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Prodi : Pendidikan Fisika (B)

⌚ Tugas Mekanika ⌚

(2.1) a). $F_x = m\ddot{x} = F_0 + Ct$

$$m \frac{dv}{dt} = F_0 + Ct$$

$$m \cdot dv = (F_0 + Ct) dt$$

$$m \int_{v_0}^v dv = \int_0^t (F_0 + Ct) dt$$

$$m(v - v_0) = F_0(t - t_0) + \frac{C}{2}(t^2 - t_0^2)$$

karena $v_0 = 0$ dan $t_0 = 0$, maka:

$$mv = F_0 t + \frac{Ct^2}{2}$$

$$v(t) = \frac{1}{m} \left(F_0 t + \frac{Ct^2}{2} \right) = \frac{dx}{dt}$$

$$\int_{x_0}^x dx = \int_{t_0}^t \frac{1}{m} \left(F_0 t + \frac{Ct^2}{2} \right) dt$$

$$x - x_0 = \frac{1}{m} \left(\frac{F_0}{2}(t^2 - t_0^2) + \frac{C}{6}(t^3 - t_0^3) \right)$$

$$x = \frac{1}{m} \left[\frac{F_0 t^2}{2} + \frac{Ct^3}{6} \right]$$

b). $F_x = m \frac{dv}{dt} = F_0 \sin Ct$

$$\int_0^v dv = \frac{F_0}{m} \int_0^t \sin Ct dt$$

$$v(t) = -\frac{F_0}{mC} \cos Ct$$

$$x(t) = \int_0^x dx$$

$$x(t) = -\frac{F_0}{mC} \int_0^t \cos Ct dt$$

$$x(t) = -\frac{F_0}{mC^2} \sin Ct$$

$$c). F_x = F_0 e^{ct}$$

$$v(t) = \frac{F_0}{m} \int_0^t e^{ct} dt$$

$$v(t) = \frac{F_0}{cm} e^{ct}$$

$$x(t) = \frac{F_0}{c^2 m} e^{ct}$$

2.2 a) $F_x = F_0 + Cx$

$$F(x) = mv \frac{dv}{dx} = F_0 + Cx$$

$$\int_{v_0}^v mv dv = \int_0^x (F_0 + Cx) dx$$

$$\frac{mv^2}{2} = F_0 x + \frac{C}{2} x^2$$

$$v = \sqrt{\frac{2}{m} \left(F_0 x + \frac{C}{2} x^2 \right)}$$

b). $F(x) = mv \frac{dv}{dx} = F_0 e^{-cx}$

$$\frac{1}{2} mv^2 = -\frac{F_0}{C} e^{-cx}$$

$$v = \frac{2}{m} \left[-\frac{F_0}{C} e^{-cx} \right]^{1/2}$$

c). $F(x) = mv \frac{dv}{dx} = F_0 \cos cx$

$$\frac{1}{2} mv^2 = \int_0^x F_0 \cos cx dx$$

$$= \frac{F_0}{C} \sin cx$$

$$v = \frac{2}{m} \left[\frac{F_0}{C} \sin cx \right]^{1/2}$$

2.3 a). $F_x = -\frac{du}{dx} = F_0 + Cx$

$$u(x) = - \left[F_0 x + \frac{C}{2} x^2 \right] + C$$

b). $F_x = -\frac{du}{dx} = F_0 e^{-cx}$

$$\int_{u(x_0)}^{u(x)} du = -F_0 \int_{x_0}^x e^{-cx} dx$$



$$u(x) - u(x_0) = \frac{F_0}{C} \left[e^{-cx} - e^{-cx_0} \right]$$

c). $F_x = -\frac{du}{dx} = F_0 \cos cx$

$$\int_{u(x_0)}^{u(x)} du = -F_0 \int_{x_0}^x \cos(cx) dx$$

$$u(x) - u(x_0) = -\frac{F_0}{C} \left[\sin cx - \sin cx_0 \right]$$

2.4 $V(x) = - \int f(x) dx$

a). $V(x) = - \int (-kx) dx$

$$V(x) = \frac{kx^2}{2}$$

Jadi, energi potensial $V(x) = \frac{kx^2}{2}$

b). dan c).

$$E = T_0 = \frac{1}{2} kA^2$$

$$E = T(x) + V(x)$$

$$T(x) = E - V(x)$$

$$T(x) = \frac{1}{2} kA^2 - \frac{1}{2} kx^2$$

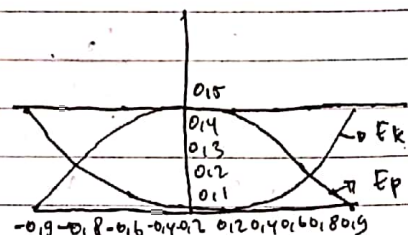
$$T(x) = \frac{k}{2} (A^2 - x^2)$$

d). $E = V(x)$

$$\frac{1}{2} kA^2 = \frac{1}{2} kx^2$$

$$x = \pm A$$

e).



Hasilnya = a) $V(x) = \frac{kx^2}{2}$

b) $T(x) = \frac{k}{2} (A^2 - x^2)$

c) $E = T_0 = \frac{1}{2} kA^2$

2.8 $v = \frac{b}{x^3}$

$$F = m \frac{dv}{dt} = m \frac{dv}{dx} \frac{dx}{dt}$$

$$= m v \frac{dv}{dx}$$

$$\frac{dv}{dx} = -\frac{3b}{x^4}$$

$$F = m \left(\frac{b}{x^3} \right) \left(-\frac{3b}{x^4} \right) = -\frac{3mb^2}{x^7}$$

$$\int_{v_0}^v \frac{v dv}{-g - kv^2} = \int_0^x dx$$

$$y = -g - kv^2$$

$$dy = -2kv dv$$

$$v dv = -\frac{1}{2k} dy$$

$$\int_{v_0}^v \frac{-1 dy}{2ky} = \int_0^x dx$$

$$-\frac{1}{2k} \ln(y) \Big|_{v_0}^v = x \Big|_0^x$$

$$-\frac{1}{2k} \ln(-g - kv^2) \Big|_{v_0}^v = x$$

$$-\frac{1}{2k} \ln \left(\frac{-g - kv^2}{-g - kv_0^2} \right) = x$$

$$\frac{-g - kv^2}{-g - kv_0^2} = e^{-2kx}$$

$$-g - kv^2 = (-g - kv_0^2) e^{-2kx}$$

$$kv^2 = (g + kv_0^2) e^{-2kx} - \frac{g}{k}$$

$$A = \frac{g}{k} + v_0^2$$

$$v^2 = A e^{-2kx} - \frac{g}{k}$$

2.11 $F(v) = -cv^{3/2}$

$$= m \frac{dv}{dt} = m \frac{dv}{dx} \frac{dx}{dt}$$

$$= m v \frac{dv}{dx}$$

$$-cv^{3/2} = m v \frac{dv}{dx}$$

$$\int_0^x dx = \int_{v_0}^v \frac{-mv}{cv^{3/2}} dv$$

$$x = -\frac{m}{c} \int_{v_0}^v \frac{dv}{\sqrt{v}} = -\frac{m}{c} \int_{v_0}^v v^{-1/2} dv$$

$$= -\frac{m}{c} \frac{v^{1/2}}{1/2} \Big|_{v_0}^v$$

$$= \frac{2m}{c} (\sqrt{v_0} - \sqrt{v})$$

Partikel berhenti, maka $v=0$, jadi,

$$x_{\max} = \frac{2m\sqrt{v_0}}{c}$$

$$m a_x = \sum F_x$$

$$m a_x = -mg + c_2 v^2$$

$$a_x = \frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt} = \frac{dv}{dx} \cdot v$$

$$m v \frac{dv}{dx} = -mg + c_2 v^2$$

$$c \frac{dv}{dx} = -g + \frac{c_2}{m} v^2$$

$$k = \frac{c_2}{m}$$

$$v \frac{dv}{dx} = -g + kv^2$$

$$\int_0^v \frac{v dv}{-g + kv^2} = \int_0^x dx$$

$$y = -g + kv^2$$



$$dy = 2k v dv$$

$$v dv = \frac{1}{2k} dy$$

$$\int_0^v \frac{1}{2k} \frac{dy}{y} = \int_{x_0}^x dx$$

$$\frac{1}{2k} \ln(y) \Big|_0^v = x \Big|_{x_0}^x$$

$$\frac{1}{2k} \ln(-g + kv^2) \Big|_0^v = x - x_0$$

$$\frac{1}{2k} \ln \left(\frac{-g + kv^2}{-g} \right) = x - x_0$$

$$\frac{-g + kv^2}{-g} = e^{2k(x-x_0)}$$

$$-g + kv^2 = -g e^{2k(x-x_0)}$$

$$kv^2 = -g e^{2k(x-x_0)} + g$$

$$v^2 = \frac{-g}{k} e^{2kx_0} e^{2kx} + \frac{g}{k}$$

$$B = \frac{g}{k} e^{2kx_0}$$

$$v^2 = \frac{g}{k} - B e^{2kx}$$

2.13 $v^2 = \left(\frac{g}{k} + v_0^2 \right) e^{-2kx} - \frac{g}{k} \rightarrow \text{keatas}$

$$v^2 = \frac{-g}{k} e^{-2kx_0} e^{2kx} + \frac{g}{k} \rightarrow \text{kebah}$$

$v=0$ pada gerak keatas

$$0 = \left(\frac{g}{k} + v_0^2 \right) e^{-2kx} - \frac{g}{k}$$

$$e^{-2kx} = \frac{\frac{g}{k}}{\frac{g}{k} + v_0^2}$$

h adalah posisi awal untuk gerakan ke bawah dan (*) $a=0$,

$$v^2 = \frac{-g}{k} e^{-2kx} e^{2kx_0} + \frac{g}{k} \rightarrow \text{sub } x=0$$

$$v^2 = \frac{-g}{k} \cdot \frac{g}{k} \cdot \frac{1 + \frac{g}{k}}{\frac{g}{k} + v_0^2}$$

$$v_t^2 = \frac{g}{k}$$

$$v^2 = -v_t^2 \cdot \frac{v_t^2}{v_t^2 + v_0^2} + v_t^2$$

$$v^2 = v_t^2 \left(\frac{1 - v_t^2}{v_t^2 + v_0^2} \right)$$

$$v^2 = v_t^2 \cdot \left(\frac{v_t^2 + v_0^2 - v_t^2}{v_t^2 + v_0^2} \right)$$

$$v^2 = \frac{v_t^2 v_0^2}{v_t^2 + v_0^2}$$

$$v = \frac{v_t v_0}{\sqrt{v_t^2 + v_0^2}}$$

2.14 $\text{Max} = \sum F_x$

$$\text{Max} = -kx^{-2}$$

$$a_x = \frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt} = \frac{dv}{dx} \cdot v$$

$$mv \frac{dv}{dx} = -kx^{-2}$$

$$v \frac{dv}{dx} = -\frac{k}{m} x^{-2}$$

$$\int_0^v v dv = -\frac{k}{m} \int_b^x x^{-2} dx$$

$$\frac{v^2}{2} \Big|_0^v = -\frac{k}{m} (-1) x^{-1} \Big|_0^x$$

$$\frac{v^2}{2} = \frac{k}{m} (x^{-1} - b^{-1})$$

$$v^2 = \frac{2k}{m} \left(\frac{b-x}{bx} \right)$$

$$v = \frac{dx}{dt}$$

$$\sqrt{\frac{2k}{m} \left(\frac{b-x}{m} \right)} = \frac{dx}{dt}$$

$$\int_b^0 \frac{dx}{\sqrt{\frac{2k}{m} \left(\frac{b-x}{m} \right)}} = \int_0^t dt$$

$$\int_b^0 \frac{dx}{\sqrt{\frac{2k}{m} \left(\frac{b-x}{m} \right)}} = \sqrt{\frac{mb}{2k}} \int_b^0 \frac{dx}{\sqrt{b-x}}$$

$$u = \sqrt{b-x}$$

$$du = \frac{1}{2\sqrt{b-x}} (-1) \cdot \sqrt{x} dx = -\frac{\sqrt{x}}{2\sqrt{b-x}}$$

$$dx = \frac{du}{-\frac{\sqrt{x}}{2\sqrt{b-x}}} = -\frac{2\sqrt{b-x}}{\sqrt{x}} du$$

$$= -\frac{2}{\sqrt{x}} du = -\frac{2}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} du = -\frac{1}{x} du$$

$$= -\frac{du}{1+u^2}$$

$$= -\frac{2x u du}{u^2 + 1}$$

$$\sqrt{b-x} = u \sqrt{x}$$

$$b-x = u^2 x$$

$$b = x^2 (u^2 + 1)$$

$$x = \frac{b}{u^2 + 1}$$

$$dx = -\frac{2b u du}{(u^2 + 1)^2}$$

$$\sqrt{\frac{mb}{2k}} \int_b^0 \frac{\sqrt{x}}{\sqrt{b-x}} dx = \sqrt{\frac{mb}{2k}} \int_b^0 \frac{1}{u} \cdot \frac{(-1) 2b u du}{(u^2 + 1)^2}$$

$$= \sqrt{\frac{mb}{2k}} \int_b^0 \frac{2b du}{(u^2 + 1)^2}$$

$$\int \frac{du}{(u^2 + 1)^2} = \frac{2n-3}{2b(n-1)} \int \frac{du}{(u^2 + 1)^{n-1}} + \frac{u}{2b(n-1)(u^2 + 1)^{n-1}}$$

$$= \frac{u}{2(u^2 + 1)} + \frac{1}{2} \int \frac{du}{u^2 + 1}$$

$$\int_b^0 \frac{du}{(u^2 + 1)^2} = \frac{u}{2(u^2 + 1)} \Big|_b^0 + \frac{1}{2} \int_b^0 \frac{du}{u^2 + 1}$$

$$\int_b^0 \frac{du}{(u^2 + 1)^2} = \frac{u}{2(u^2 + 1)} \Big|_b^0 + \frac{1}{2} \arctan u \Big|_b^0$$

$$\sqrt{\frac{mb}{2k}} \int_b^0 \frac{2b du}{(u^2 + 1)^2} = -2b \sqrt{\frac{mb}{2k}} \left(\frac{u}{2(u^2 + 1)} \Big|_b^0 + \frac{1}{2} \arctan u \Big|_b^0 \right)$$

$$= -2b \sqrt{\frac{mb}{2k}} \left(\frac{u}{2(u^2 + 1)} \Big|_0^{-\infty} + \frac{1}{2} \arctan u \Big|_0^{-\infty} \right)$$

$$= -2b \sqrt{\frac{mb}{2k}} \cdot \left(0 - 0 + \frac{1}{2} \cdot \left(-\frac{\pi}{2} - 0 \right) \right)$$

$$= \pi \sqrt{\frac{mb^3}{8k}}$$

$$\pi \sqrt{\frac{mb^3}{8k}} = \int_0^t dt$$

$$= t - 0$$

$$t = \pi \sqrt{\frac{mb^3}{8k}}$$

2.15

$$m_a = \sum F$$

$$m_a = m_g - C_1 v - C_2 v^2$$

$$m_a = 0$$

$$m_g - C_1 v - C_2 v^2 = 0$$

$$C_2 v^2 - C_1 v - m_g = 0$$

$$v_{1,2} = \frac{-C_1 \pm \sqrt{C_1^2 + 4C_2 m_g}}{2C_2}$$

pilih (+)

$$v_1 = \frac{-C_1 + \sqrt{C_1^2 + 4C_2 m_g}}{2C_2}$$

$$v_1 = v_t = \frac{-C_1}{2C_2} + \frac{\sqrt{\left(\frac{C_1}{2C_2}\right)^2 + \frac{m_g}{C_2}}}{1}$$

$$v_t = \frac{-C_1}{2C_2} + \sqrt{\left(\frac{C_1}{2C_2}\right)^2 + \frac{m_g}{C_2}}$$