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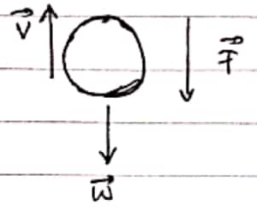
Prodi : Pendidikan Fisika (B)
MK : Mekanika

Diskusi : gerak Vertikal $f = -bv^2$



$$\Sigma F = m.a$$
$$-\vec{w} - \vec{F} = m.a$$
$$-mg(-bv^2) = m.a$$
$$\int_{v_0}^{v_t} \frac{dv}{-g-\frac{bv^2}{m}} = \int_0^t dt$$
$$\int_{v_0}^{v_t} \frac{t}{v} \frac{dv}{-g-\frac{bv}{m}} = \int_0^t dt$$

$$\Rightarrow \quad -g - \frac{bv}{m} = u$$

$$-\frac{b}{m}\, dv = du$$

$$dv = \frac{m}{b}.du$$

$$\int \frac{1}{y}\left(\frac{-m}{b}.du\right) = -\frac{m}{b}\int \frac{1}{u}\, dv = \frac{m}{b}\ln(v)$$

$$\int_{v_0}^{v_t} \frac{1}{-g-\frac{bv^2}{m}}\, dv = \int_0^t dt$$

$$\frac{-m}{bv}\ln(v) = t$$

$$\frac{-m}{bv}\ln\left(-g.\frac{bv^2}{m}\right)\Big]_{v_0}^{v_t} = t$$

$$-\frac{m}{bv}\left(\ln\left[\frac{-g.\frac{bv^2}{m}}{-g.\frac{bv^2}{m}}\right]\right)^{v} = t$$

$$\frac{-g.-\frac{bvt}{m}}{-g-\frac{bv_0}{m}} = e^{-\frac{bv^2}{m}}$$

$$-g-\frac{b}{m}vt = \left(-g-\frac{b}{m}v_0\right)e^{-\frac{bv^2}{m}}$$

$$vt = -\frac{mg}{bv} - \left(-\frac{mg}{bv} - v_0\right)e^{-\frac{bv^2}{m}}$$

$F = -W \;;\; bv = mg \;;\; vt = \frac{mg}{bv}$

$v(t) = v_T - (-v_T - v_0)\, e^{\frac{-bvt}{m}}$

$\Rightarrow a(t) = \frac{dv(t)}{dt}$

$a(t) = \frac{d}{dt}\left(v_T - (-v_0 + v_T)\, e^{\frac{-bvt}{m}}\right)$

$a(t) = 0 - (v_0 + v_T)\left(-\frac{bv}{m}\right) e^{-\frac{bvt}{m}}$

$a(t) = -\frac{bv}{m}\left(v_0 + \frac{bv}{m} \cdot v_T\right) e^{-\frac{bvt}{m}}$

$a(t) = \left(-g \cdot \frac{-b}{m} \cdot v_0\right) e^{-\frac{bvt}{m}}$

$\Rightarrow v(t) = \frac{dy(t)}{dt}$

$\Rightarrow \int_{y_0}^{yx} dy(t) = \int_0^t v(t)\, dt$

$y(t) = \int_0^t v_T - (-v_0 + v_T)\, e^{-\frac{bvt}{m}}$

$y(t) = \left[v_T t - (-v_0 + v_T)\left(-\frac{m}{bv}\right) e^{-\frac{bvt}{m}}\right]$

$y(t) = \left[v_T t - (v_0 - v_T) - \left(\frac{m}{bv}\right) e^{-\frac{bvt}{m}}\right] - \left[(-v_0 - v_T)\left(-\frac{m}{bv}\right)\right]$

$y(t) = v_T t - \left(-\frac{m}{bv}\right)(-v_0 + v_T)\left(e^{-\frac{bvt}{m}}\right)$

$y(t) = v_T t - \frac{m}{bv}(-v_0 + v_T)\left(1 - e^{-\frac{bvt}{m}}\right)$

$y(t) = v_T t - \frac{m}{bm}(-v_0 - v_T)\left(1 - e^{-\frac{bvt}{m}}\right)$

$\Rightarrow v(t) = v_T - (-v_0 + v_t)\, e^{-\frac{bvt}{m}}$

$\Rightarrow a(t) = \left(-g - \frac{bv}{m} \cdot v_0\right) e^{-\frac{bvt}{m}}$

Aproksimasi

$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$

$e^{-\frac{bvt}{m}} = 1 - \frac{bvt}{m} + \frac{1}{2}\left(\frac{bvt}{m}\right)^2 + \dots$

- $y(t) = v_T t - \frac{m}{bv}(-v_0 - v_T)\left(1 - e^{-\frac{bvt}{m}}\right)$

$y(t) = v_0 t + (-v_0 + v_T)\left(t - \frac{1}{2} \cdot \frac{bv}{m} t^2\right)$

$y(t) = v_0 t + \frac{1}{2} \cdot \frac{bv}{m}$

$y(t) = v_0 t^2 + \frac{1}{2} \cdot \frac{bv}{m}\left(-\frac{mg}{bv}\right) t^2$

* Untuk $b = 0$

$y(t) = v_0 t - \frac{1}{2} g t^2$

$\Rightarrow v(t) = v_T - (-v_0 + v_t)\, e^{-\frac{bvt}{m}}$

$v(t) = v_T - (-v_0 + v_T)\left(1 - \left(-\frac{bvt}{m}\right)\right)$

$v(t) = v_0 + \frac{bv}{m} \cdot v_0 t - gt$

* Untuk $b = 0$

$a(t) = \left(-g - \frac{bv}{m} \cdot v_0\right) e^{-\frac{bvt}{m}}$

$a(t) = \left(-g \cdot \frac{-bv}{m} \cdot v_0\right)\left(1 - \frac{bvt}{m}\right)$

$a(t) = -g + g \cdot \frac{bvt}{m} v_0 + \frac{b^2 v^2}{nt^2}$

* Untuk $b = 0$

$v_t = -g$