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2.1

a) $F_x = m\ddot{x} = F_0 + Ct$

$$m\frac{dv}{dt} = F_0 + Ct$$

$$m.dv = (F_0 + Ct)dt$$

$$m\int_{V_0}^{V} dv = \int_0^t (F_0 + Ct)dt$$

$$m(V - V_0) = F_0(t - t_0) + \frac{C}{2}(t^2 - t_0^2)$$

karena, $V_0 = 0$ dan $t_0 = 0$ maka:

$$mV = F_0 t + \frac{Ct^2}{2}$$

$$V(t) = \frac{1}{m}\left(F_0 t + \frac{Ct^2}{2}\right) = \frac{dx}{dt}$$

$$\int_{x_0}^{x} dx = \int_{t_0}^{t} \frac{1}{m}\left(F_0 t + \frac{Ct^2}{2}\right)dt$$

$$x - x_0 = \frac{1}{m}\left(\frac{F_0}{2}(t^2 - t_0^2) + \frac{C}{6}(t^3 - t_0^3)\right)$$

$$x = \frac{1}{m}\left[\frac{F_0 t^2}{2} + \frac{Ct^3}{6}\right]$$

b) $F_x = m\frac{dv}{dt} = F_0 \sin ct$

$$\int_0^V dv = \frac{F_0}{m}\int_0^t \sin ct \, dt$$

$$V(t) = -\frac{F_0}{mc}\cos ct$$

$$x(t) = \int_0^x dt$$

$$x(t) = -\frac{F_0}{mc}\int_0^t \cos ct \, dt$$

$$x(t) = -\frac{F_0}{mc^2}\sin ct$$

c) $F_x = F_0 e^{ct}$

$$V(t) = \frac{F_0}{m}\int_0^t e^{ct} dt$$

$$V(t) = \frac{F_0}{cm} e^{ct}$$

$$x(t) = \frac{F_0}{c^2 m} e^{ct}$$

2.2

a) $F_x = F_0 + cx$

$$F(x) = mv\frac{du}{dx} = F_0 + cx$$

$$\int_{V_0}^{V} mv \, du = \int_0^x (F_0 + cx)dx$$

$$\frac{mv^2}{2} = F_0 x + \frac{c}{2}x^2$$

$$V = \sqrt{\frac{2}{m}\left(F_0 x + \frac{c}{2}x^2\right)}$$

b) $F(x) = mv\frac{du}{dx} = F_0 e^{-cx}$

$$\frac{1}{2}mv^2 = -\frac{F_0}{c}e^{-cx}$$

$$V = \frac{2}{m}\left[-\frac{F_0}{c}e^{-cx}\right]^{1/2}$$

c) $F(x) = mv\frac{du}{dx} = F_0 \cos cx$

$$\frac{1}{2}mv^2 = \int_0^x F_0 \cos cx$$

$$= \frac{F_0}{c}\sin cx$$

$$V = \frac{2}{m}\left[\frac{F_0}{c}\sin cx\right]^{1/2}$$

(2.3)

a) $F_x = -\frac{du}{dx} = F_0 + cx$

$$U(x) = -\left[F_0 x + \frac{c}{2}x^2\right] + C$$

b) $F_x = -\frac{du}{dx} = F_0 e^{-cx}$

$$\int_{U(x_0)}^{U(x)} du = -F_0 \int_{x_0}^{x} e^{-cx}\,dx$$

$$U(x) - U(x_0) = \frac{F_0}{c}\left[e^{-cx} - e^{-cx_0}\right]$$

c) $F_x = -\frac{du}{dx} = F_0 \cos cx$

$$\int_{U(x_0)}^{U(x)} du = -F_0 \int_{x_0}^{x} \cos(cx)\,dx$$

$$U(x) - U(x_0) = \frac{-F_0}{c}\left[\sin cx - \sin(cx_0)\right]$$

(2.4) $V(x) = -\int F(x)\,dx$

$$V(x) = -\int(-kx)\,dx$$

$$V(x) = \frac{kx^2}{2}$$

Jadi, energi potensial: $V(x) = \frac{kx^2}{2}$

b) dan c)

$E = T_0 = \frac{1}{2}kA^2$

$E = T(x) + V(x)$

$T(x) = E - V(x)$

$T(x) = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$

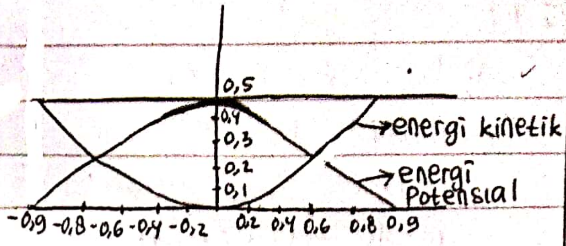
$T(x) = \frac{k}{2}(A^2 - x^2)$

d) $E = V(x)$

$\frac{1}{2}kA^2 = \frac{1}{2}kx^2$

$x = \pm A$

e)



Hasilnya: a) $V(x) = \frac{kx^2}{2}$

b) $T(x) = \frac{k}{2}(A^2 - x^2)$

c) $E = T_0 = \frac{1}{2}kA^2$

(2.8) $v = \frac{b}{x^3}$

$$F = m\frac{dv}{dt} = m\frac{dv}{dx}\frac{dx}{dt}$$

$$= mv\frac{dv}{dx}$$

$$\frac{dv}{dx} = \frac{-3b}{x^4}$$

$$F = m\left(\frac{b}{x^3}\right)\left(\frac{-3b}{x^4}\right) = \frac{-3mb^2}{x^7}$$

(2.11)

$$F(v) = -cv^{3/2}$$

$$= m\frac{dv}{dt} = m\frac{dv}{dx}\frac{dx}{dt}$$

$$= mv\frac{dv}{dx}$$

$$-cv^{3/2} = mv\frac{dv}{dx}$$

$$\int_0^x dx = \int_{v_0}^{v} -\frac{mv}{cv^{3/2}}\,dv$$

$$x = -\frac{m}{c}\int_{v_0}^{v}\frac{dv}{\sqrt{v}} = -\frac{m}{c}\int_{v_0}^{v} v^{-1/2}\,dv$$

$$= -\frac{m}{c}\left.\frac{v^{1/2}}{1/2}\right|_{v_0}^{v}$$

$$= \frac{2m}{c}\left(\sqrt{v_0} - \sqrt{v}\right)$$

Partikel berhenti, maka $v = 0$,

Jadi, $x_{max} = \frac{2m\sqrt{v_0}}{c}$

2.12

$$m.a_x = \sum F_x$$
$$m.a_x = -mg - C_2 v^2$$
$$a_x = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dx}.v$$
$$mv\frac{dv}{dx} = -mg - C_2 v^2$$
$$v.\frac{dv}{dx} = -g - \frac{C_2}{m}v^2$$
$$k = \frac{C_2}{m}$$
$$\frac{v\,dv}{dx} = -g - kv^2$$
$$\int_{v_0}^{v}\frac{v\,dv}{-g-kv^2} = \int_0^x dx$$
$$y = -g - kv^2$$
$$dy = -2kv\,dv$$
$$v\,dv = -\frac{1}{2k}dy$$
$$\int_{v_0}^{v}\frac{-1\,dy}{2ky} = \int_0^x dx$$
$$-\frac{1}{2k}\ln(y)\Big|_{v_0}^{v} = x\Big|_0^x$$
$$-\frac{1}{2k}\ln(-g-kv^2)\Big|_{v_0}^{v} = x$$
$$-\frac{1}{2k}\ln\left(\frac{-g-kv^2}{-g-kv_0^2}\right) = x$$
$$\frac{-g-kv^2}{-g-kv_0^2} = e^{-2kx}$$
$$-g-kv^2 = (-g-kv_0^2)e^{-2kx}$$
$$kv^2 = (g+kv_0^2)e^{-2kx} - g$$
$$v^2 = \left(\frac{g}{k}+v_0^2\right)e^{-2kx} - \frac{g}{k}$$
$$A = \frac{g}{k} + v_0^2$$
$$\boxed{v^2 = Ae^{-2kx} - \frac{g}{k}}$$

$$m.a_x = \sum F_x$$
$$m.a_x = -mg + C_2 v^2$$
$$a_x = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dx}.v$$
$$mv\frac{dv}{dx} = -mg + C_2 v^2$$
$$c\frac{dv}{dx} = -g + \frac{C_2}{m}v^2$$
$$k = \frac{C_2}{m}$$
$$v\frac{dv}{dx} = -g + kv^2$$
$$\int_0^v \frac{v\,dv}{-g+kv^2} = \int_0^x dx$$
$$y = -g + kv^2$$
$$dy = 2kv\,dv$$
$$v\,dv = \frac{1}{2k}dy$$
$$\int_0^v \frac{1}{2k}\frac{dy}{y} = \int_{x_0}^{x} dx$$
$$\frac{1}{2k}\ln(y)\Big|_0^v = x\Big|_{x_0}^{x}$$
$$\frac{1}{2k}\ln(-g+kv^2)\Big|_0^v = x - x_0$$
$$\frac{1}{2k}\ln\left(\frac{-g+kv^2}{-g}\right) = x - x_0$$
$$\frac{-g+kv^2}{-g} = e^{2k(x-x_0)}$$
$$-g+kv^2 = -ge^{2k(x-x_0)}$$
$$kv^2 = -ge^{2k(x-x_0)} + g$$
$$v^2 = -\frac{g}{k}e^{-2kx_0}e^{2kx} + \frac{g}{k}$$
$$B = \frac{g}{k}e^{-2kx_0}$$
$$\boxed{v^2 = \frac{g}{k} - Be^{2kx}}$$

(2.13) $V^2 = \left(\frac{g}{k} + V_0^2\right)e^{-2kx} - \frac{g}{k} \rightarrow$ ke atas

$V^2 = -\frac{g}{k}e^{-2kx_0}e^{2kx} + \frac{g}{k} \rightarrow$ ke bawah

$V=0$ pada gerak ke atas

$$0 = \left(\frac{g}{k} + V_0^2\right)e^{-2kx} - \frac{g}{k}$$

$$e^{-2kh} = \frac{\frac{g}{k}}{\frac{g}{k} + V_0^2}$$

h adalah posisi awal untuk gerakan ke bawah dan posisi akhir $a=0$, maka

$$V^2 = -\frac{g}{k}e^{-2kh}e^{2k0} + \frac{g}{k} \rightarrow \text{Substitusi } x=0$$

$$V^2 = -\frac{g}{k} \cdot \frac{\frac{g}{k}}{\frac{g}{k} + V_0^2} \cdot 1 + \frac{g}{k}$$

$$V_t^2 = \frac{g}{k}$$

$$V^2 = -V_t^2 \cdot \frac{V_t^2}{V_t^2 + V_0^2} + V_t^2$$

$$V^2 = V_t^2 \cdot \left(1 - \frac{V_t^2}{V_t^2 + V_0^2}\right)$$

$$V^2 = V_t^2 \cdot \left(\frac{V_t^2 + V_0^2 - V_t^2}{V_t^2 + V_0^2}\right)$$

$$V^2 = \frac{V_t^2 V_0^2}{V_t + V_0^2}$$

$$\boxed{V = \frac{V_t V_0}{\sqrt{V_t^2 + V_0}}}$$

(2.14) $m.a_x = \sum F_x$

$$m.a_x = -kx^{-2}$$

$$a_x = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dx}.v$$

$$mv\frac{dv}{dx} = -kx^{-2}$$

$$v\frac{dv}{dx} = -\frac{k}{m}x^{-2}$$

$$\int_0^v v\,dv = -\frac{k}{m}\int_b^x x^{-2}\,dx$$

$$\frac{v}{2}\Big|_0^v = -\frac{k}{m}(-1)x^{-1}\Big|_b^x$$

$$\frac{V^2}{2} = \frac{k}{m}(x^{-1} - b^{-1})$$

$$V^2 = \frac{2k}{m}\left(\frac{b-x}{bx}\right)$$

$$V = \frac{dx}{dt}$$

$$\sqrt{\frac{2k}{m}\left(\frac{b-x}{m}\right)} = \frac{dx}{dt}$$

$$\int_b^0 \frac{dx}{\sqrt{\frac{2k}{m}\cdot\left(\frac{b-x}{bx}\right)}} = \int_0^t dt$$

$$\int_b^0 \frac{dx}{\sqrt{\frac{2k}{m}\left(\frac{b-x}{bx}\right)}} = \sqrt{\frac{mb}{2k}}\int_b^0 \sqrt{\frac{x}{b-x}}\,dx$$

$$U = \frac{\sqrt{b-x}}{\sqrt{x}}$$

$$du = \frac{\frac{1}{2\sqrt{b-x}}(-1).\sqrt{x}\,dx - \frac{1}{2\sqrt{x}}}{x}$$

$$dx = \frac{du}{-\frac{1}{2\sqrt{x}.U\sqrt{x}} - \frac{U\sqrt{x}}{2x^{3/2}}}$$

$$= \frac{du}{-\frac{1}{2Ux} - \frac{U\sqrt{x}}{2x^{3/2}}}$$

$$= \frac{du}{-\frac{1}{2ux} - \frac{U}{2x}}$$

$$= -\frac{du}{\frac{1+U^2}{2xU}}$$

$$= -\frac{2xUdU}{U^2+1}$$

$$\sqrt{b-x} = U\sqrt{x}$$

$$b-x = U^2x$$

$$b = x(U^2+1)$$

$$x = \frac{b}{U^2+1}$$

$$dx = -\frac{2bUdU}{(U^2+1)^2}$$

$$\sqrt{\frac{mb}{2k}}\int_b^0\sqrt{\frac{x}{b-x}}\,dx = \sqrt{\frac{mb}{2k}}\int_b^0 \frac{1}{u}(-1)\frac{2bUdU}{(U^2+1)^2}$$

$$= \sqrt{\frac{mb}{2k}}\int_b^0 \frac{2bUdU}{(U^2+1)^2}$$

$$\int \frac{du}{(au+b)^n} = \frac{2n-3}{2b(n-1)} \int \frac{du}{(au^2+b)^{n-1}} + \frac{u}{2b(n-1)(au^2+b)^{n-1}}$$

$$= \frac{u}{2(u^2+1)} + \frac{1}{2}\int \frac{du}{u^2+1}$$

$$\int_b^0 \frac{du}{(u^2+1)^2} = \frac{u}{2(u^2+1)}\Big|_b^0 + \frac{1}{2}\int_b^0 \frac{du}{u^2+1}$$

$$\int_b^0 \frac{du}{(u^2+1)^2} = \frac{u}{2(u^2+1)}\Big|_b^0 + \frac{1}{2} \arctan u \Big|_b^0$$

$$-\sqrt{\frac{mb}{2k}} \int_b^0 \frac{2b\,du}{(u^2+1)^2} = -2b\sqrt{\frac{mb}{2k}} \cdot \left(\frac{u}{2(u^2+1)}\Big|_b^0 + \frac{1}{2}\arctan u \Big|_b^0\right)$$

$$= -2b\sqrt{\frac{mb}{2k}} \cdot \left(\frac{u}{2(u^2+1)}\Big|_0^{-\infty} + \frac{1}{2}\arctan u \Big|_0^{-\infty}\right)$$

$$= -2b\sqrt{\frac{mb}{2k}} \cdot \left(0 - 0 + \frac{1}{2} \cdot \left(-\frac{\pi}{2} - 0\right)\right)$$

$$= \pi\sqrt{\frac{mb^3}{8k}}$$

$$\pi\sqrt{\frac{mb^3}{8k}} = \int_0^t dt$$

$$= t - 0$$

$$\boxed{t = \pi\sqrt{\frac{mb^3}{8k}}}$$

2.15 $ma = \sum F$

$$ma = mg - C_1 V - C_2 V^2$$

$$ma = 0$$

$$mg - C_1 V - C_2 V^2 = 0$$

$$C_2 V^2 + C_1 V - mg = 0$$

$$V_{1,2} = \frac{-C_1 \pm \sqrt{C_1^2 + 4C_2 mg}}{2C_2}$$

pilih (+)

$$V_1 = \frac{-C_1 + \sqrt{C_1^2 + 4C_2 mg}}{2C_2}$$

$$V_1 = V_t = -\frac{C_1}{2C_2} + \sqrt{\left(\frac{C_1}{2C_2}\right)^2 + \frac{mg}{C_2}}$$

$$\boxed{V_t = -\frac{C_1}{2C_2} + \sqrt{\left(\frac{C_1}{2C_2}\right)^2 + \frac{mg}{C_2}}}$$