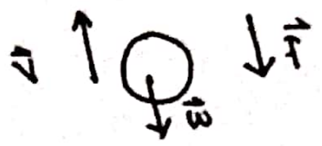
Nama : Indah Sina Tyas

Npm : 2013022042

Prodi : Pendidikan Fisika

Kelas : B

Meninjau benda yang bergerak vertikal ke atas dengan pengaruh gaya gesekan kuadratik $f = -bv^2$



* $\Sigma \vec{F} = m \cdot \vec{a}$

$-\vec{w} - \vec{f} = m \cdot \vec{a}$

$-mg - bv^2 = m \cdot a \quad | \times \frac{1}{m}$

$-g - \frac{b}{m} v^2 = a$

$-g - \frac{b}{m} v^2 = \frac{dv}{dt}$

$\int_{v_0}^{v(t)} \frac{dv}{-g - \frac{b}{m} v^2} = \int_0^t dt$

$\int_{v_0}^{v(t)} \frac{f}{v} \cdot \frac{dv}{-g - \frac{bv^2}{m}} = \int_0^t dt$

misal : $-g - \frac{bv^2}{m} = u$

$-\frac{b}{m} dv = du$

$dv = -\frac{m}{b} du$

$\int \frac{1}{u} dv = \int \frac{1}{u} - \frac{m}{b} du$

$\int \frac{1}{u} \left(-\frac{m}{b} du\right) = -\frac{m}{b} \int \frac{1}{u} du$

$= -\frac{m}{b} \ln |u|$

$$\int_{v_0}^{v(t)} \frac{1}{-g-\frac{bv^2}{m}}\,dv = \int_0^t dt$$

$$-\frac{m}{bv} \ln |u| \Big]_{v_0}^{v(t)} = t$$

$$-\frac{m}{bv} \ln \left[-g-\frac{bv^2}{m}\right]_{v_0}^{v(t)} = t$$

$$-\frac{m}{bv} \left(\ln \left| \frac{-g-\frac{bv^2}{m}}{-g-\frac{bv^2}{m}} \right| \right)^{v} = t$$

$$\frac{-g-\frac{bv(t)}{m}}{-g-\frac{bv_0}{m}} = e^{-\frac{bv^2}{m}}$$

$$-g-\frac{b}{m}v(t) = \left(-g-\frac{b}{m}v_0\right)e^{-\frac{bv^2}{m}}$$

$$v(t) = \left(-\frac{mg}{bv}\right) - \left(-\frac{mg}{bv} - v_0\right) e^{-\frac{bv(t)}{m}}$$

karna: $f = -w$

$bv = mg$

$v_T = \frac{mg}{bv}$

maka, $v(t) = v_T - (v_T - v_0)\, e^{-\frac{bv(t)}{m}}$

* $a(t) = \frac{dv(t)}{dt}$

$$= \frac{d}{dt}\left(v_T - (v_0 + v_T)\, e^{-\frac{bv(t)}{m}}\right)$$

$$= 0 - (-v_0 + v_T)\left(-\frac{bv}{m}\right) e^{-\frac{bv(t)}{m}}$$

$$= \frac{-bv}{m}(-v_0 + v_T)\, e^{-\frac{bv(t)}{m}}$$

$$= -\frac{bv}{m}\left(V_0 + \frac{bv}{m} V_T\right) e^{-\frac{bv}{m}}$$

$V(t) = \frac{dy(t)}{dt}$

$$\int_0^{y(t)} dy(t) = \int_0^t V(t)\, dt$$

$$y(t) = \int_0^t \left(V_T - (-V_0 + V_T) e^{-\frac{bv(t)}{m}}\right) dt$$

$$= V_T t - (-V_0 + V_T)\left(-\frac{m}{bv}\right) e^{-\frac{bv(t)}{m}} \Big]_0^t$$

$$= \left[V_T t - (-V_0 + V_T)\left(-\frac{m}{bv}\right) e^{-\frac{bv(t)}{m}}\right] \left[(-V_0 + V_T)\left(-\frac{m}{bv}\right)\right]$$

$$= V_T t - \left(\frac{m}{bv}\right)(-V_0 + V_T)\left(e^{-\frac{bv(t)}{m}}\right)$$

$$= V_T t - \frac{m}{bv}(-V_0 + V_T)\left(1 - e^{-\frac{bv(t)}{m}}\right)$$

$$\boxed{\begin{aligned} y(t) &= V_T t - \frac{m}{b}(-V_0 - V_T)\left(1 - e^{-\frac{bv(t)}{m}}\right) \\ V(t) &= V_T - (-V_0 + V_T) e^{-\frac{bv(t)}{m}} \\ a(t) &= \left(-g - \frac{bv}{m} V_0\right) e^{-\frac{bv(t)}{m}} \end{aligned}}$$

Diaproksimasi

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{x!} + \dots$$

$$e^{-\frac{bv(t)}{m}} = 1 - \frac{bv(t)}{m} + \frac{1}{2}\left(\frac{bv(t)}{m}\right)^2 + \dots$$

$y(t) = V_T t - \frac{m}{bv}(-V_0 - V_T)\left(1 - e^{-\frac{bv(t)}{m}}\right)$

$$y(t) = V_T t - (-V_0 + V_T)\left(t - \frac{1}{2}\frac{bv}{m} t^2\right)$$

$$y(t) = V_0 t + \frac{1}{2} \cdot \frac{bv}{m} \cdot V_0 t^2 + \frac{1}{2}\frac{bv}{m}\left(\frac{-mg}{bv}\right) t^2$$

untuk $b = 0$

maka, $y(t) = V_0 t - \frac{1}{2} g t^2$

* $v(t) = V_T - (-V_0 + V_T)\, e^{-\frac{bv(t)}{m}}$

$= V_T - (-V_0 + V_T)\left(1 - \left(-\frac{bv(t)}{m}\right)\right)$

$= V_0 t + \frac{bv}{m} V_0 t - gt$

Untuk $b = 0$

maka, $V(t) = V_0 - gt$

* $a(t) = \left(-g - \frac{bv}{m} V_0\right) e^{-\frac{bv(t)}{m}}$

$a(t) = \left(-g - \frac{bv}{m} V_0\right)\left(1 - \frac{bv(t)}{m}\right)$

$= -g + g \cdot \frac{bv(t)}{m} - \frac{bv}{m} V_0 + \frac{b^2 v^2}{m^2} V_0 t$

Untuk $b = 0$

maka, $V(t) = -g$