given by

$$V_t = [(mg/C_2) + (C_1/2C_2)^2]^{1/2} - (C_1/2C_2)$$

When both the linear and the quadratic terms in the drag force are taken into account.

Penyelesaian

2.1 (a). $F_x = m\ddot{x} = F_0 + ct$

Diketahui

$V_0 = 0$, $t_0 = 0$ dan $x_0 = 0$

$F = -\frac{dv}{dx}$.

$= m\frac{dv}{dt} = F_0 + ct$

$m\,dv = (F_0 + ct)\,dt$

$m\int_{v_0}^{v} dv = \int_{t_0}^{t} (F_0 + ct)\,dt$

$m(V - V_0) = F_0(t - t_0) + \frac{c}{2}(t^2 - t_0^2)$

$V_0 = 0$. $t_0 = 0$

$mV = F_0 t + \frac{ct^2}{2}$

$v(t) = \frac{1}{m}\left(F_0 t + \frac{ct^2}{2}\right) = \frac{dx}{dt}$

$\int_{x_0}^{x} dx = \int_{t_0}^{t} \frac{1}{m}\left(F_0 t + \frac{ct^2}{2}\right) dt$

$X - X_0 = \frac{1}{m}\left(\frac{F_0}{2}(t^2 - t_0^2) + \frac{c}{6}(t^3 - t_0^3)\right)$

$= \frac{F_0}{2m}t^2 + \frac{c}{6m}t^3$

(b). $F_x = m\frac{dv}{dt} = F_0 \sin ct$

$\int_{v_0}^{v} dv = \frac{F_0}{m}\int_0^t \sin ct\,dt$

$V(t) = \frac{-F_0}{mc}\cos ct$

$x(t) = \int_0^x dx = -\frac{F_0}{mc}\int_0^t \cos ct\,dt$

(c) $F_x = m\frac{dv}{dt} = F_0 e^{ct}$

$$v(t) = \frac{F_0}{m}\int_0^t e^{ct}\,dt = \frac{F_0}{cm}\left(e^{ct}-1\right)dt$$

$$x(t) = \frac{F_0}{c^2 m}\left(e^{ct} - 1 - ct\right)$$

2.2 (a) $F_x = -\frac{dv}{dx} = F_0 + cx$

$$F(x) = mv\frac{dv}{dx} = F_0 + cx$$

$$\int_{v_0}^{v} mv\,dv = \int_0^x (F_0 + cx)\,dx$$

$$\frac{m}{2}v^2 = F_0 x + \frac{c}{2}x^2$$

$$v(x) = \sqrt{\frac{2}{m}\left(F_0 x + \frac{c}{2}x^2\right)}$$

(b) $F(x) = mv\frac{dv}{dx} = F_0 e^{-cx}$

$$\frac{1}{2}mv^2 = -\frac{F_0}{c}e^{-cx}$$

$$V = \frac{2}{m}\left[-\frac{F_0}{c}e^{-cx}\right]^{1/2}$$

(c) $F(x) = mv\frac{dv}{dx} = F_0 \cos cx$

$$\frac{1}{2}mv^2 = \int_0^x F_0 \cos cx = \frac{F_0}{c}\sin cx$$

$$V = \frac{2}{m}\left[\frac{F_0}{c}\sin cx\right)^{1/2}$$

2.3 (a). $F_x = -\frac{dv}{dx} = F_0 + cx$

$$V(x) = -\left[F_0 x + \frac{c}{2}x^2\right] + c$$

(b). $F_x = -\frac{dv}{dx} = F_0 e^{-cx}$

$$\int_{V(x_0)}^{V(x)} dV = -F_0 \int_{x_0}^{x} e^{-cx}\, dx$$

$$V(x) - V(x_0) = \frac{F_0}{c}\left[e^{-cx} - e^{-cx_0}\right]$$

(c). $F_x = -\dfrac{dV}{dx} = F_0 \cos cx$

$$\int_{V(x_0)}^{V(x)} dV = -F_0 \int_{x_0}^{x} \cos(cx)\, dx$$

$$V(x) - V(x_0) = \frac{-F_0}{c}\left[\sin cx - \sin cx_0\right]$$

2.4 (a). $V(x) = -\int F(x)\, dx$

$$= -\int (-kx)\, dx$$

$$= \frac{kx^2}{2}$$

(b). $E = T_0 = \frac{1}{2}kA^2$

$E = T(x) + V(x)$

$T(x) = E - V(x)$

$= \frac{1}{2}kA^2 - \frac{1}{2}kx^2$

$T(x) = \frac{k}{2}(A^2 - x^2)$

(c) Diawal kita hanya mempunyai energi kinetik, jadi itu adalah energi total $E = T_0$ $= \frac{1}{2}kA^2$

(d) titik balik bisa diperoleh dari $E = V(x)$

$$\frac{1}{2}kA^2 = \frac{1}{2}kx^2$$

$$x = \pm A$$

No.

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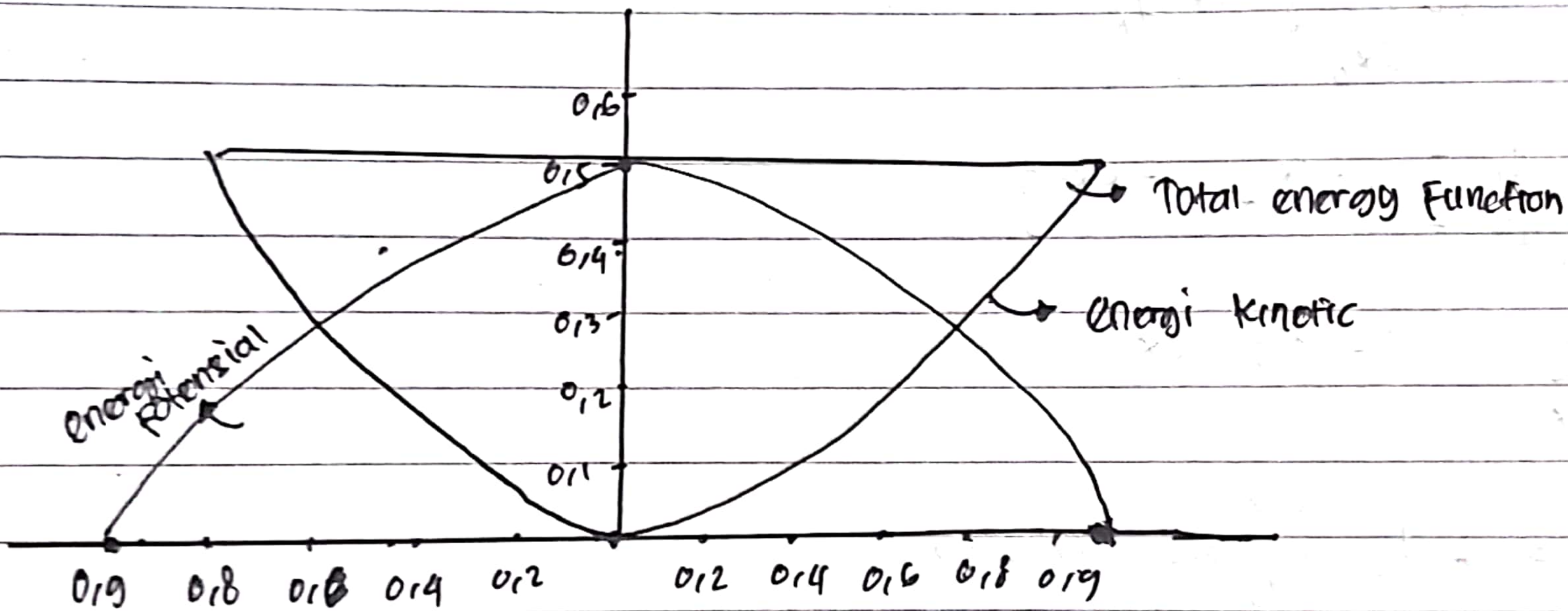
(e)



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Total energy Function
energi kinetic
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2.8 Given $v = \frac{b}{x^3}$

Sln $F = m\frac{dv}{dt} = m\frac{dv}{dx}\frac{dx}{dt}$

$= mv\frac{dv}{dx}$

$\frac{dv}{dx} = \frac{-3b}{x^4}$

$F = m\left(\frac{b}{x^3}\right)\left(\frac{-3b}{x^4}\right) = \frac{-3mb^2}{x^7}$

2.11 Block sliding under force

$f(v) = -cv^{3/2}$

$= m\frac{dv}{dt} = m\frac{dv}{dx}\frac{dx}{dt} = mv\frac{dv}{dx}$

the distance travelled is (-)

$-cv^{3/2} = mv\frac{dv}{dx}$

$\int_0^x dx = \int_{v_0}^{v} -\frac{mv}{cv^{3/2}}\,dv$

$x = -\frac{m}{c}\int_{v_0}^{v}\frac{dv}{\sqrt{v}} = -\frac{m}{c}\int_{v_0}^{v} v^{-1/2}\,dv$

$= -\frac{m}{c}\left.\frac{v^{1/2}}{1/2}\right|_{v_0}^{v}$

$= -\frac{2m}{c}\left(\sqrt{v_0} - \sqrt{v}\right)$

the particle is stopped when $v = 0$, so the maximum distance travelled is

$x_{max} = \frac{2m\sqrt{v_0}}{c}$

2.12 ~~[illegible]~~

Upwark motion

Hukum 2 Newton

$ma_x = \Sigma F_x$

$ma_x = -mg - C_2V^2$

$a_x = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt}$

$mv\frac{dv}{dx} = -mg - C_2V^2$

$v \cdot \frac{dv}{dx} = -g - \frac{C_2}{m}V^2$

$k = \frac{C_2}{m}$

$v\frac{dv}{dx} = -g - kv^2$

$\int_{v_0}^{v} \frac{v\,dv}{-g-kv^2} = \int_0^x dx$

$y = -g - kv^2$

$dy = -2kv\,dv$

$v\,dv = -\frac{1}{2k}dy$

$\int_{v_0}^{v} -\frac{1}{2k}\frac{dy}{y} = \int_0^x dx$

$-\frac{1}{2k}\ln(y)\Big|_{v_0}^{v} = x\Big|_0^x$

$-\frac{1}{2k}\ln\left(\frac{-g-kv^2}{-g-kv_0^2}\right) = x$

$\frac{-g-kv^2}{-g-kv_0^2} = e^{-2kx}$

$-g-kv^2 = (-g-kv_0^2)e^{-2}$

$kv^2 = (g+kv_0^2)e^{-2kx}$

$V^2 = \left(\frac{g}{k}+v_1^2\right)e^{-2kx}$

$A = \frac{g}{k}+v_0^2$

$V^2 = Ae^{-2kx} - \frac{g}{k}$

downwark motion

$ma_x = \Sigma F_x$

$ma_x = -mg + C_2V^2$

$a_x = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dx} \cdot v$

$mv\frac{dv}{dx} = -mg + C_2V^2$

$v\frac{dv}{dx} = -g + \frac{C_2}{m}V^2$

$k = \frac{C_2}{m}$

$v\frac{dv}{dx} = -g + kv^2$

$\int_0^v \frac{v\,dv}{-g+kv^2} = \int_{x_0}^x dx$

$y = -g + kv^2$

$dy = 2kv\,dv$

$v\,dv = \frac{1}{2k}dy$

$\int_0^v \frac{1}{2k}\frac{dy}{y} = \int_{x_0}^x dx$

$\frac{1}{2k}\ln(y)\Big|_0^v = x\Big|_{x_0}^x$

$\frac{1}{2k}\ln(-g+kv^2)\Big|_0^v = x - x_0$

$\frac{1}{2k}\ln\left(\frac{-g+kv^2}{g}\right) = x - x_0$

$\frac{-g+kv^2}{-g} = e^{2k(x-x_0)}$

$-g+kv^2 = -ge^{2k(x-x_0)}$

$kv^2 = -ge^{2k(x-x_0)}+g$

$V^2 = -\frac{g}{k}e^{-2kx_0}e^{2kx},$

$B = \frac{g}{k}e^{-2kx_0}$

$V^2 = \frac{g}{k} - Be^{2kx}$

2.13 $V^2 = \left(\frac{g}{k} + V_0^2\right) e^{-2kx} - \frac{g}{k}$ (upward motion)

$V^2 = -\frac{g}{k} e^{-2kx_1} e^{2kx} + \frac{g}{k}$ (downward motion)

$0 = \left(\frac{g}{k} + V_0^2\right) e^{-2kh} - \frac{g}{k}$

$e^{-2kh} = \frac{g/k}{g/k + V_0^2}$

$V^2 = -\frac{g}{k} e^{-2kh} e^{2k \cdot 0} + \frac{g}{k}$

$= -\frac{g}{k} \cdot \frac{g/k}{g/k + V_0^2} \cdot 1 + \frac{g}{k}$

$V_t^2 = \frac{g}{k}$

$V^2 = -V_t^2 \cdot \frac{V_t^2}{V_t^2 + V_0^2} + V_t^2$

$= V_t^2 \cdot \left(\frac{V_t^2 + V_0^2 - V_t^2}{V_t^2 + V_0^2}\right) = \frac{V_t^2 V_0^2}{V_t^2 + V_0^2}$

$V = \frac{V_t^2 \cdot V_0^2}{\sqrt{V_t^2 + V_0^2}}$

2.14

$F(x) = -kx^{-2}$

$m \frac{d\dot{x}}{dt} = -kx^{-2}$

$\frac{d\dot{x}}{dt} = \frac{dx}{dt} \frac{d\dot{x}}{dx}$

$= \dot{x} \frac{d\dot{x}}{dx}$

$m\dot{x} \frac{d\dot{x}}{dx} = -kx^{-2}$

$\int_0^{\dot{x}} \dot{x}\, dx = -\frac{k}{m} \int_b^x \frac{dx}{x^2}$

$\frac{1}{2} \dot{x}^3 = \frac{k}{m}\left(\frac{1}{x} - \frac{1}{b}\right)$

$\dot{x} = \sqrt{\frac{2k}{mb}\left(\frac{b-x}{x}\right)}$

$\frac{dx}{dt} = \sqrt{\frac{2k}{mb}\left(\frac{b-x}{x}\right)}$

$\int_0^t dt = \sqrt{\frac{mb}{2k}} \int_b^0 \left(\frac{x}{b-x}\right)^{1/2} dx$

$t = \sqrt{\frac{mb}{2k}} \left(b \int_b^b \frac{dx}{b-x} - \int_b^0 dx \right)$

$\int_0^t dt = \sqrt{\frac{mb^3}{2k}} \int_b^0 \left(\frac{x/b}{1 - x/b}\right)^{1/2} d\left(\frac{x}{b}\right)$

$t = \sqrt{\frac{mb^3}{2k}} \int_{-\pi/2}^0 \frac{\sin\theta\,(2\sin\theta\cos\theta)}{\cos\theta}$

$t = \sqrt{\frac{2mb^3}{k}} \int_{-\pi/2}^0 \sin^2\theta\, d\theta$

$t = \pi \sqrt{\frac{mb^3}{8k}}$

2.15 Hukum 2 Newton

$$ma = \Sigma F$$

$$ma = mg - C_1 V - C_2 V^2$$

$$ma = 0$$

$$mg - C_1 V - C_2 V^2 = 0$$

$$C_2 V^2 + C_1 V - mg = 0$$

$$V_{1,2} = \frac{-C_1 \pm \sqrt{C_1^2 + 4C_2 mg}}{2C_2}$$

$$V_1 = \frac{-C_1 + \sqrt{C_1^2 + 4C_2 mg}}{2C_2}$$

$$V_1 = V_t = -\frac{C_1}{2C_2} + \sqrt{\left(\frac{C_1}{2C_2}\right)^2 + \frac{mg}{C_2}}$$

$$V_t = -\frac{C_1}{2C_2} + \sqrt{\left(\frac{C_1}{2C_2}\right)^2 + \frac{mg}{C_2}}$$