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2.1 a) $F_x = m\ddot{x} = F_0 + Ct$

$$m\frac{dv}{dt} = F_0 + Ct$$

$$m\,dv = (F_0 + Ct)\,dt$$

$$m\int_{v_0}^{v} dv = \int_{t_0}^{t} (F_0 + Ct)\,dt$$

$$m(v - v_0) = F_0(t - t_0) + \frac{C}{2}(t^2 - t_0^2)$$

$$v_0 = 0 \quad , \quad t_0 = 0$$

$$mv = F_0 t + \frac{Ct^2}{2}$$

$$v(t) = \frac{1}{m}\left(F_0 t + \frac{Ct^2}{2}\right) = \frac{dx}{dt}$$

$$\int_{x_0}^{x} dx = \int_{t_0}^{t} \frac{1}{m}\left(F_0 t + \frac{Ct^2}{2}\right) dt$$

$$x - x_0 = \frac{1}{m}\left(\frac{F_0}{2}(t^2 - t_0^2) + \frac{C}{6}(t^3 - t_0^3)\right)$$

$$x = \frac{1}{m}\left[\frac{F_0 t^2}{2} + \frac{Ct^3}{6}\right]$$

b) $F_x = m\frac{dv}{dt} = F_0 \sin ct$

$$\int_0^v dv = \frac{F_0}{m}\int_0^t \sin ct \; dt$$

$$v(t) = -\frac{F_0}{mc}\cos ct$$

$$x(t) = \int_0^x dx = -\frac{F_0}{mc}\int_0^t \cos ct \; dt$$

$$= -\frac{F_0}{mc^2}\sin ct$$

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c) $F_x = m\frac{dv}{dt} = F_0 e^{ct}$

$$v(t) = \frac{F_0}{m}\int_0^t e^{ct}\,dt = \frac{F_0}{cm}e^{ct}$$

$$x(t) = \frac{F_0}{c^2 m}e^{ct}$$

(2.2) a) $F_x = -\frac{du}{dx} = F_0 + cx$

$$F(x) = mv\frac{dv}{dt} = F_0 + cx$$

$$\int_{v_0}^{v} mv\,dv = \int_0^x (F_0 + cx)\,dx$$

$$\frac{m}{2}v^2 = F_0 x + \frac{c}{2}x^2$$

$$v = \sqrt{\frac{2}{m}\left(F_0 x + \frac{c}{2}x^2\right)}$$

b) $F(x) = mv\frac{dv}{dx} = F_0 e^{-cx}$

$$\frac{1}{2}mv^2 = -\frac{F_0}{c}e^{-cx}$$

$$v = \frac{2}{m}\left[-\frac{F_0}{c}e^{-cx}\right]^{1/2}$$

c) $F(x) = mv\frac{dv}{dx} = F_0 \cos cx$

$$\frac{1}{2}mv^2 = \int_0^x F_0 \cos cx = \frac{F_0}{c}\sin cx$$

$$v = \frac{2}{m}\left[\frac{F_0}{c}\sin cx\right]^{1/2}$$

(2.3) a) $F_x = -\frac{du}{dx} = F_0 + cx$

$$V(x) = -\left[F_0 x + \frac{c}{2}x^2\right] + C..$$

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b) $F_x = -\frac{du}{dx} = F_0 e^{-cx}$

$$\int_{V(x_0)}^{V(x)} du = -F_0 \int_{x_0}^{x} e^{-cx}\, dx$$

$$V(x) - V(x_0) = \frac{F_0}{c}\left[e^{-cx} - e^{-cx_0} \right]$$

c) $F_x = -\frac{du}{dx} = F_0 \cos cx$

$$\int_{V(x_0)}^{V(x)} du = -F_0 \int_{x_0}^{x} \cos (cx)\, dx$$

$$V(x) - V(x_0) = -\frac{F_0}{c}\left[\sin cx - \sin cx_0 \right]$$

2.4 (a) $F(x) = -\frac{dV(x)}{dx} = -kx$

$$V(x) = \int_0^x kx\, dx = \frac{1}{2}kx^2$$

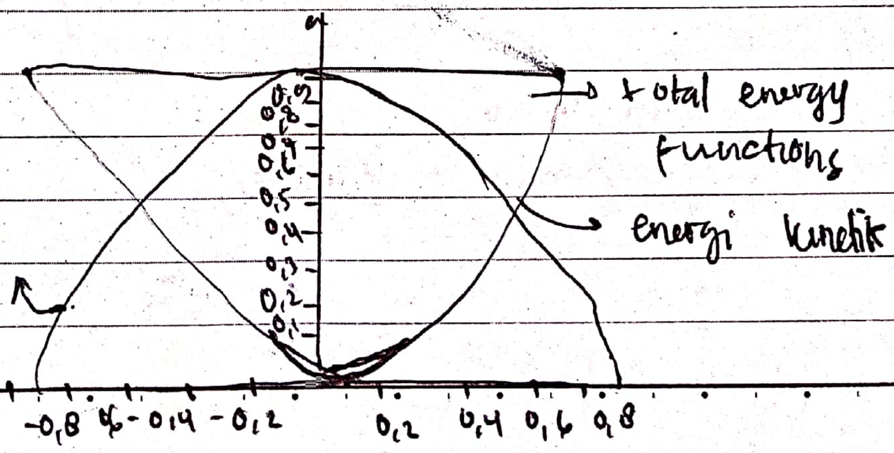
(b) $T_0 = T(x) + V(x)$

$$T(x) = T_0 - V(x) = \frac{1}{2}k(A - x^2)$$

(c) $E = T_0 = \frac{1}{2}kA^2$

(d) Turning points @ $T(x_1) \rightarrow 0 \quad \therefore x = \pm A$

(e)



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2.8 $v = bx^{-3}$

$$F = m\frac{dv}{dt} = m\cdot\frac{dv}{dx}\cdot\frac{dx}{dt}$$

$$= mv\frac{dv}{dx}$$

$$\frac{dv}{dx} = \frac{-3b}{x^4}$$

$$F = m\left(\frac{b}{x^3}\right)\left(-\frac{3b}{x^4}\right) = -\frac{3mb^2}{x^7}$$

2.11

$$f(v) = -c.v^{3/2}$$

$$= m.\frac{dv}{dt} = m.\frac{dv}{dx}.\frac{dx}{dt} = mv\frac{dv}{dx}$$

The distance travelled is -

$$-cv^{3/2} = mv.\frac{dv}{dx}$$

$$\int_0^x dx = \int_{v_0}^{v} -\frac{mv}{cv^{3/2}}\,dv$$

$$x = -\frac{m}{c}\int_{v_0}^{v}\frac{dv}{\sqrt{v}} = -\frac{m}{c}\int_{v_0}^{v} v^{-1/2}\,dv$$

$$= -\frac{m}{c}\left.\frac{v^{1/2}}{1/2}\right|_{v_0}^{v}$$

$$= +\frac{2m}{c}\left(\sqrt{v_0} - \sqrt{v}\right)$$

Saat $v = 0$,

$$x_{max} = \frac{2m\sqrt{v_0}}{c}$$

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(2.13) $V^2 = \left(\frac{g}{k} + V_0^2\right) e^{-2kx} - \frac{g}{k}$

$V^2 = -\frac{g}{k} e^{-2kx_0} e^{2kx} + \frac{g}{k}$

•) $v = 0$ (upward motion)

$0 = \left(\frac{g}{k} + V_0^2\right) e^{-2kh} - g/k$

$e^{-2kh} = \dfrac{g/k}{\frac{g}{k} + V_0^2}$

•) $x = 0$ (downward motion)

$V^2 = -\frac{g}{k} e^{-2kh} e^{2k.0} + \frac{g}{k}$

$= -g/k \cdot \dfrac{g/k}{\frac{g}{k} + V_0^2} \cdot 1 + \frac{g}{k}$

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$$V_t^2 = g/k$$

$$V^2 = -V_t^2 \cdot \frac{V_t^2}{V_t^2 + V_0^2} + V_t^2$$

$$= V_t^2 \cdot \left(1 - \frac{V_t^2}{V_t^2 + V_0^2}\right)$$

$$= V_t^2 \cdot \left(\frac{V_t^2 + V_0^2 - \cancel{V_t^2}}{V_t^2 + V_0^2}\right)$$

$$= \frac{V_t^2 V_0^2}{V_t^2 + V_0^2}$$

$$v = \frac{V_t V_0}{\sqrt{V_t^2 + V_0^2}}$$

(2) Hk. 2 Newton

$$\cdot) \; m a_x = \Sigma F_x$$

$$m a_x = -mg - C_2 V^2$$

$$a_x = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt}$$

$$m v \cdot \frac{dv}{dx} = -mg - C_2 V^2$$

$$v \cdot \frac{dv}{dx} = -g - \frac{C_2}{m} V^2$$

$$k = C_2/m$$

$$v \cdot \frac{dv}{dx} = -g - kv^2$$

$$\int_{V_0}^{V} \frac{v\,dv}{-g - kv^2} = \int_0^x dx$$

$$y = -g - kv^2$$

$$dy = -2kv\,dv$$

$$v\,dv = -\tfrac{1}{2k}\,dy$$

$$\int_{V_0}^{V} -\frac{1}{2k} \frac{dy}{y} = \int_0^x dx$$

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$$-\frac{1}{2k}\ln(y)\Big|_{v_0}^{v} = x\Big|_0^x$$

$$-\frac{1}{2k}\ln(-g-kv^2)\Big|_{v_0}^{v} = x$$

$$-\frac{1}{2k}\ln\left(\frac{-g-kv^2}{-g-kv_0^2}\right) = x$$

$$\frac{-g-kv^2}{-g-kv_0^2} = e^{-2kx}$$

$$-g-kv^2 = (-g-kv_0^2)e^{-2kx}$$

$$kv^2 = (g+kv_0^2)\,e^{-2kx}$$

$$v^2 = \left(\frac{g}{k}+v_0^2\right)e^{-2kx}$$

$$A = \frac{g}{k}+v_0^2$$

$$v^2 = Ae^{-2kx} - \frac{g}{k}$$

(4) $F(x) = -kx^{-2}$

$$m\frac{d\dot{x}}{dt} = -kx^{-2}$$

$$\frac{d\dot{x}}{dt} = \frac{dx}{dt}\cdot\frac{d\dot{x}}{dx}$$

$$= \dot{x}\frac{d\dot{x}}{dx}$$

$$m\dot{x}\frac{d\dot{x}}{dx} = -kx^{-2}$$

$$\int_0^{\dot{x}} \dot{x}\,d\dot{x} = -k/m\int_b^x dx/x^2$$

$$\frac{1}{2}\dot{x}^2 = \frac{k}{m}\left(\frac{1}{x}-\frac{1}{b}\right)$$

$$\dot{x} = \sqrt{\frac{2k}{mb}\left(\frac{b-x}{x}\right)}$$

$$\frac{dx}{dt} = \sqrt{\frac{2k}{mb}\left(\frac{b-x}{x}\right)}$$

$$dt = \sqrt{\frac{mb}{2k}\left(\frac{x}{b-x}\right)}\,dx$$

$$\int_0^t dt = \sqrt{\frac{mb}{2k}}\int_b^0\left(\frac{x}{b-x}\right)^{\frac{1}{2}}dx$$

$$b = \sqrt{\frac{mb}{2k}}\left(b\int_b^0\frac{dx}{b-x} - \int_b^0 dx\right)$$

$$\int_0^t dt = \sqrt{\frac{mb^3}{2k}}\int_b^0\left(\frac{x/b}{1-x/b}\right)^{\frac{1}{2}}d\left(\frac{x}{b}\right)$$

$$t = \sqrt{\frac{mb^3}{2k}}\int_{-\pi/2}^0\frac{\sin\theta\,(2\sin\theta\cos\theta)\,d\theta}{\cos\theta}$$

$$t = \sqrt{\frac{2mb^3}{k}}\int_{-\pi/2}^0 \sin^2\theta\,d\theta$$

$$t = \pi\sqrt{\frac{mb^3}{8k}}$$

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(15) $m \cdot \dfrac{dv}{dt} = mg - C_1 v - C_2 v^2$

$$\int_0^t \frac{dt}{m} = \int_0^v \frac{dv}{mg - C_1 v - C_2 v^2}$$

Using $\displaystyle\int \frac{dx}{a + bx + cx^2} = \frac{1}{\sqrt{b^2 - 4ac}} \ln \frac{2cx + b - \sqrt{b^2 - 4ac}}{2cx + b + \sqrt{b^2 - 4ac}}$

$$\frac{t}{m} = \frac{1}{\sqrt{C_1^2 + 4mgC_2}} \ln \left. \frac{-2C_2 v - C_1 - \sqrt{C_1^2 + 4mgC_2}}{-2C_2 v - C_1 + \sqrt{C_1^2 + 4mgC_2}} \right|_0^v$$

$$\frac{t}{m}\left(C_1^2 + 4mgC_2\right)^{1/2} = \ln \frac{\left(2C_2 v + C_1 + \sqrt{C_1^2 + 4mgC_2}\right)\left(C_1 - \sqrt{C_1^2 - 4mgC_2}\right)}{\left(2C_2 v + C_1 - \sqrt{C_1^2 + 4mgC_2}\right)\left(C_1 + \sqrt{C_1^2 + 4mgC_2}\right)}$$

as $t \to \infty$, $2C_2 V_t + C_1 - \sqrt{C_1^2 + 4mgC_2} = 0$

$$V_t = -\frac{C_1}{2C_2} + \left[\left(\frac{C_1}{2C_2}\right)^2 + \frac{mg}{C_2}\right]^{1/2}$$

When $v = V_t$

$$m\frac{dv}{dt} = 0 = mg - C_1 V_t - C_2 V_t^2$$

$$V_t = -\frac{C_1}{2C_2} + \left[\left(\frac{C_1}{2C_2}\right)^2 + \frac{mg}{C_2}\right]^{1/2}$$

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