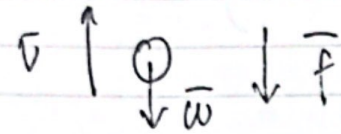
Nama = Fadiyah Farah Khairunnisaa
Npm = 2013022016
Prodi = Pendidikan Fisika
Kelas = B

Gerak vertikal keatas di bawah pengaruh gaya gesekan



$$\Sigma f = m.\vec{a}$$
$$-\vec{w} - \vec{f} = m.\vec{a}$$
$$-mg - bv^2 = m.a$$
$$-g - \frac{b}{m}v^2 = a$$
$$-g - \frac{b}{m}v^2 = \frac{dv}{dt}$$

$$\int_{v_0}^{v(t)} \frac{dv}{-g - b/m v^2} = \int_0^t dt$$

Misal : $-g - \frac{b}{m}v^2 = u$

$$-\frac{b}{m}\,dv = du$$
$$dv = -\frac{m}{b}\,du$$

$$\int \frac{1}{u}\,dv = \int \frac{1}{u} - \frac{m}{b}.du$$
$$\int \frac{1}{u}\left(\frac{-m}{b}\,dv\right) = -\frac{m}{b}\int \frac{1}{u}\,du$$
$$= -\frac{m}{b}\ln|u|$$

$$\int_{v_0}^{v(t)} \frac{1}{-g - \frac{b}{m}v^2}\,dv = \int_0^t dt$$
$$\frac{-m}{bv}\ln|u|\Big|_{v_0}^{v(t)} = t$$
$$-\frac{m}{bv}\ln\left[-g - \frac{bv^2}{m}\right]_{v_0}^{v(t)} = t$$
$$-\frac{m}{bv}\left[\ln\left[\frac{-g - bv^2/m}{-g - bv^2/m}\right]\right]^{v} = t$$

$$\frac{-g-\frac{bv}{m}(t)}{-g-\frac{bv_0}{m}} = e^{-\frac{bv}{m}t}$$

$$-g-\frac{b}{m}v(t) = \left(-g-\frac{b}{m}v_0\right)e^{-\frac{bv}{m}t}$$

$$v(t) = \left(\frac{-mg}{bv}\right)-\left(\frac{-mg}{bv}-v_0\right)e^{-\frac{bv(t)}{m}}$$

Karena
$f = -w$
$bv = mg$
$v_T = \frac{mg}{bv}$

maka, $v(t) = v_T-(v_T-v_0)e^{-\frac{bv(t)}{m}}$

- $a(t) = \frac{dv(t)}{dt}$

$$= \frac{d}{dt}\left(v_T-(-v_0+v_T)e^{-\frac{bv(t)}{m}}\right)$$

$$= 0-(-v_0+v_T)\left(-\frac{bv}{m}\right)e^{-\frac{bv(t)}{m}}$$

$$= -\frac{bv}{m}(-v_0+v_T)e^{-\frac{bv(t)}{m}}$$

$$= \frac{-bv}{m}\left(v_0+\frac{bv}{m}v_T\right)e^{-\frac{bv(t)}{m}}$$

- $v(t) = \frac{dy(t)}{dt}$

$$\int_0^{y(t)} dy(t) = \int_0^t v(t)\,dt$$

$$y(t) = \int_0^t \left(v_T-(-v_0+v_T)e^{-\frac{bv(t)}{m}}\right)dt$$

$$= v_Tt-(-v_0+v_T)\left(-\frac{m}{bv}\right)e^{-\frac{bv(t)}{m}}\Big|_0^t$$

$$= \left[v_Tt-(-v_0+v_T)\left(\frac{-m}{bv}\right)e^{-\frac{bv(t)}{m}}\right]-\left[(-v_0+v_T)\left(\frac{-m}{bv}\right)\right]$$

$$= v_T t - \left(\frac{m}{bv}\right) - (v_0 + v_T)\left(e^{\frac{-bv(t)}{m}}\right)$$
$$= v_T t - \frac{m}{bv}(-v_0 + v_T)\left(1 - e^{\frac{-bv(t)}{m}}\right)$$

- $y(t) = v_T t - \frac{m}{b}(-v_0 - v_T)\left(1 - e^{\frac{-bv(t)}{m}}\right)$
- $v(t) = v_t = (-v_0 + v_T)\, e^{\frac{-bv(t)}{m}}$
- $a(t) = \left(-g - \frac{bv}{m} v_0\right) e^{\frac{-bv(t)}{m}}$

Diaproksimasi :

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$
$$e^{\frac{-bv(t)}{m}} = 1 - \left(\frac{bv(t)}{m}\right) + \frac{1}{2}\left(\frac{bv(t)}{m}\right)^2 + \dots$$

- $y(t) = v_T t - \frac{m}{bv}(-v_0 - v_T)\left(1 - e^{\frac{-bv(t)}{m}}\right)$

$$y(t) = v_T t - (-v_0 + v_T)\left(t - \frac{1}{2}\frac{bv}{m}t^2\right)$$
$$y(t) = v_0 t + \frac{1}{2}\frac{bv}{m} v_0 t^2 + \frac{1}{2}\frac{bv}{m}\left(\frac{-mg}{bv}\right) t^2$$

Untuk $b = 0$ maka, $y(t) = v_0 t = \frac{1}{2} g t^2$

$$v(t) = v_T - (-v_0 + v_T)\, e^{\frac{-bv(t)}{m}}$$
$$= v_T - (-v_0 + v_T)\left(1 - \left(-\frac{bv(t)}{m}\right)\right)$$
$$= v_0 t + \frac{bv}{m} v_0 t - g t$$

Untuk $b = 0$ maka, $v(t) = v_0 - g t$

$$a(t) = \left(-g - \frac{bv}{m} v_0\right) e^{\frac{-bv(t)}{m}}$$
$$a(t) = \left(-g - \frac{b}{m} v_0\right)\left(1 - \frac{bv(t)}{m}\right)$$
$$= -g + g \cdot \frac{bv(t)}{m} - \frac{bv}{m} v_0 + \frac{b^2 v^2}{m^2} v_0 t$$

Untuk $b = 0$
maka, $v(t) = -g$.