Nama : Ririn Oriska
Npm : 2013022010
Kelas : B

"Tugas Mekanika"

2.1 a) $F_x = m\ddot{x} = F_0 + Ct$

$$m\frac{dv}{dt} = F_0 + Ct$$

$$m\,dv = (F_0 + Ct)\,dt$$

$$m\int_{v_0}^{v} dv = \int_0^t (F_0 + Ct)\,dt$$

$$m(v - v_0) = F_0(t - t_0) + \frac{C}{2}(t^2 - t_0^2)$$

karena $v_0 = 0$ dan $t_0 = 0$, maka =

$$v(t) = \frac{1}{m}\left(F_0 + \frac{Ct^2}{2}\right) = \frac{dx}{dt}$$

$$\int_{x_0}^{x} dx = \int_{t_0}^{t}\frac{1}{m}\left(F_0 t + \frac{Ct^2}{2}\right)dt$$

$$x - x_0 = \frac{1}{m}\left(\frac{F_0}{2}(t^2 - t_0^2) + \frac{C}{6}(t^3 - t_0^3)\right)$$

$$x = \frac{1}{m}\left[\frac{F_0 t^2}{2} + \frac{Ct^2}{6}\right]$$

b) $F_x = m\frac{dv}{dt} = F_0 \sin Ct$

$$\int_0^v dv = \frac{F_0}{m}\int_0^t \sin Ct \cdot dt$$

$$v(t) = -\frac{F_0}{mC}\cos Ct$$

$$x(t) = \int_0^x dt$$

$$x(t) = -\frac{F_0}{mC}\int_0^t \cos Ct\,dt$$

$$x(t) = -\frac{F_0}{mC^2}\sin Ct$$

c) $F_x = F_0 e^{Ct}$

$$v(t) = \frac{F_0}{m}\int_0^t e^{Ct}\,dt$$

$$v(t) = \frac{F_0}{Cm}e^{Ct}$$

$$x(t) = \frac{F_0}{C^2 m}e^{Ct}$$

2.2 a) $F_x = F_0 + C_x$

$$F(x) = mv\frac{dv}{dx} = F_0 + C_x$$

$$\int_0^v mv\,dv = \int_0^x (F_0 + (x)\,dx)$$

$$\frac{mv^2}{2} = F_0 x + \frac{C}{2}x^2$$

$$v = \sqrt{\frac{2}{m}\left(F_0 x + \frac{C}{2}x^2\right)}$$

b) $F(x) = mv\frac{dv}{dx} = F_0 e^{-Cx}$

$$\frac{1}{2}mv^2 = -\frac{F_0}{C}e^{-Cx}$$

$$v = \frac{2}{m}\left[-\frac{F_0}{C}e^{-Cx}\right]^{1/2}$$

c) $F(x) = mv\frac{dv}{dx} = F_0 \cos Cx$

$$\frac{1}{2}mv^2 = \int_0^x F_0 \cos Cx$$

$$= \frac{F_0}{C}\sin Cx$$

$$v = \frac{2}{m}\left[\frac{F_0}{C}\sin Cx\right]^{1/2}$$

2.3 a) $F_x = -\frac{du}{dx} = F_0 + Cx$

$$u(x) = -\left[F_0 x + \frac{C}{2}x^2\right] + C$$

b) $F_x = -\frac{du}{dx} = F_0 e^{-Cx}$

$$\int_{u(x_0)}^{u(x)} du = -F_0\int_{x_0}^{x} e^{-Cx}\,dx$$

$$u(x) - u(x_0) = \frac{F_0}{C}\left[e^{-Cx} - e^{-Cx_0}\right]$$

c) $F_x = -\frac{du}{dx} = F_0 \cos Cx$

$$\int_{u(x_0)}^{u(x)} du = -F_0\int_{x_0}^{x}\cos(x)\,dx$$

$$u(x) - u(x_0) = -\frac{F_0}{C}\left[\sin Cx - \sin Cx_0\right]$$

2.4 $U(x) = -\int f(x)\,dx$

a) $U(x) = -\int (-kx)\,dx$

$U(x) = \frac{kx^2}{2}$

Jadi, energi potensial $V(x) = \frac{kx^2}{2}$

b) dan c)

$E = T_0 = \frac{1}{2}kA^2$

$E = T(x) + U(x)$

$T(x) = E - U(x)$

$T(x) = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$

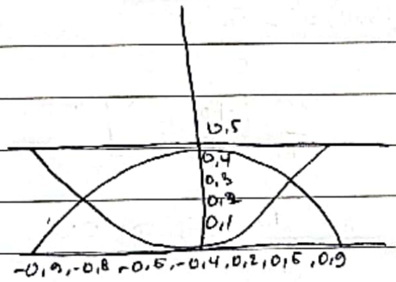
$T(x) = \frac{k}{2}(A^2 - x^2)$

d) $E = V_c x$

$\frac{1}{2}kA^2 = \frac{1}{2}kx^2$

$x = \pm A$

e)



Hasilnya = a) $V(x) = kx^2/2$

b) $T(x) = \frac{k}{2}(A^2 - x^2)$

c) $E = T_0 = \frac{1}{2}kA^2$

2.8 $V = b/x^3$

$F = m\frac{dv}{dt} = m\frac{dv}{dx}\cdot\frac{dx}{dt}$

$= mv\frac{dv}{dx}$

$\frac{dv}{dx} = -\frac{3b}{x^4}$

$F = m\left(\frac{b}{x^3}\right)\left(\frac{-3b}{x^4}\right) = -3mb^2/x^7$

2.11 $F(v) = -cv^{3/2}$

$= m\frac{dv}{dt} = m\frac{dv}{dx}\cdot\frac{dx}{dt}$

$= m\frac{dv}{dx}$

$\int_0^x dx = \int_{v_0}^{v}\frac{-mv}{cv^{3/2}}\,dv$

$x = -\frac{m}{c}\int_{v_0}^{v}\frac{dv}{\sqrt{v}} = -\frac{m}{c}\int_{v_0}^{v} v^{-1/2}\,dv$

$= -\frac{m}{c}\frac{v^{1/2}}{1/2}\Big|_{v_0}^{v}$

$= \frac{2m}{c}\left(\sqrt{v_0} - \sqrt{v}\right)$

Partikel berhenti, maka $v = 0$, Jadi ~~x mak~~ =

$x_{max} = \frac{2m\sqrt{v_0}}{c}$

2.12 $m.a_x = \Sigma F_x$

$m.a_x = -mg - c_2v^2$

$a_x = \frac{dv}{dt} = \frac{dv\,dx}{dx\,dt} = \frac{dv}{dt} = v$

$m\,dv/dx = -mg = c_2v^2$

$v.\,dv/dx = -g - \frac{c_2}{m}v^2$

$k = c^2/m$

$\int_{v_0}^{v}\frac{v\,dv}{-g-kv^2} = \int_0^x dx$

$y = -g - kv^2$

$dy = -2kv\,dv$

$v\,dv = -\frac{1}{2k}dy$

$\int_{v_0}^{v} -\frac{dy}{2ky} = \int_0^x dx$

$-\frac{1}{2k}\ln(y)\Big|_{v_0}^{v} = x\Big|_0^x$

$-\frac{1}{2k}\ln(-g-kv^2)\Big|_{v_0}^{v} = x$

$-\frac{1}{2k}\ln\left(\frac{-g-kv^2}{-g-kv_0^2}\right) = x$

$\frac{-g-kv^2}{-g-kv_0^2} = e^{-2kx}$

$-g-kv^2 = (-g-kv_0^2)e^{-2kx}$

$kv^2 = (g+kv_0^2)e^{-2kx} - \frac{g}{k}$

$A = \frac{g}{k} + v_0^2$

$\boxed{v^2 = Ae^{-2kx} - \frac{g}{k}}$

$ma_x = \Sigma F_x$

$m.a_x = -mg + c_2v^2$

$$a_x = dv/dt = dv/dx \cdot dx/dt = dv/dx \cdot v$$

$$mv\frac{dv}{dx} = -mg + C_2 v^2$$

$$\circ \quad dv/dx = -g + C_2/m \cdot v^2$$

$$k = C_2/m$$

$$v\,dv/dx = -g + kv^2$$

$$\int_0^v \frac{v\,dv}{-g+kv^2} = \int_0^x dx$$

$$y = -g + kv^2$$

$$dy = 2kv\,dv$$

$$v\,dv = \frac{1}{2k}dy$$

$$\int_0^v \frac{1}{2k}\frac{dy}{y} = \int_{x_0}^x dx$$

$$\frac{1}{2k}\ln(y)\Big|_0^v = x\Big|_{x_0}^x$$

$$\frac{1}{2k}\ln(-g+kv^2)\Big|_0^v = x - x_0$$

$$\frac{1}{2k}\ln\left(\frac{-g+kv^2}{-g}\right) = x - x_0$$

$$\frac{-g+kv^2}{-g} = e^{2k(x-x_0)}$$

$$-g + kv^2 = -ge^{2k(x-x_0)}$$

$$kv^2 = -ge^{2k(x-x_0)} + g$$

$$v^2 = -\frac{g}{k}e^{2kx_0}e^{2kx} + \frac{g}{k}$$

$$\beta = \frac{g}{k}e^{2kx_0}$$

$$\boxed{v^2 = \frac{g}{k} - \beta e^{2kx}}$$

2.13 $V^2 = \left(\frac{g}{k} + V_0^2\right)e^{-2kx} - \frac{g}{k} \rightarrow$ keatas

$$V^2 = -g/k \cdot e^{-2kx_0}e^{2kx} + \frac{g}{k} \rightarrow \text{kebawah}$$

- $v = 0$ pada gerak keatas

$$0 = (g/k + V_0^2)e^{-2kx} - g/k$$

$$e^{-2kx} = \frac{g/k}{g/k + V_0^2}$$

- h adalah posisi awal untuk gerakan kebawah dan (t) $a = 0$,

$$V^2 = -g/k \cdot e^{-2kh}e^{2k0} + g/k \rightarrow \text{sub } x = 0$$

$$V^2 = -g/k \cdot \frac{g/k}{g/k + V_0^2} \cdot 1 + g/k$$

$$V^2 = -g/k$$

$$= -V_t^2 \cdot \frac{V_t^2}{V_t^2 + V_0^2} + V_t^2$$

$$= V_t^2\left(1 - \frac{V_t^2}{V_t^2 + V_0^2}\right)$$

$$V^2 = \frac{V_t^2 V_0^2}{V_t^2 + V_0^2}$$

$$V = \frac{V_t V_0}{\sqrt{V_t^2 + V_0^2}}$$

2.14 $ma_x = \Sigma F_x$

$$ma_x = -kx^{-2}$$

$$a_x = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dx}\cdot v$$

$$mv\frac{dv}{dx} = \frac{-k}{m}\int_b^x x^{-2}dx$$

$$\frac{v^2}{2} = \frac{k}{m}(x^{-1} - b^{-1})$$

$$v^2 = \frac{2k}{m}\left(\frac{b-x}{bx}\right)$$

$$v = dx/dt$$

$$\sqrt{\frac{2k}{m}\left(\frac{b-x}{m}\right)} = \frac{dx}{dt}$$

$$\int_b^0 \frac{dx}{\sqrt{\frac{2k}{m}\cdot\left(\frac{b-x}{bx}\right)}} = \int_0^t dt$$

$$\int_b^0 \frac{dx}{\sqrt{\frac{2k}{m}\left(\frac{b-x}{bx}\right)}} = \sqrt{\frac{mb}{2k}}\int_b^0\sqrt{\frac{x}{b-x}}\,dx$$

$$u = \frac{\sqrt{b-x}}{\sqrt{x}}$$

$$du = \frac{\frac{1}{2\sqrt{b-x}}(-1)\cdot\sqrt{x}\,dx - \frac{1}{2\sqrt{x}}}{x}$$

$$dx = \frac{du}{\frac{-1}{2\sqrt{x}\cdot u\sqrt{x}} - \frac{u\sqrt{x}}{2x^{3/2}}}$$

$$= \frac{du}{-\frac{1}{2ux} - \frac{u}{2x}}$$

$$= -\frac{du}{\frac{1+v^2}{2xu}}$$

$$= -\frac{2xv\,du}{v^2+1}$$

$$\sqrt{b-x} = v\sqrt{x}$$

$$b - x = v^2x$$

$$b = x^2(v^2+1)$$

$$x = b/u^2+1$$

$$dx = -\frac{2bv\,du}{(v^2+1)^2}$$

$$\sqrt{\frac{mb}{2k}}\int_b^0 \sqrt{\frac{x}{b-x}}\,dx = \sqrt{\frac{mb}{2k}}\int_b^0 (\mp)\,\frac{2bv\,du}{(v^2+1)^2}$$

$$= \sqrt{\frac{mb}{2k}}\int_b^0 \frac{2bv\,du}{(v^2+1)^2}$$

$$\int \frac{dv}{(av+b)^n} = \frac{2n-3}{2b(n-1)}\int \frac{dv}{(av^2+b)^{n+1}} + \frac{v}{2b(n-1)(av^2+b)^{n-1}}$$

$$= \frac{u}{2(u^2+1)} + \frac{1}{2}\int \frac{du}{v^2+1}$$

$$\int_b^0 \frac{du}{(v^2+1)^2} = \frac{v}{2(v^2+1)}\Big|_b^0 + \frac{1}{2}\int_b^0 \frac{du}{v^2+1}$$

$$\int_b^0 \frac{du}{(v^2+1)^2} = \frac{v}{2(v^2+1)}\Big|_b^0 + \frac{1}{2}\arctan v\Big|_b^0$$

$$-\sqrt{\frac{mb}{2k}}\int_b^0 \frac{2b\,du}{(v^2+1)^2} = -2b\sqrt{\frac{mb}{2k}}\left(\frac{v}{2(v^2+1)}\Big|_b^0 + \frac{1}{2}\arctan u\Big|_b^0\right)$$

$$= -2b\sqrt{\frac{mb}{2k}}\left(\frac{v}{2(v^2+1)}\Big|_0^{-\infty} + \frac{1}{2}\arctan v\Big|_0^{-\infty}\right)$$

$$= -2b\sqrt{\frac{mb}{2k}}\left(0-0+\frac{1}{2}\cdot\left(-\frac{\pi}{2}-0\right)\right)$$

$$= \pi\sqrt{\frac{mb^3}{8k}}$$

$$\pi\sqrt{\frac{mb^3}{8k}} = \int_0^t dt$$

$$t = \pi\sqrt{\frac{mb^3}{8k}}$$

2.15

$$ma = \Sigma F$$

$$ma = mg - C_1 v - C_2 v^2$$

$$mg - C_1 v - C_2 v^2 = 0$$

$$C_2 v^2 - C_1 v - mg = 0$$

$$v_{1,2} = \frac{-C_1 \pm \sqrt{C_1^2 + 4C_2 mg}}{2C_2}$$

pilih (+)

$$v_1 = \frac{-C_1 + \sqrt{C_1^2 + 4C_2 mg}}{2C_2}$$

$$v_1 = v_t = \frac{-C_1}{2C_2} + \sqrt{\left(\frac{C_1}{2C_2}\right)^2 + \frac{mg}{C_2}}$$

$$\boxed{v_t = \frac{-C_1}{2C_2} + \sqrt{\left(\frac{C_1}{2C_2}\right)^2 + \frac{mg}{C_2}}}$$