

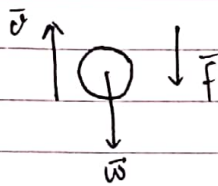
Nama : Windu Dwi Safitri

NPM : 2013022018.

Kelas : B.

Prodi : Pendidikan Fisika.

Diskusi gerak vertikal ke atas dipengaruhi (hambatan udara $f = -bv^2$).



$$f = -bv^2$$

$$\Sigma F = m \cdot a$$

$$-w - f = m \cdot a$$

$$-mg - bv^2 = m \cdot a \quad | \times \frac{1}{m}$$

$$-g - \frac{bv^2}{m} = a$$

$$-g - \frac{bv^2}{m} = \frac{dv}{dt}$$

$$\int_{v_0}^{v_t} \frac{dv}{-g - \frac{b}{m}v^2} = \int_0^t dt$$

$$\int_{v_0}^{v_t} \frac{1}{v} \cdot \frac{dv}{-g - \frac{b}{m}v^2} = \int_0^t dt$$

$$\int_{v_0}^{v_t} \frac{1}{-g - \frac{b}{m}v^2} \cdot dv = \int_0^t dt$$

$$\frac{-m}{bv} \ln u \Big|_{v_0}^{v_t} = t$$

$$\frac{-m}{bv} \ln \left[-g - \frac{bv^2}{m} \right] \Big|_{v_0}^{v_t} = t$$

misalkan.

$$-g - \frac{bv^2}{m} = u$$

$$-\frac{b}{m} \cdot dv = du$$

$$dv = \frac{-m}{b} \cdot du$$

$$\int \frac{1}{u} dv = \int \frac{1}{u} \left(\frac{-m}{b} \right) \cdot du$$

$$= \frac{-m}{b} \int \frac{1}{u} du$$

$$= \frac{-m}{b} \ln u$$

$$\frac{-m}{bv} \left(\ln \left(\frac{-g - \frac{bv^2}{m}}{-g - \frac{bv_0^2}{m}} \right) \right) = t$$

$$\frac{-g - \frac{bv^2}{m}}{-g - \frac{bv_0^2}{m}} = e^{-\frac{bv^2}{m}}$$

$$-g - \frac{bv^2}{m} = \left(-g - \frac{bv_0^2}{m} \right) e^{-\frac{bv^2}{m}}$$

$$Vt = \left(\frac{mg}{bv} \right) - \left(\frac{mg}{bv} - v_0 \right) e^{-\frac{btv}{m}}$$

karena $V_T = \frac{mg}{bv}$

maka-

$$V(t) = V_T - (V_T - v_0) e^{-\frac{btv}{m}}$$

a). $a(t) = \frac{dV(t)}{dt}$

$$= \frac{d}{dt} \left(V_T - (V_T - v_0) e^{-\frac{btv}{m}} \right)$$

$$= 0 - (-v_0 + V_T) \left(-\frac{bv}{m} \right) e^{-\frac{btv}{m}}$$

$$= -\frac{bv}{m} (-v_0 + V_T) e^{-\frac{btv}{m}}$$

$$= -\frac{bv}{m} \left(v_0 + \frac{bv}{m} V_T \right) e^{-\frac{btv}{m}}$$

• $V(t) = \frac{dy(t)}{dt}$

$$\int_0^t dy(t) = \int_0^t V(t) \cdot dt$$

$$y(t) = \int_0^t \left(V_T - (-v_0 + V_T) e^{-\frac{btv}{m}} \right) dt$$

$$= V_T t - (-v_0 + V_T) \left(-\frac{m}{bv} \right) e^{-\frac{btv}{m}} \Big|_0^t$$

$$\begin{aligned}
 &= \left[V_T t - (-V_0 + V_T) \left(\frac{-m}{bV} \right) e^{-\frac{bVt}{m}} \right] - \left[(-V_0 + V_T) \left(\frac{-m}{bV} \right) \right] \\
 &= V_T t - \left(\frac{m}{bV} \right) (-V_0 + V_T) \left(e^{-\frac{bVt}{m}} \right) \\
 &= V_T t - \frac{m}{bV} (-V_0 + V_T) (1 - e^{-\frac{bVt}{m}})
 \end{aligned}$$

$$\begin{aligned}
 y(t) &= V_T t - \frac{m}{b} (V_0 - V_T) (1 - e^{-\frac{bVt}{m}}) \\
 V(t) &= V_T - (-V_0 + V_T) \left(e^{-\frac{bVt}{m}} \right) \\
 a(t) &= \left(-g - \frac{bV}{m} V_0 \right) e^{-\frac{bVt}{m}}
 \end{aligned}$$

Aproksimasi: $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!}$

$$e^{-\frac{bVt}{m}} = 1 - \frac{bVt}{m} + \frac{1}{2} \left(\frac{bVt}{m} \right)^2$$

$$\Rightarrow y(t) = V_T t - \frac{m}{bV} (-V_0 + V_T) (1 - e^{-\frac{bVt}{m}})$$

$$y(t) = V_T t - (-V_0 + V_T) \left(t - \frac{1}{2} \frac{bV}{m} t^2 \right)$$

$$y(t) = V_0 t + \frac{1}{2} \frac{bV}{m} V_0 t^2 + \frac{1}{2} \frac{bV}{m} \left(\frac{mg}{bV} \right) t^2$$

$$y(t) = V_0 t - \frac{1}{2} g t^2 + \frac{1}{2} \frac{bV}{m} V_0 t^2$$

Untuk $b=0$, maka $y(t) = V_0 t - \frac{1}{2} g t^2$

$$\begin{aligned}
 \Rightarrow V(t) &= V_T - (-V_0 + V_T) e^{-\frac{bVt}{m}} \\
 &= V_T - (-V_0 + V_T) \left(1 - \frac{bVt}{m} \right) \\
 &= \cancel{V_T} + V_0 - \cancel{V_T} - \frac{bVt V_0}{m} + \frac{bVt V_T}{m} \\
 &= V_0 - \frac{bVt mg}{bVm} - \frac{bVt V_0}{m}
 \end{aligned}$$

$$= V_0 - gt - \frac{b t v V_0}{m}$$

Jika $b=0$ maka $v(t) = V_0 - gt$.

$$\begin{aligned} \therefore a(t) &= \left(-g - \frac{b v V_0}{m}\right) e^{-\frac{b t v}{m}} \\ &= \left(-g - \frac{b v V_0}{m}\right) \left(1 - \frac{b v(t)}{m}\right) \\ &= -g + g \frac{b t v}{m} - \frac{b v V_0}{m} + \frac{b^2 v^2 V_0 t}{m^2} \end{aligned}$$

Untuk $b=0$ maka $a = -g$.