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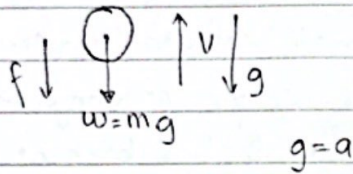
"Gerak Vertikal yang dipengaruhi Hambatan Udara (Air Resistance)"

Gaya gesekan pada hambatan udara,  $f = -bv$

posisi,  $y(t)$

kecepatan,  $v(t)$

percepatan,  $a(t)$



Berdasarkan Hukum II Newton

$$\sum \vec{F} = m\vec{a}$$

$$-W - f = m a$$

$$-mg - bv = m a$$

$$-g - \frac{b}{m} v = a$$

$$-g - \frac{b}{m} v = \frac{dv}{dt}$$

$$\int_{v_0}^{v(t)} \frac{dv}{-g - \frac{b}{m} v} = \int_0^t dt \quad \dots 1)$$

$$\text{Misal} = -g - \frac{b}{m} v = u$$

$$-\frac{b}{m} dv = du$$

$$dv = -\frac{m}{b} du \quad \dots 2)$$

Subst.  $dv = -\frac{m}{b} du$  ke pers. 1) dan  $u$  ke pers (2)

$$\int_{v_0}^{v(t)} \frac{1}{u} dv = \int_{v_0}^{v(t)} \frac{1}{u} -\frac{m}{b} du$$

$$= -\frac{m}{b} \int_{v_0}^{v(t)} \frac{1}{u} du$$

$$= -\frac{m}{b} \ln u \quad \dots 2)$$

$$\text{Subst.} \int_0^{v(t)} \frac{dv}{u} = -\frac{m}{b} \ln u \text{ ke pers. 1)}$$

$$\int_{v_0}^{v(t)} \frac{dv}{(-g - \frac{b}{m} v)} = \int_0^t dt$$



$$\begin{aligned} \frac{-m}{b} \ln \left[ \frac{-g - \frac{b}{m} v}{-g - \frac{b}{m} v_0} \right] \Big|_{v_0}^{v_t} &= t \\ \frac{-m}{b} \ln \left( \frac{-g - \frac{b}{m} v}{-g - \frac{b}{m} v_0} \right) \Big|_{v_0}^{v_t} &= t \\ \frac{-m}{b} \left[ \ln \left( \frac{-g - \frac{b}{m} v_t}{-g - \frac{b}{m} v_0} \right) - \ln \left( \frac{-g - \frac{b}{m} v_0}{-g - \frac{b}{m} v_0} \right) \right] &= t \end{aligned}$$

sifat logaritma

$$\ln a - \ln b = \ln \frac{a}{b}$$

$$\ln \left( \frac{-g - \frac{b}{m} v_t}{-g - \frac{b}{m} v_0} \right) = \frac{-b}{m} t$$

$$\left( \ln \left| \frac{-g - \frac{b}{m} v_t}{-g - \frac{b}{m} v_0} \right| \right) = \frac{-b}{m} t$$

$$\frac{-g - \frac{b}{m} v_t}{-g - \frac{b}{m} v_0} = e^{-\frac{bt}{m}}$$

$$-g - \frac{b}{m} v_t = \left( -g - \frac{b}{m} v_0 \right) e^{-\frac{bt}{m}}$$

$$\frac{-b}{m} v_t = g + \left( g - \frac{b}{m} v_0 \right) e^{-\frac{bt}{m}}$$

$$v_t = \left( \frac{mg}{b} + v_0 \right) e^{-\frac{bt}{m}} - \frac{mg}{b}$$

Karena  $f = -w$ 

$$f = -mg$$

$$b v = -mg$$

$$v_t = -\frac{mg}{b}$$

$$v(t) = \left( -v_t - v_0 \right) e^{-\frac{bt}{m}} + v_t$$

$$v(t) = \left( v_0 - v_t \right) e^{-\frac{bt}{m}} + v_t$$

sebelum aproksimasi

Persamaan  $dt$ :

$$a(t) = \frac{dv(t)}{dt}$$

$$= \frac{d}{dt} \left( v_0 - v_t \right) e^{-\frac{bt}{m}} + v_t$$

$$= -\frac{b}{m} \left( v_0 - v_t \right) e^{-\frac{bt}{m}} + 0$$

$$= \left( \frac{b}{m} v_t - \frac{b}{m} v_0 \right) e^{-\frac{bt}{m}}$$

$$= \left( \frac{b}{m} \left( \frac{-mg}{-b} \right) - \frac{b}{m} v_0 \right) e^{-\frac{bt}{m}}$$

$$a(t) = \left( -g - \frac{b}{m} v_0 \right) e^{-\frac{bt}{m}}$$

persamaan  $a(t)$  sebelum aproksimasiPersamaan  $y(t)$ :

$$v(t) = \frac{dy(t)}{dt}$$

$$\int_0^{y(t)} dy(t) = \int_0^t v(t) dt$$

$$y(t) = \int_0^t \left( v_0 - v_t \right) e^{-\frac{bt}{m}} + v_t dt$$

$$= v_t t - \frac{m}{b} \left( v_0 - v_t \right) e^{-\frac{bt}{m}} \Big|_0^t$$

$$y(t) = \left( v_t t - \frac{m}{b} \left( v_0 - v_t \right) e^{-\frac{bt}{m}} \right) - \left( -\frac{m}{b} \left( v_0 - v_t \right) \right)$$

$$y(t) = v_t t + \frac{m}{b} \left( v_0 - v_t \right) \left( 1 - e^{-\frac{bt}{m}} \right)$$

persamaan  $y(t)$  sebelum aproksimasi

Tahap Aproksimasi

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$\text{Maka } e^{-\frac{bt}{m}} = 1 + \left( -\frac{bt}{m} \right) + \frac{1}{2} \left( \frac{bt}{m} \right)^2 + \dots$$

$$\text{Subst. pada pers. } v(t) = \left( v_0 - v_t \right) e^{-\frac{bt}{m}} + v_t$$

$$v_t = v_t + \left( v_0 - v_t \right) \left( 1 - \frac{bt}{m} \right)$$

$$= v_t = v_0 - v_0 \frac{b}{m} t + v_t + \frac{b}{m} v_t t$$

$$= v_0 - v_0 t \frac{b}{m} + \frac{b}{m} \left( \frac{-mg}{b} \right) t$$

$$= v_0 - g t - v_0 t \frac{b}{m}$$

$$v(t) = v_0 - g t - \frac{b}{m} v_0 t$$

pers.  $v(t)$  gerak vertikal dengan gesekan udara



Jika kita substitusikan  $b=0$  maka akan diperoleh persamaan

$$V(t) = V_0 - gt$$

Subst.  $e^{-\frac{bt}{m}} = 1 + \left(-\frac{bt}{m}\right) + \frac{1}{2} \left(\frac{bt}{m}\right)^2$

$$y(t) = V_T t + \frac{m}{b} (V_0 - V_T) \left(1 - e^{-\frac{bt}{m}}\right)$$

$$y(t) = V_T t + \frac{m}{b} (V_0 - V_T) \left(1 - \left(1 - \frac{bt}{m} + \frac{1}{2} \left(\frac{bt}{m}\right)^2\right)\right)$$

$$= V_T t + \frac{m}{b} (V_0 - V_T) \left(\frac{bt}{m} - \frac{1}{2} \left(\frac{bt}{m}\right)^2\right)$$

$$y(t) = V_T t + (V_0 - V_T) \left(t - \frac{1}{2} \frac{b}{m} t^2\right)$$

$$= V_T t + V_0 t - V_T t - \frac{1}{2} \frac{b}{m} V_0 t^2 + \frac{1}{2} \frac{b}{m} V_T t^2$$

$$= V_0 t - \frac{1}{2} \frac{b}{m} V_0 t^2 + \frac{1}{2} \frac{b}{m} V_T t^2$$

$$= V_0 t - \frac{1}{2} g t^2 - \frac{1}{2} \frac{b}{m} V_0 t^2$$

Jadi persamaan  $y(t)$  untuk gerak vertikal ke atas dengan pengaruh gaya gesek udara adalah

$$y(t) = V_0 t - \frac{1}{2} g t^2 - \frac{1}{2} \frac{b}{m} V_0 t^2$$

Jika di substitusikan  $b=0$  maka akan diperoleh

$$y(t) = V_0 t - \frac{1}{2} g t^2 \text{ (gerak vertikal ke atas)}$$

Substitusikan  $e^{-\frac{bt}{m}} = 1 - \frac{bt}{m}$  pada pers.

$$a(t) = \left(-g - \frac{b}{m} V_0\right) e^{-\frac{bt}{m}}$$

$$a(t) = \left(-g - \frac{b}{m} V_0\right) \left(1 - \frac{bt}{m}\right)$$

$$a(t) = -g + \frac{b}{m} g t - \frac{b}{m} V_0 + \frac{b^2}{m^2} V_0 t$$

$$a(t) = -g - \frac{b}{m} V_0 + \left(\frac{b^2}{m^2} V_0 + \frac{b}{m} g\right) t$$

Jadi, persamaan  $a(t)$  untuk gerak vertikal ke atas dengan pengaruh gaya gesek udara :

$$a(t) = -g - \frac{b}{m} V_0 + \left(\frac{b^2}{m^2} V_0 + \frac{b}{m} g\right) t$$

Jika substitusi  $b=0$  maka akan diperoleh  $a(t) = -g$  sama seperti gerak vertikal ke atas.

**Kesimpulan :**

Rumus  $y(t)$  gerak vertikal ke atas dipengaruhi gaya gesekan udara :

$$y(t) = V_0 t - \frac{1}{2} g t^2 - \frac{1}{2} \frac{b}{m} V_0 t^2$$

Rumus  $V(t)$  gerak vertikal ke atas dipengaruhi gaya gesekan udara :

$$V(t) = V_0 - g t - \frac{b}{m} V_0 t$$

Rumus  $a(t)$  gerak vertikal ke atas dipengaruhi gaya gesekan udara :

$$a(t) = -g - \frac{b}{m} V_0 + \left(\frac{b^2}{m^2} V_0 + \frac{b}{m} g\right) t$$

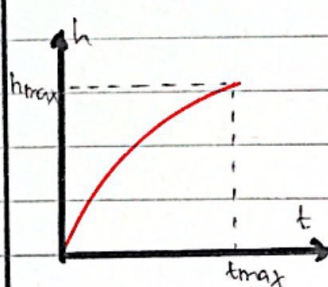


Gráfico y-t

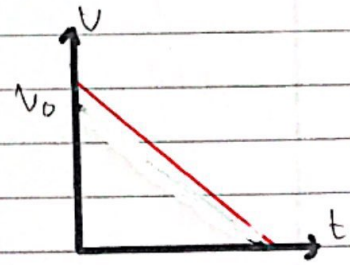


Gráfico v-t

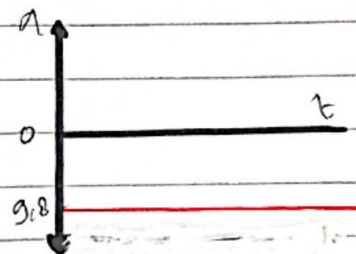


Gráfico a-t