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Date :

Nama : Mita Safira
NPM : 2013022044
Kelas : B

Mekanika
Chapter 2

2.1 $x = 0$
$t = 0$
Tentukan :
a. $fx = fo + ct$
b. $fx = fo \sin ct$
c. $fx = fo\, e^{ct}$
Dimana fo dan c adalah konstan
Jawab :

a.) $\dot{x} = \frac{1}{m} (fo + ct)$

$\dot{x} = \int_0^x \frac{1}{m} (f + ct)\, dt = \frac{fo}{m} t + \frac{s}{2m} t^2$

$x = \int_0^x \left(\frac{fo}{m} t + \frac{c}{2m} t^2 \right) = \frac{f}{m} t^2 + \frac{c}{6m} t^3$

b.) $\ddot{x} = \frac{f}{m} \sin ct$

$\ddot{x} = \int_0^x \frac{fo}{m} \sin ct\ dt = \frac{fo}{cm} \cos ct \Big|_0^t$

$x = \int_0^t \frac{fo}{cm} (1 - \cos ct)\, dt = \frac{fo}{cm} \left(1 - \frac{1}{c} \sin ct\right)$

c.) $\ddot{x} = \frac{fo}{m} e^{ct}$

$\ddot{x} = \frac{fo}{cm} e^{ct} \Big|_0^t = \frac{fo}{cm} \left(e^{ct} - 1\right)$

$x = \frac{fo}{cm} \left(\frac{1}{c} e^{ct} - \frac{1}{c} - 1\right)$

$= \frac{fo}{c^2 m} \left(e^{ct} - 1 - ct\right)$

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2.2 a.) $\ddot{x} = \frac{d\dot{x}}{dt} = \frac{d\dot{x}}{dx} \cdot \frac{dx}{dt} = \frac{\dot{x}\,d\dot{x}}{dx}$

$\frac{\dot{x}\,d\dot{x}}{dx} = \frac{1}{m}\,(f_0 + cx)$

$\frac{\dot{x}\,d\dot{x}}{dx} = \frac{1}{m}\,(f + cx)\,dx$

$\frac{1}{2}\dot{x}^2 = \frac{1}{m}\left(f_0 x + \frac{cx^2}{2}\right)$

$\dot{x} = \left(\frac{x}{m}\,(2f_0 + cx)\right)^{1/2}$

b.) $\dot{x} = \frac{\dot{x}\,d\dot{x}}{dx} = \frac{1}{m}\,f_0 e^{-ct}$

$\dot{x}dx = \frac{f_0}{cm}\,(e^{-cx} - 1) = \frac{f_0}{cm}\,(1 - e^{-cx})$

$\frac{1}{2}x^2 = \frac{f_0}{cm}\,(e^{-cx} - 1) = \frac{f_0}{cm}\,(1 - e^{-cx})$

$\dot{x} = \left[\frac{2f_0}{cm}\,(1 - e^{-cx})\right]^{1/2}$

c.) $\ddot{x} = \dot{x}\frac{d\dot{x}}{dx} = \frac{1}{m}\,(f \cos cx)$

$\dot{x}d\dot{x} = \frac{f_0}{m}\cos cx\,dx$

$\frac{1}{2}\dot{x}^2 = \frac{f_0}{cm}\sin cx$

$\dot{x} = \left(\frac{2f_0}{cm}\sin cx\right)^{1/2}$

2.3 a.) $V(x) = \int_0^x (f_0 + cx)\,dx$

$= f_0 x - \frac{cx^2}{2} + C$

b.) $V(x) = -\int_c^x f_0 e^{-ct}\,dx$

$= \frac{f_0}{c}\sin cx + C$

c.) $V(x) = -\int_x^x f_0 \cos cx\,dx$

$= \frac{f_0}{c}\sin cx + C$

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2.4 a.) $V(x) = -\int f(x)\,dx$

$V(x) = -\int(-kx)\,dx$

$V(x) = \frac{kx^2}{2}$

b.) $E = T_0 = \frac{1}{2}kA^2$

$= T(x) + V_c \times 1$

$T(x) = E - V(x)$

$= \frac{1}{2}kA^2 - \frac{1}{2}kx^2$

$= \frac{k}{2}(A^2 - x^2)$

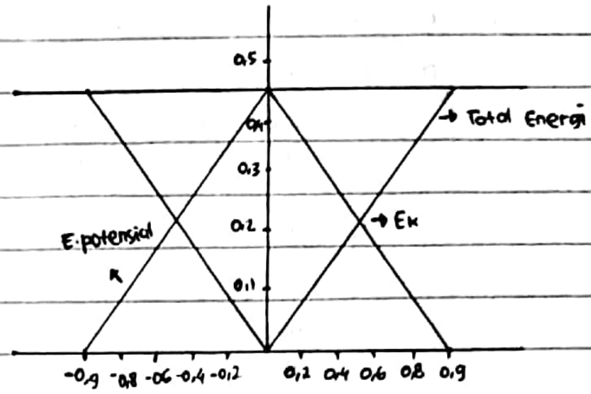
c.) $E = T_0 = \frac{1}{2}kA^2$

d.) $E = V(x)$

$\frac{1}{2}kA^2 = \frac{1}{2}kx^2$

$x = \frac{1}{2}A$

e.)



2.8 $v = \frac{b}{x^2}$

$f = m\frac{dv}{dt} = m\frac{dv}{dt} \cdot \frac{dx}{dt}$

$= mv\frac{dv}{dx}$

$\frac{dv}{dx} = -\frac{3b}{x^4}$

$f = m\left(\frac{b}{m^3}\right)\left(-\frac{3b}{x^4}\right)$

$= -\frac{3mb^2}{x^7}$

2.11 $a = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = v\frac{dv}{dx} = -\frac{c}{m}v^{3/2}$

$= v^{-t}\,dv = -\frac{c}{m}\,dx$

$= \int_v^v v^{-1/2}\,dx = \int_0^x -\frac{c}{m}\,dx$

$= -2v_0^t \cdot -\frac{c}{m}\,x\,max$

$x_{max} = \frac{2mv_0^{1/2}}{c}$

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2.12 $m \cdot a_x = + f_x = \Sigma f_x$

$m \cdot a_x = mg - C_2 V^2$

$a_x = \frac{dv}{dx} = \frac{dv\,dx}{dx\,dt} = \frac{dv}{dt}$

$m \cdot \frac{dv}{dx} = -mg = -C_2 V^2$

$v = \frac{dv}{dx} = -g - \frac{C_2 V^2}{m}$

$k = \frac{C^2}{m}$

$\int_{B}^{V} \frac{v\,dv}{-g - kv^2} = \int_{B}^{A} dx$

$y = -u - kv^2$

$dy = -2kv\,dv$

$v\,dv = \frac{1}{2} k\,dy$

$\int_{VB}^{V} \cdot \frac{dy}{2ky} = \int_{0}^{X} dy$

$-\frac{1}{2} k \ln (-g - kv^2) \Big|_{V_0}^{V} = x$

$\Rightarrow \frac{1}{2k} \ln \left(\frac{-g - kv^2}{-g - kv_0^2} \right) = x$

$\frac{-g - kv^2}{-g - kV_0^2} = e^{-2kx}$

$-g - kv^2 = (-g - kV_0^2) e^{-2kx}$

$kv^2 = (g + kV_0^2) e^{-2kx} - \frac{g}{k}$

$A = \frac{g}{k} + V_0^2$

$V^2 = Ae^{-kx} - \frac{g}{k}$

$Ma_x = \Sigma f_x$

$m \cdot a_x = -mg + C_2 V^2$

$a_x = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = \frac{dv}{dx} \cdot v$

$mv = \frac{dv}{dx} - mg + C_2 V^2$

$\frac{dv}{dx} = -g + \frac{C_2}{m} \cdot v^2$

$k = \frac{C_2}{m}$

$v \frac{dv}{dx} = -g + kv^2$

$\int_0^V \frac{v\,dv}{-g + kv^2} = \int_0^X dx$

$y = -g + kv^2$

$v\,dv = 2kv\,dv$

$v\,dv = \frac{1}{2} k\,dy$

$\int_0^V \frac{1}{2} k \frac{dy}{y} = \int_{X_0}^{X} dx$

$\frac{1}{2k} \ln (y) \Big|_0^V = x \Big|_{X_0}^{X}$

$\frac{1}{2k} \ln (-g + kv^2) \Big|_0^V = x - x_0$

$\frac{1}{2k} \ln \left(\frac{-g + kv^2}{g} \right) = x - x_0$

$\frac{-g + kv^2}{g} = e^{2k(x + x_0)}$

$\Rightarrow g + kv^2 = -g e^{2k(x - x_0)}$

$kv^2 = -g e^{2k(x - x_0)}$

$V^2 = -\frac{g}{k} e^{2kx_0} e^{2kx} + \frac{g}{k}$

$B = \frac{g}{k} e^{2kx_0}$

$V^2 = \frac{g}{k} - Be^{2kx}$

PAPERLINE

2.13 $V^2 = \left(\frac{g}{k} + V_0^2\right) e^{-2kx} - \frac{g}{k} \Rightarrow$ ke atas

$V^2 = -\frac{g}{k} \cdot e^{-2kx_0} e^{2kx} + \frac{g}{k} \Rightarrow$ ke bawah

$V = 0 \Rightarrow$ gerak ke atas

$0 = \left(\frac{g}{k} + V_0^2\right) e^{-2kx} - \frac{g}{k}$

$e^{-2kx} = \dfrac{\frac{g}{k}}{\frac{g}{k} + V_0^2}$

h posisi awal untuk gerakan ke bawah (t) $a > 0$

$V^2 = \frac{-g}{k} \cdot e^{-2kh} e^{2k0} + \frac{g}{b}$

$V^2 = \dfrac{\frac{-g}{k} \cdot \frac{g}{k}}{\frac{g}{k} + V_0^2} \cdot 1 + \frac{g}{k}$

$V^2 = \frac{g}{k}$

$= -Vt^2 - \dfrac{Vt^2}{Vt + V_0^2} + Vt^2$

$= Vt^2 \left(1 - \dfrac{Vt^2}{Vt^2 + V_0^2}\right)$

$V^2 = \dfrac{V_0^2 V_0^2}{Vt + V_0^2}$

$V = \dfrac{V_t V_0}{\sqrt{Vt^2 + V_0}}$

$\sqrt{\frac{2k}{m}\left(\frac{b-x}{m}\right)} = \frac{dx}{dt}$

$\displaystyle\int_0^0 \frac{dx}{\sqrt{\frac{2k}{m} \cdot \left(\frac{b-x}{bx}\right)}} = \int_0^x dt$

$\displaystyle\int_0^0 \frac{dx}{\sqrt{\frac{2k}{m}\left(\frac{b-x}{bx}\right)}} = \sqrt{\frac{mb}{2k}} \int_0^0 \sqrt{\frac{x}{b-x}}\, dx$

$u = \sqrt{\dfrac{b-x}{\sqrt{x}}}$

$dv = \dfrac{\frac{1}{2\sqrt{bx}} (-1) \cdot \sqrt{x}\, dx \cdot \frac{1}{\sqrt{x}}}{x}$

$dx = \dfrac{dv}{\frac{1}{-2\sqrt{2} \cdot v\sqrt{x}} - \frac{v\sqrt{x}}{2x}}$

$= \dfrac{dv}{-\frac{1}{2\sqrt{x}} - \frac{v}{2x}}$

$= \dfrac{dv}{\frac{1+v^2}{2xv}}$

$= \sqrt{b-x} = v\sqrt{x}$

$b - x = v^2 x$

$b = x^2 (v^2 + 1)$

$r = \frac{b}{v^2} + 1$

$dx = \dfrac{-2bv\, dv}{(v^2+1)^2}$

$\displaystyle\sqrt{\frac{mb}{2k}} \int_b^0 \sqrt{\frac{x}{b-x}}\, dx = \sqrt{\frac{mb}{2k}} \int_b^0 f(x) \frac{2bv\,dv}{(v^2+1)^2}$

$\displaystyle= \sqrt{\frac{mb}{2k}} \int_b^0 \frac{2bv\,dv}{(v^2+1)^2}$

$\displaystyle\int \frac{dv}{(av+b)^n} = \frac{2n-3}{2b(n-1)} \int \frac{dv}{(av^2+b)^{n+1}} + \frac{v}{2b(n-1)(av^2+b)^{n-1}}$

$\displaystyle= \frac{v}{2(v^2+1)} + \frac{1}{2} \int \frac{dv}{b^2+1}$

$\displaystyle\int_b^0 \frac{av}{(v^2+1)^2} = \frac{v}{2(v^2+1)} \Big|_b^0 + \frac{1}{2} \arctan v \Big|_b^0$

$\displaystyle-\sqrt{\frac{mb}{2k}} \int_b^0 \frac{2b\,dv}{(v^2+1)^2} = -2b \sqrt{\frac{mb}{2k}} \left(\frac{v}{2(v^2+1)}\Big|_b^0 + \frac{1}{2} \arctan v \Big|_b^0\right)$

$\displaystyle= -2b \sqrt{\frac{mb}{2k}} \left(\frac{v}{2(v^2+1)}\Big|_0^\infty + \frac{1}{2} \arctan v \Big|_0^\infty\right)$

$\displaystyle= -2b \sqrt{\frac{mb}{2k}} \left(0 - 0 + \frac{1}{2} \cdot \left(-\frac{\pi}{0} - 0\right)\right) = \pi \sqrt{\frac{mb^3}{8k}}$

$\displaystyle\pi \sqrt{\frac{mb^3}{8k}} = \int_0^t dt$

$t = \pi \sqrt{\dfrac{mb^3}{8k}}$

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2.15 $ma = \Sigma F$

$ma = mg - V - C_2 V^2$

$mg - C_1 v - C_2 V^2 = 0$

$C_2 V^2 - C_1 v - mg = 0$

$$V_{1,2} = \frac{-C \pm \sqrt{C_1^2 + 4(2mg)}}{2C_2}$$

Pilih (+)

$$V_1 = \frac{-C + \sqrt{C_1^2 + 4(2mg)}}{2C_2}$$

$$V_1 = V_t = \frac{-C_1}{2C_2} + \sqrt{\left(\frac{C_1}{2\sqrt{2}}\right)^2 + \frac{mg}{C_2}}$$

$$V_t = \frac{-C_1}{2C_2} + \sqrt{\left(\frac{C_1}{2\sqrt{2}}\right)^2 + \frac{mg}{C_2}}$$