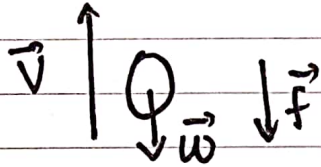


Nama : Putri Asnau Karimah
 Npm : 2013022014
 Prodi : Pendidikan Fisika
 Kelas : B

Gerak Vertikal keatas dibawah pengaruh gaya gesekan
 Kuadratik : $f = -bv^2$



$$\begin{aligned} \sum \vec{F} &= m \cdot \vec{a} \\ -\vec{w} - \vec{f} &= m \cdot \vec{a} \\ -mg - bv^2 &= m \cdot a & | \times \frac{1}{m} \\ -g - \frac{b}{m} v^2 &= a \end{aligned}$$

$$-g - \frac{b}{m} v^2 = \frac{dv}{dt}$$

$$\int_{v_0}^{v(t)} \frac{dv}{-g - \frac{b}{m} v^2} = \int_0^t dt$$

Misalkan : $-g - \frac{b}{m} v^2 = u$

$$-\frac{b}{m} dv = du$$

$$dv = -\frac{m}{b} du$$

$$\int \frac{1}{u} dv = \int \frac{1}{u} - \frac{m}{b} \cdot du$$

$$\begin{aligned} \int \frac{1}{u} \left(-\frac{m}{b} du \right) &= -\frac{m}{b} \int \frac{1}{u} du \\ &= -\frac{m}{b} \ln |u| \end{aligned}$$

$$\int_{v_0}^{v(t)} \frac{1}{-g - \frac{b}{m}v^2} dv = \int_0^t dt$$

$$\left. \frac{-m}{bv} \ln |u| \right|_{v_0}^{v(t)} = t$$

$$\frac{-m}{bv} \ln \left[-g - \frac{bv^2}{m} \right] \bigg|_{v_0}^{v(t)} = t$$

$$\frac{-m}{bv} \left(\ln \left| \frac{-g - \frac{bv^2}{m}}{-g - \frac{bv_0^2}{m}} \right| \right) = t$$

$$\frac{-g - \frac{bv(t)}{m}}{-g - \frac{bv_0}{m}} = e^{-\frac{bv^2}{m}}$$

$$-g - \frac{bv(t)}{m} = \left(-g - \frac{bv_0}{m} \right) e^{-\frac{bv^2}{m}}$$

$$v(t) = \left(\frac{-mg}{bv} \right) - \left(\frac{-mg}{bv} - v_0 \right) e^{-\frac{bv(t)}{m}}$$

Karena $f = -w$

$$bv = mg$$

$$V_T = \frac{mg}{bv}$$

$$\text{Maka, } v(t) = V_T - (V_T - v_0) e^{-\frac{bv(t)}{m}}$$

$$\rightarrow a(t) = \frac{dv(t)}{dt}$$

$$= \frac{d}{dt} \left[V_T - (-v_0 + V_T) e^{-\frac{bv(t)}{m}} \right]$$

$$= 0 - (-v_0 + V_T) \left(-\frac{bv}{m} \right) e^{-\frac{bv(t)}{m}}$$

$$= -\frac{bv}{m} (-v_0 + V_T) e^{-\frac{bv(t)}{m}}$$

$$= -\frac{bv}{m} \left(v_0 + \frac{bvV_T}{m} \right) e^{-\frac{bv(t)}{m}}$$

$$\Rightarrow V(t) = \frac{dy(t)}{dt}$$

$$\int_0^{y(t)} dy(t) = \int_0^t V(t) dt$$

$$y(t) = \int_0^t \left(V_T - (-V_0 + V_T) e^{-\frac{bV(t)}{m}} \right) dt$$

$$= V_T t - (-V_0 + V_T) \left(-\frac{m}{bV} \right) e^{-\frac{bV(t)}{m}} \Bigg|_0^t$$

$$= \left[V_T t - (-V_0 + V_T) \left(-\frac{m}{bV} \right) e^{-\frac{bV(t)}{m}} \right] - \left[(-V_0 + V_T) \left(-\frac{m}{bV} \right) \right]$$

$$= V_T t - \left(\frac{m}{bV} \right) - (-V_0 + V_T) \left(e^{-\frac{bV(t)}{m}} \right)$$

$$= V_T t - \frac{m}{bV} (-V_0 + V_T) (1 - e^{-\frac{bV(t)}{m}})$$

$$\Rightarrow y(t) = V_T t - \frac{m}{b} (-V_0 - V_T) (1 - e^{-\frac{bV(t)}{m}})$$

$$\Rightarrow V(t) = V_T - (-V_0 + V_T) e^{-\frac{bV(t)}{m}}$$

$$\Rightarrow a(t) = \left(-g - \frac{bV}{m} V_0 \right) e^{-\frac{bV(t)}{m}}$$

Daproksimasi :

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$e^{-\frac{bV(t)}{m}} = 1 - \left(\frac{bV(t)}{m} \right) + \frac{1}{2} \left(\frac{bV(t)}{m} \right)^2 + \dots$$

$$\Rightarrow y(t) = V_T t - \frac{m}{bV} (-V_0 - V_T) (1 - e^{-\frac{bV(t)}{m}})$$

$$y(t) = V_T t - (-V_0 + V_T) \left(t - \frac{1}{2} \frac{bV}{m} t^2 \right)$$

$$y(t) = V_0 t + \frac{1}{2} \frac{bV}{m} V_0 t^2 + \frac{1}{2} \frac{bV}{m} \left(-\frac{mg}{bV} \right) t^2$$

Untuk $b=0$

$$\text{maka, } y(t) = V_0 t - \frac{1}{2} g t^2$$

$$\begin{aligned}
 \bullet) v(t) &= v_T - (-v_0 + v_T) e^{-\frac{bv(t)}{m}} \\
 &= v_T - (-v_0 + v_T) \left(1 - \left(-\frac{bv(t)}{m}\right)\right) \\
 &= v_0 t + \frac{bv}{m} v_0 t - gt
 \end{aligned}$$

untuk $b=0$

maka $v(t) = v_0 - gt$

$$\bullet) a(t) = \left(-g - \frac{bv}{m} v_0\right) e^{-\frac{bv(t)}{m}}$$

$$\begin{aligned}
 a(t) &= \left(-g - \frac{b}{m} v_0\right) \left(1 - \frac{bv(t)}{m}\right) \\
 &= -g + g \cdot \frac{bv(t)}{m} - \frac{bv}{m} v_0 + \frac{b^2 v^2}{m^2} v_0 t
 \end{aligned}$$

untuk $b=0$

maka, $v(t) = -g$.