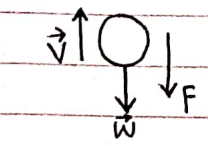
Nama : Pita Nadia
NPM : 2013022008
Kelas : B

Gerak Vertikal keatas dibawah pengaruh gaya gesekan kuadratik $f = -bv^2$



Berdasarkan hukum 2 newton

$\Sigma F = m.a$

$\vec{w} + \vec{F} = -m.a \longrightarrow -\vec{w} - \vec{F} = m.a$

$-mg - bv^2 = ma \longrightarrow$ dikali $\frac{1}{m}$

$-g - \frac{bv^2}{m} = a$

$-g - \frac{bv^2}{m} = \frac{dv}{dt}$

$\int_{V_0}^{V_t} \frac{dv}{-g - \frac{bv^2}{m}} = \int_0^t dt$

misalkan $-g - \frac{bv}{m} = u$

$-\frac{b}{m} dv = du$

$dv = \frac{m}{b} du$

$\int_{V_0}^{V(t)} \frac{1}{u} dv = \int_0^t dt$

$\int_{V_0}^{V(t)} \frac{1}{-g - \frac{bv^2}{m}} dv = \int_0^t dt$

$-\frac{m}{bv} \ln \left[-g - \frac{bv^2}{m} \right]_{V_0}^{V(t)} = t$

$\ln \left| \frac{-g - \frac{bv^2}{m} V_t}{-g - \frac{bv^2}{m} V_0} \right| = \frac{-bv^2}{m}$

$\frac{-g - \frac{bV_t}{m}}{-g - \frac{bV_0}{m}} = e^{-\frac{bv^2}{m}}$

$-g - \frac{b}{m} V_t = \left(-g - \frac{b}{m} V_0\right) e^{-\frac{bv^2}{m}}$

$V_t = \frac{-mg}{bv} - \left(\frac{-mg}{bv} - V_0\right) e^{-\frac{bV_t}{m}}$

karena $F = w$, $bv = mg$, $V_T = \frac{mg}{b}$

$$\boxed{V(t) = V_T - (V_T - V_0) e^{-\frac{bV_t}{m}}}$$

$V(t) = V_T - (-V_0 + V_T) e^{-\frac{bV_t}{m}}$

$V(t) = V_T - (-V_0 + V_T)\left(1 - \frac{bV_t}{m}\right)$

$$\boxed{V(t) = V_0 + \frac{bv}{m} V_0 t - gt}$$

Untuk $b = 0$ (tanpa gesekan udara)

$V(t) = V_0 - gt$

$a(t) = \frac{dV(t)}{dt}$

$= \frac{d}{dt}\left(V_T - (V_0 + V_T)\right) e^{-\frac{bV_t}{m}}$

$= \frac{bv}{m}\left(-V_0 + V_T\right) e^{-\frac{bV_t}{m}}$

$= -\frac{b}{v}\left(V_0 + \frac{bv}{m} V_T\right) e^{-\frac{bV_t}{m}}$

$= \left(\frac{bv}{m} - \frac{mg}{bv} - \frac{bv}{m} V_0\right) e^{-\frac{bV_t}{m}}$

$$\boxed{a(t) = \left(-g - \frac{b}{m} V_0\right) e^{-\frac{bV_t}{m}}}$$

$a(t) = \left(-g - \frac{bv}{m} V_0\right) e^{-\frac{bV_t}{m}}$

$a(t) = \left(-g - \frac{bv}{m} V_0\right)\left(1 - \frac{bV_t}{m}\right)$

$$\boxed{a(t) = -g + g\frac{bV_t}{m} - \frac{bv}{m} V_0 + \frac{b^2 v^2}{m^2} V_0 t}$$

Jika tanpa gesekan udara ($b = 0$)

$a(t) = -g$

$V(t) = \frac{dy(t)}{dt}$

$\int_{y(0)}^{y(x)} dy(t) = \int_0^t V(t)\, dt$

$y(t) = \int_0^t \left(V_T - (-V_0 + V_T) e^{-\frac{bV_t}{m}}\right)$

$y(t) = \left[V_T - (-V_0 + V_T)\left(-\frac{m}{bv}\right) e^{-\frac{bV_t}{m}}\right]_0^t$

$y(t) = \left[\left(V_{Tt} - (V_0 + V_T)\left(\frac{m}{bv}\right) e^{-\frac{bV_t}{m}}\right) - \left((-V_0 - V_T)\left(-\frac{m}{bv}\right)\right)\right]$

$y(t) = V_{Tt} - \left(-\frac{m}{bv}\right)(-V_0 + V_T)\left(e^{-\frac{bV_t}{m}}\right)$

$$\boxed{y(t) = V_{Tt} - \frac{m}{bv}\left(-V_0 + V_T\right)\left(1 - e^{-\frac{bV_t}{m}}\right)}$$

Aproksimasi

$$e^{-\frac{bV_t}{m}} = 1 - \frac{bV_t}{m} + \frac{1}{2}\left(\frac{bV_t}{m}\right)^2 + \dots$$

$$y(t) = V_T t - \frac{m}{bv}\left(-V_0 - V_T\right)\left(1 - e^{-\frac{bV_t}{m}}\right)$$

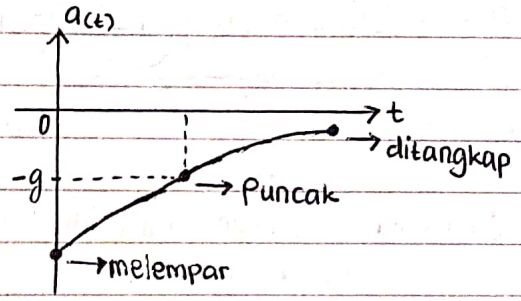
$$y(t) = V_T t - \left(-V_0 + V_T\right)\left(t - \frac{1}{2}\frac{bv}{m}t^2\right)$$

$$\boxed{y(t) = V_0 t + \frac{1}{2}\frac{bv}{m}V_0 t^2 + \frac{1}{2}\frac{bv}{m}\left(-\frac{mg}{bv}\right)t^2}$$

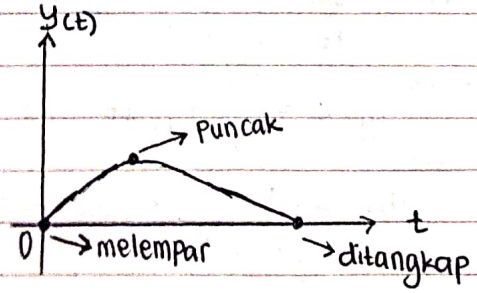
Jika tanpa gesekan udara ($b = 0$)

$$\boxed{y(t) = V_0 t - \frac{1}{2}gt^2}$$

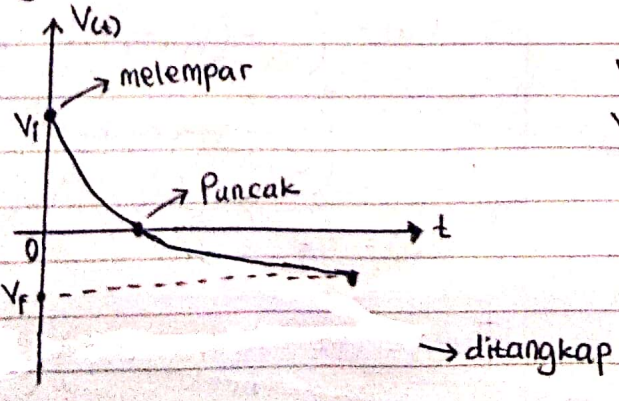
Grafik fungsi $a(t)$ terhadap waktu



Grafik fungsi $y(t)$ terhadap waktu



Grafik fungsi $V(t)$ terhadap waktu



V_i = kecepatan awal

V_f = kecepatan saat ditangkap.