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2.1 Find the velocity $\dot{x}$ and the position x as functions of the time f for a particle of mass m, which start from rest at $x = 0$ and $t = 0$, subject to the following force functions:

a) $F_x = F_0 + ct$

b) $F_x = F_0 \sin ct$

c) $F_x = F_0 e^{ct}$

where F_0 and c are positive constans.

Answer

a) $F_x = m\ddot{x} = F_0 + ct$

$$m\frac{dv}{dt} = F_0 + ct$$

$$m\,dv = (F_0 + ct)\,dt$$

$$m\int_{v_0}^{v} dv = \int_{t_0}^{t} (F_0 + ct)\,dt$$

$$m(v - v_0) = F_0(t - t_0) + \frac{c}{2}(t^2 - t_0^2)$$

$$v_0 = 0 \quad \text{at} \quad t_0 = 0$$

$$mv = F_0 t + \frac{ct^2}{2}$$

$$v(t) = \frac{1}{m}\left(F_0 t + \frac{ct^2}{2}\right) = \frac{dx}{dt}$$

$$\int_{x_0}^{x} dx = \int_{t_0}^{t} \frac{1}{m}\left(F_0 t + \frac{ct^2}{2}\right) dt$$

$$x - x_0 = \frac{1}{m}\left(\frac{F_0}{2}(t^2 - t_0^2) + \frac{c}{6}(t^3 - t_0^3)\right)$$

$$x = \frac{1}{m}\left[\frac{F_0}{2}t^2 + \frac{ct^3}{6}\right]$$

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b) $F_x = m\frac{dv}{dt} = F_0 \sin ct$

$$\int_0^v dv = \frac{F_0}{m}\int_0^t \sin ct \, dt$$

$$v(t) = -\frac{F_0}{mc}\cos ct$$

$$x(t) = \int_0^x dx = -\frac{F_0}{mc}\int_0^t \cos ct \, dt$$

$$= -\frac{F_0}{mc^2}\sin ct$$

c) $F_x = m\frac{dv}{dt} = F_0 e^{ct}$

$$v(t) = \frac{F_0}{m}\int_0^t e^{ct} dt$$

$$= \frac{F_0}{cm} e^{ct}$$

$$x(t) = \frac{F_0}{c^2 m} e^{ct}$$

2.2) Find the volocity $\dot{x}$ as a function of the displacement x for a Particle of mass from rest at $x = 0$, subject to the pollowing force functions:

a) $F_x = F_0 + cx$
b) $F_x = F_0 e^{-cx}$
c) $F_x = F_0 \cos cx$

where F_0 and c are positive constants.

Answer

a) $F_x = -\frac{dv}{dx} = F_0 + cx$

$$F(x) = mv\frac{dv}{dx} = F_0 + cx$$

$$\int_{v_0}^{v} mv\,dv = \int_0^x (F_0 + cx)\,dx$$

$$\frac{m}{2}v^2 = F_0 x + \frac{c}{2}x^2$$

$$v = \sqrt{\frac{2}{m}\left(F_0 x + \frac{c}{2}x^2\right)}$$

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b) $F(x) = mv\frac{dv}{dx} = F_0 e^{-cx}$

$\frac{1}{2}mv^2 = -\frac{F_0}{c}e^{-cx}$

$v = \frac{2}{m}\left[-\frac{F_0}{c}e^{-cx}\right]^{1/2}$

c) $F(x) = mv\frac{dv}{dx} = F_0 \cos cx$

$\frac{1}{2}mv^2 = \int_0^x F_0 \cos cx = \frac{F_0}{c}\sin cx$

$v = \frac{2}{m}\left[\frac{F_0}{c}\sin cx\right]^{1/2}$

2.3) Find the potential energy function $V(x)$ for each of the forces in problem 2.2.

Answer.

a) $F_x = -\frac{dV}{dx} = F_0 + cx$

$V(x) = -\left[F_0 x + \frac{c}{2}x^2\right] + c$

b) $F_x = -\frac{dV}{dx} = F_0 e^{-cx}$

$\int_{V(x_0)}^{V(x)} dV = -F_0\int_{x_0}^{x} e^{-cx}\, dx$

$V(x) - V(x_0) = \frac{F_0}{c}\left[e^{-cx} - e^{-cx_0}\right]$

c) $F_x = -\frac{dV}{dx} = F_0 \cos cx$

$\int_{V(x_0)}^{V(x)} dV = -F_0\int_{x_0}^{x} \cos(cx)\, dx$

$V(x) - V(x_0) = -\frac{F_0}{c}\left[\sin cx - \sin cx_0\right]$

2.4) A particle of mass m is constrained to lie along a friction-less horizontal plane subject to a force given by the expression $F(x) = -kx$. it is projected from $x=0$ to the right along the positive x direction with initial kinetic energy

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$T_0 = \frac{1}{2}kA^2$. k and A positive constants. Find.

a) The potential energy function $V(x)$ for this force
b) The kinetic energy, and
c) The total energy of the particle as a function of its position
d) Find the turning points of the motion
e) Sketch the potential, kinetic, and total energy functions (optional: use Mathcad or mathematica to plot there functions set k dan A each equal to 1)

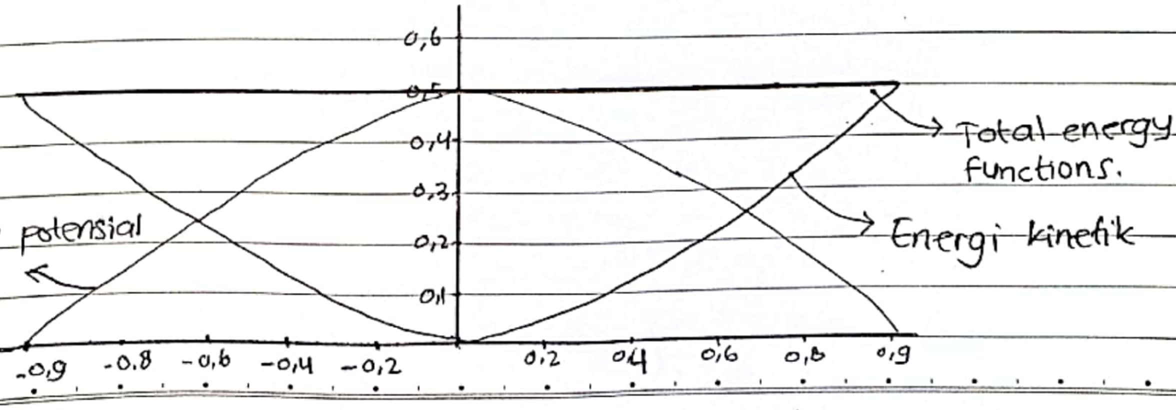
Answer

a) $V(x) = -\int f(x)\,dx$
$= -\int (-kx)\,dx$
$= \frac{kx^2}{2}$

b) $E = T_0 = \frac{1}{2}kA^2$
$E = T(x) + V(x)$
$T(x) = E - V(x)$
$T(x) = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$
$T(x) = \frac{k}{2}(A^2 - x^2)$

c) Diawal kita hanya memiliki energi kinetik, jadi itu adalah energi total $E = T_0 = \frac{1}{2}kA^2$

d) titik balik dapat kita temukan dari $E = V(x)$
$E = V(x)$
$\frac{1}{2}kA^2 = \frac{1}{2}kx^2$
$x = \pm A$

e)


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2.8) Given that the velocity of a particle in rectilinear motion varies with displacement x according for the equation
$$\dot{x} = bx^{-3}$$
where b is a positive constant find the force acting on the particle as a function of x (hint : $F = m\ddot{x} = m\dot{x}\, d\dot{x}/dx$)

Answer :

Given $v = \frac{b}{x^3}$

$$F = m\frac{dv}{dt}$$

$$= m\frac{dv}{dx}\cdot\frac{dx}{dt}$$

$$= mv\frac{dv}{dx}$$

$$\frac{dv}{dx} = -\frac{3b}{x^4}$$

$$F = m\left(\frac{b}{x^3}\right)\left(\frac{-3b}{x^4}\right)$$

$$= -\frac{3mb^2}{x^7}$$

2.11) A metal block of mass m slide off on a horizontal surface that has been lubricated with a heavy oil so that the block suffers a viscous resistance that varies as the $\frac{3}{2}$ power of the speed : $F(v) = -cv^{3/2}$
if the initial speed of the block is V_0 at $x = 0$, show that the block cannot travel farther then $2mV_0^{1/2}/c$

Answer :

Block silinder under force

$$f(v) = -cv^{3/2}$$

$$= m\frac{dv}{dt} = m\frac{dv}{dx}\frac{dx}{dt} = mv\frac{dv}{dx}$$

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Jarak yang ditempuh

$$-CV^{3/2} = mV\frac{dV}{dx}$$

$$\int_0^x dx = \int_{V_0}^{V} -\frac{mV}{CV^{3/2}}\, dV$$

$$x = -\frac{m}{c}\int_{V_0}^{V}\frac{dV}{\sqrt{V}}$$

$$= -\frac{m}{c}\int_{V_0}^{V} V^{-1/2}\, dV$$

$$= -\frac{m}{c}\,\frac{V^{1/2}}{1/2}\Big|_{V_0}^{V}$$

$$= \frac{2m}{c}\left(\sqrt{V_0}-\sqrt{V}\right)$$

partikel berhenti ketika $V=0$, maka jarak maksimal yang ditempuh adalah $X_{max} = \frac{2m\sqrt{V_0}}{c}$

2.12) A gun is fired straight up. Assuming that the air drag on the bullet varies quadratically with speed, show that the speed varies with height according to the equations

$$V^2 = Ae^{-2kx} - \frac{g}{k} \quad \text{(upwark motion)}$$

$$V^2 = \frac{g}{k} - Be^{2kx} \quad \text{(downwark motion)}$$

In which A dan B are constants of integration, g is the acceleration of gravity, and $k = C_2/m$ where C_2 is the drag constant and m is the mass of the bullet. (note: x is measured positive upward, and the gravitational force is assumed to be constant)

Answer:

* Upwark motion

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$$ma_x = \Sigma F_x$$

$$ma_x = -mg - C_2V^2$$

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$$a_x = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt}$$

$$mv \frac{dv}{dx} = -mg - c_2 v^2$$

$$v \frac{dv}{dx} = -g - \frac{c_2}{m} v^2$$

$$k = \frac{c_2}{m}$$

$$v \frac{dv}{dx} = -g - kv^2$$

$$\int_{v_0}^{v} \frac{v\,dv}{-g-kv^2} = \int_0^x dx$$

$$y = -g - kv^2$$

$$dy = -2kv\,dv$$

$$v\,dv = -\frac{1}{2k}\,dy$$

$$\int_{v_0}^{v} \frac{-1}{2k} \frac{dy}{y} = \int_0^x dx$$

$$-\frac{1}{2k} \ln(y)\Big|_{v_0}^{v} = x\Big|_0^x$$

$$-\frac{1}{2k} \ln\left(\frac{-g-kv^2}{-g-kv_0^2}\right) = x$$

$$\frac{-g-kv^2}{-g-kv_0^2} = e^{-2kx}$$

$$-g-kv^2 = (-g-kv_0^2)e^{-2}$$

$$kv^2 = (g+kv_0^2)e^{-2kx}$$

$$v^2 = \left(\frac{g}{k} + v_i^2\right)e^{-2kx}$$

$$A = \frac{g}{k} + v_0^2$$

$$\boxed{v^2 = Ae^{-2kx} - \frac{g}{k}}$$

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* downwark motion

$$ma_x = \Sigma F_x$$

$$ma_x = -mg + C_2 V^2$$

$$a_x = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = \frac{dv}{dx} \cdot v$$

$$mv \frac{dv}{dx} = -mg + C_2 V^2$$

$$v \frac{dv}{dx} = -g + \frac{C_2}{m} V^2$$

$$k = \frac{C_2}{m}$$

$$v \frac{dv}{dx} = -g + kv^2$$

$$\int_0^v \frac{v\,dv}{-g+kv^2} = \int_{x_0}^{x} dx$$

$$y = -g + kv^2$$

$$dy = 2k\,v\,dv$$

$$v\,dv = \frac{1}{2k}\,dy$$

$$\int_0^v \frac{1}{2k} \frac{dy}{y} = \int_{x_0}^{x} dx$$

$$\frac{1}{2k} \ln(y) \Big|_0^v = x \Big|_{x_0}^{x}$$

$$\frac{1}{2k} \ln[-g+kv^2] \Big|_0^v = x - x_0$$

$$\frac{1}{2k} \ln\left(\frac{-g+kv^2}{-g}\right) = x - x_0$$

$$\frac{-g+kv^2}{-g} = e^{2k(x-x_0)}$$

$$-g+kv^2 = -ge^{2k(x-x_0)}$$

$$kv^2 = -ge^{2k(x-x_0)} + g$$

$$V^2 = -\frac{g}{k} e^{-2kx_0} e^{2kx} +$$

$$B = \frac{g}{k} e^{-2kx_0}$$

$$\boxed{V^2 = \frac{g}{k} - Be^{2kx}}$$

2.13) Use the above result to show that, when the bulet hits the ground on its return, the speed is equal to the expression $\frac{V_0 V_t}{(V_0^2+V_t^2)^{1/2}}$ in which V_0 is the initial upwark speed and $V_t = (mg/C_2)^{1/2}$ = terminal speed = $(g/k)^{1/2}$

Answer:

$$V^2 = \left(\frac{g}{k} + V_0^2\right) e^{-2kx} - \frac{g}{k} \quad \text{(upwark motion)}$$

$$V^2 = -\frac{g}{k} e^{-2kx_0} e^{2kx} + \frac{g}{k} \quad \text{(downward motion)}$$

$$0 = \left(\frac{g}{k} + V_0^2\right) e^{-2kh} - \frac{g}{k}$$

$$e^{-2kh} = \frac{\frac{g}{k}}{\frac{g}{k} + V_0^2}$$

$$V^2 = -\frac{g}{k} e^{-2kh} e^{2k\cdot 0} + \frac{g}{k}$$

$$= -\frac{g}{k} \cdot \frac{\frac{g}{k}}{\frac{g}{k}+V_0^2} \cdot 1 + \frac{g}{k}$$

$$V_t^2 = \frac{g}{k}$$

$$V^2 = -V_t^2 \cdot \frac{V_t^2}{V_t^2+V_0^2} + V_t^2$$

$$= V_t^2 \cdot \left(\frac{V_t^2+V_0^2-V_t^2}{V_t^2+V_0^2}\right) = \frac{V_t^2 V_0^2}{V_t^2+V_0^2}$$

$$V = \frac{V_t^2 \cdot V_0^2}{\sqrt{V_t^2+V_0^2}}$$

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2.14) A particle of mass is released from rest a distance b from origin of force that attracts the particle according to the inverse square law: $F(x) = -kx^{-2}$.
Show that the time required for the particle to reach the origin is $\pi \left(\frac{mb^3}{8k}\right)^{1/2}$

Answer:

$$F(x) = -kx^{-2}$$

$$m\frac{d\dot{x}}{dt} = -kx^{-2}$$

$$\frac{d\dot{x}}{dt} = \frac{dx}{dt}\cdot\frac{d\dot{x}}{dx}$$

$$= \dot{x}\frac{d\dot{x}}{dx}$$

$$m\dot{x}\frac{d\dot{x}}{dx} = -kx^{-2}$$

$$\int_0^{\dot{x}} \dot{x}\,d\dot{x} = -\frac{k}{m}\int_b^x \frac{dx}{x^2}$$

$$\frac{1}{2}\dot{x}^2 = \frac{k}{m}\left(\frac{1}{x} - \frac{1}{b}\right)$$

$$\dot{x} = \sqrt{\frac{2k}{mb}\left(\frac{b-x}{x}\right)}$$

$$\frac{dx}{dt} = \sqrt{\frac{2k}{mb}\left(\frac{b-x}{x}\right)}$$

$$dt = \sqrt{\frac{mb}{2k}\left(\frac{x}{b-x}\right)}\,dx$$

$$\int_0^t dt = \sqrt{\frac{mb}{2k}}\int_b^0 \left(\frac{x}{b-x}\right)^{\frac{1}{2}} dx$$

$$t = \sqrt{\frac{mb}{2k}}\left(b\int_b^0 \frac{dx}{b-x} - \int_b^0 dx\right)$$

$$\int_0^t dt = \sqrt{\frac{mb^3}{2k}}\int_b^0 \left(\frac{\frac{x}{b}}{1-\frac{x}{b}}\right)^{\frac{1}{2}} d\left(\frac{x}{b}\right)$$

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$$t = \sqrt{\frac{mb^3}{2k}} \int_{-\pi/2}^{0} \frac{\sin\theta\,(2\sin\theta\cos\theta)}{\cos\theta}\, d\theta$$

$$t = \sqrt{\frac{2mb^3}{k}} \int_{-\pi/2}^{0} \sin^2\theta\, d\theta$$

$$t = \pi \sqrt{\frac{mb^3}{8k}}$$

2.15) Show that the terminal speed of a falling spherical object is given by $V_t = [(mg/c_2) + (c_1/2c_2)^2]^{1/2} - (c_1/2c_2)$ when both the linear and the quadratic term in the drag force are taken into account

Answer

Hukum 2 Newton

$$ma = \Sigma F$$

$$ma = mg - c_1 v - c_2 v^2$$

$$ma = 0$$

$$mg - c_1 v - c_2 v^2 = 0$$

$$c_2 v^2 + c_1 v - mg = 0$$

$$v_{1,2} = \frac{-c_1 \pm \sqrt{c_1^2 + 4c_2 mg}}{2c_2}$$

$$v_1 = \frac{-c_1 + \sqrt{c_1^2 + 4c_2 mg}}{2c_2}$$

$$v_1 = V_t = -\frac{c_1}{2c_2} + \sqrt{\left(\frac{c_1}{2c_2}\right)^2 + \frac{mg}{c_2}}$$

$$V_t = -\frac{c_1}{2c_2} + \sqrt{\left(\frac{c_1}{2c_2}\right)^2 + \frac{mg}{c_2}}$$