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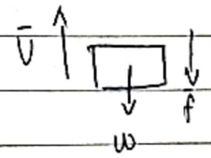
Gerak Vertikal ke atas dengan Hambatan udara $f = -bv^2$



karena $F = -w$

$bv = mg$

$V_T = \frac{mg}{b^v}$

$\Sigma F = m.a$

$-\bar{w}.\bar{F} = m.a$

$-mg - bv^2 = m.a$

$$\int_{v_0}^{v_t} \frac{dv}{-g - \frac{bv^2}{m}} = \int_0^t dt$$

$$\int_{v_0}^{v_t} \frac{f}{\bar{v}} \cdot \frac{dv}{-g - \frac{bv^2}{m}} = \int_0^t dt$$

misal :

$-g - \frac{bv}{m} = u$

$\frac{-b}{m} dv = du$

$dv = \frac{m}{b} du$

$\hookrightarrow \int \frac{1}{u}\left(-\frac{m}{b}\ -du\right) = -\frac{m}{b}\int \frac{1}{u}\ dv = \frac{m}{b}\ln(u)$

$$\int_{v_0}^{v_t} \frac{1}{-g - \frac{bv^2}{m}}\ dv = \int_0^t dt$$

$-\frac{m}{bv}\ln(u) = t$

$$-\frac{m}{bv}\ln\left[-g - \frac{bv^2}{m}\right]_{v_0}^{v_t} = t$$

$$\frac{-m}{bv}\left(\ln\left|\frac{-g \cdot \frac{bv^2}{m}}{-g \cdot \frac{bv^2}{m}}\right|\right)^{v_t} = t$$

$$\frac{-g - \frac{bv_t}{m}}{-g - \frac{bv_0}{m}} = e^{-\frac{bv^2}{m}}$$

$$-g - \frac{b}{m}v_t = \left(-g - \frac{b}{m}v_0\right)e^{-\frac{bv^2}{m}}$$

$$v_t = \left(\frac{-mg}{bv}\right) - \left(\frac{-mg}{bv} - v_0\right)e^{-\frac{bvt}{m}}$$

maka, $v(t) = V_T - (V_T - v_0)\ e^{-\frac{bvt}{m}}$

$a(t) = \frac{dv(t)}{dt}$

$= \frac{d}{dt}\left(v_T(-v_0 + v_T)\right)e^{-\frac{bvt}{m}}$

$= 0 - (-v_0 + v_T)\left(-\frac{bv}{m}\right)e^{-\frac{bvt}{m}}$

$= \frac{-bv}{m}\left(-v_0 + v_T\right)e^{-\frac{bvt}{m}}$

$= -\frac{bv}{m}\left(v_0 + \frac{bv}{m}v_T\right)e^{-\frac{bvt}{m}}$

$a(t) = \frac{dy(t)}{dt}$

$$\int_{y_0}^{y_t} dy(t) = \int_0^t v(t)\ dt$$

$$y(t) = \int_0^t v_T - (-v_0 + v_T)e^{-\frac{bvt}{m}}$$

$= \left[v_T t - (v_0 + v_T)\left(-\frac{m}{bv}\right)e^{-\frac{bvt}{m}}\right]_0^t$

$= \left[v_T t - (-v_0 + v_T)\left(-\frac{m}{bv}\right)e^{-\frac{bvt}{m}}\right]$

$- \left[(-v_0 - v_T)\left(-\frac{m}{bv}\right)\right]$

$= v_T t - \left(\frac{m}{bv}\right)(-v_0 + v_T)\left(e^{-\frac{bvt}{m}}\right)$

$= v_T t - \frac{m}{bv}(-v_0 + v_T)\left(1 - e^{-\frac{bvt}{m}}\right)$

$$\boxed{\begin{aligned} y(t) &= v_T t - \frac{m}{b}(-v_0 - v_T)\left(1 - e^{-\frac{bvt}{m}}\right) \\ v(t) &= v_T - (-v_0 + v_0)\ e^{-\frac{bvt}{m}} \\ a(t) &= \left(-g - \frac{bv}{m}v_0\right)e^{-\frac{bvt}{m}} \end{aligned}}$$

Dipindai dengan CamScanner

Diaproksimasi

$$e^{x} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$e^{-\frac{bvt}{m}} = 1 - \frac{bvt}{m} + \frac{1}{2}\left(\frac{bvt}{m}\right)^2 + \dots$$

$$y(t) = V_T t - \frac{m}{bv}(-V_0 - V_T)\left(1 - e^{-\frac{bvt}{m}}\right)$$

$$y(t) = V_T t - (-V_0 + V_T)\left(t - \frac{1}{2}\frac{bv}{m} t^2\right)$$

$$y(t) = V_0 t + \frac{1}{2} \cdot \frac{bv}{m} \cdot V_0 t^2 + \frac{1}{2} \times \frac{bv}{m}\left(\frac{-mg}{bv}\right) t^2$$

untuk $b = 0$

maka, $\boxed{y(t) = V_0 t - \frac{1}{2} g t^2}$

$$V(t) = V_T - (-V_0 + V_T) e^{-\frac{bvt}{m}}$$

$$= V_T = (-V_0 + V_T)\left(1 - \left(-\frac{bvt}{m}\right)\right)$$

$$= V_0 t + \frac{bv}{m} V_0 t - gt$$

untuk $b = 0$

maka, $\boxed{V(t) = V_0 - gt}$

$$a(t) = \left(-g - \frac{bv}{m} V_0\right) e^{-\frac{bvt}{m}}$$

$$a(t) = \left(-g - \frac{bv}{m} V_0\right)\left(1 - \frac{bvt}{m}\right)$$

$$= -g + g \cdot \frac{bvt}{m} - \frac{bv}{m} V_0 + \frac{b^2 v^2}{m^2} V_0 t$$

untuk $b = 0$

maka, $\boxed{U(t) = -g}$

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