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Prodi : Pendidikan Fisika

Kelas : B

Materi : Mekanika

Diskusi Gerak vertikal ke atas yang dipengaruhi hambatan udara

$$\vec{v} \uparrow \quad \text{O} \quad \vec{v}_0 \downarrow \quad \vec{w} \downarrow \quad \vec{F} \downarrow \quad F = -b\vec{v}$$

• Berdasarkan H. II Newton

$$\sum \vec{F} = m \cdot \vec{a}$$

$$-\vec{w} - \vec{F} = m \cdot \vec{a}$$

$$-mg - bv = m \cdot a \quad | \times \frac{1}{m}$$

$$-g - \frac{b}{m} v = a$$

$$-g - \frac{b}{m} v = \frac{dv}{dt}$$

$$\int_{v_0}^{v(t)} \frac{dv}{-g - \frac{b}{m} v} = \int_0^t dt$$

$$\text{misal: } -g - \frac{b}{m} v = u$$

$$-\frac{b}{m} dv = du$$

$$dv = -\frac{m}{b} du$$

$$\int \frac{1}{u} du = \int \frac{1}{u} - \frac{m}{b} du$$

$$\int \frac{1}{u} \left(-\frac{m}{b} du \right) = -\frac{m}{b} \int \frac{1}{u} du$$

$$= -\frac{m}{b} \ln |u|$$

$$\int_{v_0}^{v(t)} \frac{1}{-g - \frac{b}{m}v} dv = \int_0^t dt$$

$$-\frac{m}{b} \ln |u| \Big|_{v_0}^{v(t)} = t$$

$$-\frac{m}{b} \ln \left| -g - \frac{b}{m}v \right| \Big|_{v_0}^{v(t)} = t$$

$$-\frac{m}{b} \left(\ln \left| -g - \frac{b}{m}v(t) \right| - \ln \left| -g - \frac{b}{m}v_0 \right| \right) = t$$

$$\boxed{\ln a - \ln b = \ln \frac{a}{b}}$$

$$\ln \left| \frac{-g - \frac{b}{m}v(t)}{-g - \frac{b}{m}v_0} \right| = -\frac{b}{m}t$$

$$\exp \left(\ln \left| \frac{-g - \frac{b}{m}v(t)}{-g - \frac{b}{m}v_0} \right| \right) = e^{-\frac{b}{m}t}$$

$$\boxed{e^{\ln x} = x}$$

$$\frac{-g - \frac{b}{m}v(t)}{-g - \frac{b}{m}v_0} = e^{-\frac{b}{m}t}$$

$$-g - \frac{b}{m}v(t) = \left(-g - \frac{b}{m}v_0 \right) e^{-\frac{b}{m}t}$$

$$-\frac{b}{m}v(t) = g + \left(-g - \frac{b}{m}v_0 \right) e^{-\frac{b}{m}t} \quad | \times -\frac{m}{b}$$

$$v(t) = -\frac{m}{b}g - \frac{m}{b} \left(-g - \frac{b}{m}v_0 \right) e^{-\frac{b}{m}t}$$

$$v(t) = \left(\frac{m}{b}g + v_0 \right) e^{-\frac{b}{m}t} - \frac{m}{b}g$$

• karena, $f = -w$

$$f = -mg$$

$$bv = -mg$$

$$v_T = -\frac{mg}{b}$$

$$v(t) = (-v_T + v_0) e^{-\frac{b}{m}t} + v_T$$

$$\boxed{v(t) = (v_0 - v_T) e^{-\frac{b}{m}t} + v_T}$$

→ Pers $v(t)$ sebelum Aproksimasi

> Persamaan $a(t)$:

$$a(t) = \frac{dv(t)}{dt}$$

$$= \frac{d}{dt} ((v_0 - v_T) e^{-\frac{b}{m}t} + v_T)$$

$$= -\frac{b}{m} \cdot (v_0 - v_T) e^{-\frac{b}{m}t} + 0$$

$$= \left(\frac{b}{m} v_T - \frac{b}{m} v_0 \right) e^{-\frac{b}{m}t}$$

$$= \left(\frac{b}{m} \left(-\frac{mg}{b} \right) - \frac{b}{m} v_0 \right) e^{-\frac{b}{m}t}$$

$$\boxed{a(t) = \left(-g - \frac{b}{m} v_0 \right) e^{-\frac{b}{m}t}}$$

→ Pers $a(t)$ belum aproksimasi

→ Pers $y(t)$:

$$v(t) = \frac{dy(t)}{dt}$$

$$\int_0^t dy(t) = \int_0^t v(t) dt$$

$$y(t) = \int_0^t ((v_0 - v_T) e^{-\frac{b}{m}t} + v_T) dt$$

$$y(t) = v_T t - \frac{m}{b} (v_0 - v_T) e^{-\frac{b}{m}t} \Big|_0^t$$

$$y(t) = \left[v_T t - \frac{m}{b} (v_0 - v_T) e^{-\frac{b}{m}t} \right] - \left[-\frac{m}{b} (v_0 - v_T) \right]$$

$$\boxed{y(t) = v_T t + \frac{m}{b} (v_0 - v_T) (1 - e^{-\frac{b}{m}t})}$$

→ Pers $y(t)$ sebelum Aproksimasi

Aproksimasi

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$e^{-\frac{bt}{m}} = 1 + \left(\frac{bt}{m}\right) + \frac{1}{2} \left(\frac{bt}{m}\right)^2 + \dots$$

$$\bullet v(t) = (v_0 - v_T) e^{-\frac{bt}{m}} + v_T$$

$$v(t) = v_T + (v_0 - v_T) \left(1 - \frac{bt}{m}\right)$$

$$v(t) = \cancel{v_T} + v_0 - v_0 \frac{bt}{m} = \cancel{v_T} + \frac{b}{m} v_T t$$

$$v(t) = v_0 - v_0 t \frac{b}{m} + \frac{1}{m} \left(-\frac{mg}{b}\right) t$$

$$v(t) = v_0 - gt - v_0 t \frac{b}{m}$$

$$\boxed{v(t) = v_0 - gt - \frac{b}{m} v_0 t} \quad \text{Persu(t) gerak Vertikal ke atas dg Pengaruh gesekan udara}$$

Jika mensubstitusikan $b=0$ maka akan didapat Pers 1

$$v(t) = v_0 - gt \quad \text{Crumur gerak vertikal ke atas}$$

$$\bullet a(t) = \left(-g - \frac{b}{m} v_0\right) e^{-\frac{bt}{m}}$$

$$a(t) = \left(-g - \frac{b}{m} v_0\right) \left(1 - \frac{bt}{m}\right)$$

$$a(t) = -g + \frac{b}{m} gt - \frac{b}{m} v_0 + \frac{b^2}{m^2} v_0 t$$

$$a(t) = -g - \frac{b}{m} v_0 + \left(\frac{b^2}{m^2} v_0 + \frac{b}{m} g\right) t$$

$$\boxed{a(t) = -g - \frac{b}{m} v_0 + \left(\frac{b^2}{m^2} v_0 + \frac{b}{m} g\right) t}$$

↳ Pers $a(t)$ gerak Vertikal ke atas dg Pengaruh gaya gesek udara.

Jika mensubstitusikan $b=0$ maka akan didapat

$$a(t) = -g \quad \text{Crumur gerak vertikal ke atas}$$

$$y(t) = V_T t + \frac{m}{b} (V_0 - V_T) (1 - e^{-\frac{bt}{m}})$$

$$y(t) = V_T t + \frac{m}{b} (V_0 - V_T) (1 - 1 - \frac{bt}{m} + \frac{1}{2} (\frac{bt}{m})^2)$$

$$y(t) = V_T t + \frac{m}{b} (V_0 - V_T) (1 - 1 - \frac{bt}{m} + \frac{1}{2} (\frac{bt}{m})^2)$$

$$y(t) = V_T t + \frac{m}{b} (V_0 - V_T) (\frac{bt}{m} - \frac{1}{2} (\frac{bt}{m})^2)$$

$$y(t) = V_T t + (V_0 - V_T) (t - \frac{1}{2} \frac{b}{m} t^2)$$

$$y(t) = \cancel{V_T t} + V_0 t - \cancel{V_T t} - \frac{1}{2} \frac{b}{m} V_0 t^2 + \frac{1}{2} \frac{b}{m} V_T t^2$$

$$y(t) = V_0 t - \frac{1}{2} \frac{b}{m} V_0 t^2 + \frac{1}{2} \frac{b}{m} (-\frac{mg}{b}) t^2$$

$$y(t) = V_0 t - \frac{1}{2} g t^2 - \frac{1}{2} \frac{b}{m} V_0 t^2$$

$$y(t) = V_0 t - \frac{1}{2} g t^2 - \frac{1}{2} \frac{b}{m} V_0 t^2$$

↳ Pers $y(t)$ untuk gerak vertikal ke atas dg pengaruh gaya gesek udara.

Jika mensubstitusikan $b = 0$ maka akan didapat:

$$y(t) = V_0 t - \frac{1}{2} g t^2 \text{ (rumus gerak vertikal ke atas)}$$