No.

Date

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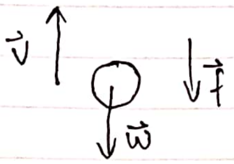
Gerak Vertikal ke atas di bawah Pengaruh gaya gesekan kuadratik

$f = -bv^2$



$$\Sigma \vec{F} = m \cdot \vec{a}$$

$$-\vec{w} - \vec{f} = m \cdot \vec{a}$$

$$-mg - bv^2 = m \cdot a \qquad \Big| \times \frac{1}{m}$$

$$-g - \frac{b}{m} v^2 = a$$

$$-g - \frac{b}{m} v^2 = \frac{dv}{dt}$$

$$\int_{v_0}^{v(t)} \frac{dv}{-g - \frac{b}{m} v^2} = \int_0^t \cdot dt$$

$$\int_{v_0}^{v(t)} \frac{f}{v} \cdot \frac{dv}{-g - \frac{bv^2}{m}} = \int_0^t dt$$

misalkan : $-g - \frac{bv^2}{m} = u$

$$-\frac{b}{m} dv = du$$

$$dv = -\frac{m}{b} du$$

$$\int \frac{1}{u} dv = \int \frac{1}{u} - \frac{m}{b} du$$

$$\int \frac{1}{u} \left(-\frac{m}{b} du \right) = -\frac{m}{b} \int \frac{1}{u} du$$

$$= -\frac{m}{b} \ln |u|$$

Dipindai dengan CamScanner

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$$\int_{V_0}^{V(t)} \frac{1}{-g-\frac{bv^2}{m}}\, dv = \int_0^t dt$$

$$-\frac{m}{bv} \ln|u| \Big]_{V_0}^{V(t)} = t$$

$$-\frac{m}{bv} \ln\left[-g-\frac{bv^2}{m}\right]_{V_0}^{V(t)} = t$$

$$-\frac{m}{bv}\left(\ln\left|\frac{-g-\frac{bv^2}{m}}{-g-\frac{bv^2}{m}}\right|\right)^{V} = t$$

$$\frac{-g-\frac{bV(t)}{m}}{-g-\frac{bV_0}{m}} = e^{-\frac{bv^2}{m}}$$

$$-g-\frac{b}{m}V(t) = \left(-g-\frac{b}{m}V_0\right) e^{-\frac{bv^2}{m}}$$

$$V(t) = \left(-\frac{mg}{bv}\right) - \left(-\frac{mg}{bv} - V_0\right) e^{-\frac{bV(t)}{m}}$$

Karena $F = -w$

$bv = mg$

$V_T = \frac{mg}{bv}$

maka, $V(t) = V_T - (V_T - V_0)\, e^{-\frac{bV(t)}{m}}$

$\circ$ $a(t) = \frac{dV(t)}{dt}$

$$= \frac{d}{dt}\left(V_T - (-V_0 + V_T)\, e^{-\frac{bV(t)}{m}}\right)$$

$$= 0 - (-V_0 + V_T)\left(-\frac{bv}{m}\right) e^{-\frac{bV(t)}{m}}$$

$$= -\frac{bv}{m}(-V_0 + V_T)\, e^{-\frac{bV(t)}{m}}$$

$$= -\frac{bv}{m}\left(V_0 + \frac{bv}{m}V_T\right) e^{-\frac{bV(t)}{m}}$$

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$$\circ)\ v(t) = \frac{dy(t)}{dt}$$

$$\int_0^{y(t)} dy(t) = \int_0^t v(t)\ dt$$

$$y(t) = \int_0^t \left(V_T - (-V_0 + V_T)\, e^{-\frac{bv(t)}{m}}\right) dt$$

$$= V_T t - (-V_0+V_T)\left(-\frac{m}{bv}\right) e^{-\frac{bv(t)}{m}} \Big]_0^t$$

$$= \left[V_T t - (-V_0+V_T)\left(-\frac{m}{bv}\right) e^{-\frac{bv(t)}{m}}\right] - \left[(-V_0+V_T)\left(-\frac{m}{bv}\right)\right]$$

$$= V_T t - \left(\frac{m}{bv}\right)(-V_0+V_T)\left(e^{-\frac{bv(t)}{m}}\right)$$

$$= V_T t - \frac{m}{bv}(-V_0+V_T)\left(1 - e^{-\frac{bv(t)}{m}}\right)$$

$$\boxed{\begin{aligned} y(t) &= V_T t - \frac{m}{b}(-V_0 - V_T)\left(1 - e^{-\frac{bv(t)}{m}}\right) \\ v(t) &= V_T - (-V_0+V_T)\, e^{-\frac{bv(t)}{m}} \\ a(t) &= \left(-g - \frac{bv}{m} V_0\right) e^{-\frac{bv(t)}{m}} \end{aligned}}$$

Diaproksimasi

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$e^{-\frac{bv(t)}{m}} = 1 - \frac{bv(t)}{m} + \frac{1}{2}\left(\frac{bv(t)}{m}\right)^2 + \dots$$

$$\circ)\ y(t) = V_T t - \frac{m}{bv}(-V_0 - V_T)\left(1 - e^{-\frac{bv(t)}{m}}\right)$$

$$y(t) = V_T t - (-V_0 + V_T)\left(t - \frac{1}{2}\frac{bv}{m}t^2\right)$$

$$y(t) = V_0 t + \frac{1}{2}\cdot\frac{bv}{m}\cdot V_0 t^2 + \frac{1}{2}\frac{bv}{m}\left(-\frac{mg}{bv}\right)t^2$$

untuk $b = 0$

maka, $\boxed{y(t) = V_0 t - \frac{1}{2}gt^2}$

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•) $v(t) = V_T - (-V_0 + V_T)\, e^{-\frac{bv(t)}{m}}$

$= V_T - (-V_0 + V_T)\left(1 - \left(-\frac{bv(t)}{m}\right)\right)$

$= V_0 t + \frac{bv}{m} V_0 t - gt$

untuk $b = 0$

maka, $\boxed{v(t) = V_0 - gt}$

•) $a(t) = \left(-g - \frac{bv}{m} V_0\right) e^{-\frac{bv(t)}{m}}$

$a(t) = \left(-g - \frac{bv}{m} V_0\right)\left(1 - \frac{bv(t)}{m}\right)$

$= -g + g \cdot \frac{bv(t)}{m} - \frac{bv}{m} V_0 + \frac{b^2 v^2}{m^2} V_0 t$

untuk $b = 0$

maka, $\boxed{v(t) = -g}$