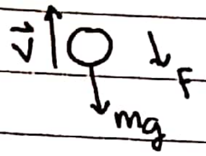
No

Date

Atikkotunnajiah

2013022056

Diskusi gerak vertikal $(F = -bv^2)$



Berdasarkan hukum 2 Newton

$$\Sigma F = m \cdot a$$

$$-mg - bv^2 = m \cdot a$$

$$-g - \frac{bv^2}{m} = a$$

$$-g - \frac{bv^2}{m} = \frac{dv}{dt}$$

$$\int_{v_0}^{v_t} \frac{dv}{-g - \frac{bv^2}{m}} = \int_0^t dt$$

kita misalkan $-g - \frac{b}{m} = u$

$$-\frac{b}{m} dv = du$$

$$dv = \frac{m}{b} du$$

$$\int_{v_0}^{v(t)} \frac{1}{u} dv = \int_0^t dt$$

$$\int_{v_0}^{v(t)} \frac{1}{-g - \frac{bv^2}{m}} dv = \int_0^t dt$$

$$-\frac{m}{b} \ln \left[-g - \frac{bv^2}{m} \right]_{v_0}^{v(t)} = t$$

$$\ln \left| \frac{-g - \frac{bv^2}{m} v_t}{-g - \frac{bv^2}{m} v_0} \right| = -\frac{bv^2}{m}$$

$$\frac{-g - \frac{bv_t}{m}}{-g - \frac{bv_0}{m}} = e^{-\frac{bv^2}{m}}$$

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$$-g - \frac{b}{m} V_t = \left(-g - \frac{b}{m} V_0\right) e^{\frac{-bv}{m}}$$

$$V_t = \frac{-mg}{bv} - \left(\frac{-mg}{bv} - V_0\right) e^{\frac{-bvt}{m}}$$

Because $F = W$, $bv = mg$, $V_T = \frac{mg}{b}$

$$V_t = V_T - (V_T - V_0) e^{\frac{-bVt}{m}}$$

$$= V_T - (-V_0 + V_T) e^{\frac{-bVt}{m}}$$

$$= V_T - (-V_0 + V_T)\left(1 - \frac{bVt}{m}\right)$$

$$= V_0 + \frac{bv}{m} V_0 t - gt$$

* $$a_{(t)} = \frac{dv(t)}{dt}$$

$$= \frac{d}{dt}\left(V_t - (V_0 + V_T)\right) e^{\frac{-bVt}{m}}$$

$$= \frac{bv}{m}(-V_0 + V_T) e^{\frac{-bVt}{m}}$$

$$= -\frac{b}{v}\left(V_0 + \frac{bv}{m} V_T\right) e^{\frac{-bVt}{m}}$$

$$= \left(\frac{bv}{m} - \frac{mg}{bv} - \frac{bv}{m} V_0\right) e^{\frac{-bVt}{m}}$$

$$= \left(-g - \frac{b}{m} V_0\right) e^{\frac{-bVt}{m}}$$

$$= \left(-g - \frac{bv}{m} V_0\right)\left(1 - \frac{bVt}{m}\right)$$

$$= -g + g\frac{bVt}{m} - \frac{bv}{m} V_0 + \frac{b^2 v^2}{m^2} V_0 t$$

* $$V_{(t)} = \frac{dy(t)}{dt}$$

$$\int_{y(0)}^{y(x)} dy(t) = \int_0^t V(t)\, dt$$

$$y(t) = \int_0^t (V_T - (-V_0 + V_T)) e^{\frac{-bVt}{m}}$$

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$$y(t) = \left[V_T - (-V_0 + V_T)\left(-\frac{m}{bV}\right) e^{-\frac{bVt}{m}} \right]_0^t$$

$$= \left[\left(V_T t - (V_0 + V_T\left(-\frac{m}{bV}\right) e^{-\frac{bVt}{m}}\right) - (-V_0 - V_T)\left(-\frac{bm}{bV}\right)\right) \right]$$

$$= V_T t - \left(-\frac{m}{bV}\right)(-V_0 + V_T)\left(e^{-\frac{bVt}{m}}\right)$$

$$= V_T t - \frac{m}{bV}(-V_0 + V_T)\left(1 - e^{-\frac{bVt}{m}}\right)$$

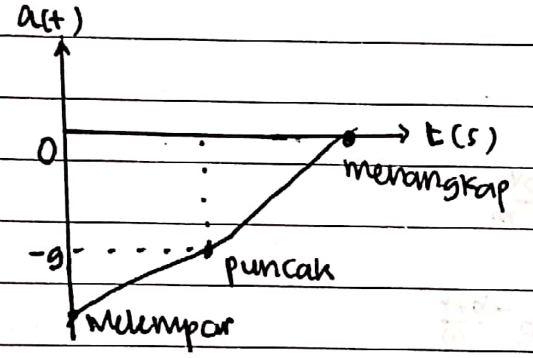
Aproksimasi

$$e^{-\frac{bVt}{m}} = 1 - \frac{bVt}{m} + \frac{1}{2}\left(\frac{bVt}{m}\right)^2 + \dots$$

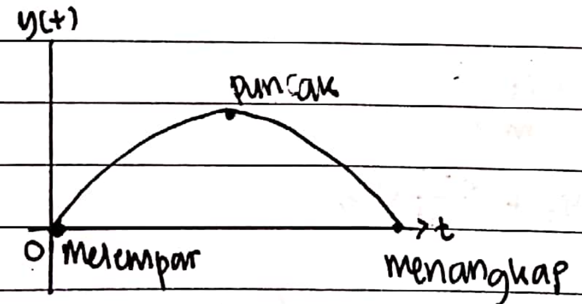
$$y(t) = V_T t - \frac{m}{bV}(-V_0 - V_T)\left(1 - e^{-\frac{bVt}{m}}\right)$$

$$= V_T t - (-V_0 + V_T)\left(t - \frac{1}{2}\frac{bV}{m}t^2\right)$$

$$y(t) = V_0 t + \frac{1}{2}\frac{bV}{m}V_0 t^2 + \frac{1}{2}\frac{bV}{m}\left(-\frac{mg}{bV}\right)t^2$$



(Grafik a(t) - t)



(Grafik y(t) - t)