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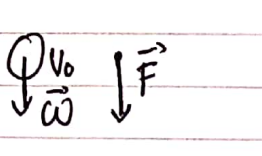
Kelas : B

Diskusi gerak vertikal keatas yang dipengaruhi Hambatan Udara



f = gaya gesek udara
$f = -bv$

Berdasarkan Hukum II Newton

$$\Sigma f = m \cdot \vec{a}$$
$$-\vec{w} - \vec{f} = m \cdot \vec{a}$$
$$-mg - bv = m \cdot a \quad | \times \frac{1}{m}$$
$$-g - \frac{b}{m} v = \frac{dv}{dt}$$

$$\int_{v_0}^{v(t)} \frac{dv}{-g - \frac{b}{m}v} = \int_0^t dt \quad \ldots (1)$$

misalkan : $-g - \frac{b}{m} v = u$

$$-\frac{b}{m} dv = du$$
$$dv = -\frac{m}{b} du$$

} substitusikan persamaan 1

$$\int \frac{1}{u} dv = -\frac{m}{b} \int \frac{1}{u} du$$
$$\int \frac{1}{u} \left(-\frac{m}{b} du\right) = -\frac{m}{b} \int \frac{1}{u} dv$$
$$\int \frac{1}{-g - \frac{b}{m} v} dv = -\frac{m}{b} \ln (u)$$

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$$\int_{V_0}^{V(t)} \frac{1}{-g-\frac{b}{m}v}\,dv = \int_0^t dt$$

$$-\frac{m}{b}\ln[u]\Big|_{V_0}^{V(t)} = t$$

$$-\frac{m}{b}\ln\left|-g-\frac{b}{m}v\right|\Big|_{V_0}^{V(t)} = t$$

$$-\frac{m}{b}\left(\ln\left|-g-\frac{b}{m}V(t)\right| - \ln\left|-g-\frac{b}{m}V_0\right|\right) = t$$

$$\ln a - \ln b = \ln\frac{a}{b}$$

$$\ln\left|\frac{-g-\frac{b}{m}V(t)}{-g-\frac{b}{m}V_0}\right| = -\frac{b}{m}t$$

$$\exp\left(\ln\left|\frac{-g-\frac{b}{m}V(t)}{-g-\frac{b}{m}V_0}\right|\right) = e^{-\frac{b}{m}t}$$

$$e^{\ln u} = u \rightsquigarrow u = \frac{-g-\frac{b}{m}V(t)}{-g-\frac{b}{m}V_0}$$

$$\frac{-g-\frac{b}{m}V(t)}{-g-\frac{b}{m}V_0} = e^{-\frac{b}{m}t}$$

$$-g-\frac{b}{m}V(t) = e^{-\frac{b}{m}t}\left(-g-\frac{b}{m}V_0\right)$$

$$-\frac{b}{m}V(t) = e^{-\frac{b}{m}t}\left(-g-\frac{b}{m}V_0\right)+g \quad \Big| \times -\frac{m}{b}$$

$$V(t) = -\frac{m}{b}\left(-g-\frac{b}{m}V_0\right)e^{-\frac{b}{m}t} - \frac{m}{b}g$$

$$V(t) = \left(\frac{m}{b}g + V_0\right)e^{-\frac{b}{m}t} - \frac{m}{b}g.$$

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Karena $f = -w$

$f = -mg$

$bv = -mg$

$v_f = \frac{-mg}{b}$

⇒ persamaan v_t dapat ditulis

$$v(t) = (-v_t + v_0)\, e^{-\frac{bt}{m}} + v_f$$

$$\boxed{v(t) = (v_0 - v_T)\, e^{-\frac{bt}{m}} + v_T}$$ ⇝ persamaan $v(t)$ sebelum aproksimasi

⇒ Persamaan $a(t)$ dapat dicari dg

$$a(t) = \frac{dv(t)}{dt}$$

$$= \frac{d}{dt}(v_0 - v_T)\, e^{-\frac{bt}{m}} + v_T$$

$$= -\frac{b}{m} \cdot (v_0 - v_t)\, e^{-\frac{bt}{m}} + 0$$

$$= \left(\frac{b}{m} v_T - \frac{b}{m} \cdot v_0\right) e^{-\frac{bt}{m}}$$

$$= \left(\frac{b}{m} \cdot \left(\frac{-mg}{b}\right) - \frac{b}{m} \cdot v_0\right) e^{-\frac{bt}{m}}$$

$$\boxed{a(t) = \left(-g - \frac{b}{m} v_0\right) e^{-\frac{bt}{m}}}$$ ⇝ persamaan $a(t)$ sebelum aproksimasi

⇒ persamaan $y(t)$ dapat dicari dg :

$$v(t) = \frac{dy(t)}{dt}$$

$$\int_0^{y(t)} dy(t) = \int v(t)\, dt \cdot$$

$$y(t) = \int_0^t (v_0 - v_t)\, e^{-\frac{bt}{m}} + v_T$$

$$y(t) = v_{Tt} - \frac{m}{b} \cdot (v_0 - v_t)\, e^{-\frac{bt}{m}} \Big]_0^t$$

$$y(t) = \left(v_{tt} - \frac{m}{b}(v_0 - v_t)\, e^{-\frac{bt}{m}}\right) - \left(\frac{-m}{b}(v_0 - v_T\right)$$

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$$y(t) = V_T t + \frac{m}{b}(V_0 - V_T)(1 - e^{-\frac{bt}{m}})$$

↝ Tahap Aproksimasi

$$e^x = 1 + \frac{x}{1!} \cdot \frac{x^2}{2!} \cdot \frac{x^3}{3!} + \dots$$

maka : $e^{-\frac{bt}{m}} = 1 + \left(\frac{-bt}{m}\right) + \frac{1}{2}\left(\frac{bt}{m}\right)^2 + \dots$

·) subsitusikan pd persamaan $V(t) = (V_0 - V_T)e^{-\frac{bt}{m}} + V_T$

$$V_t = V_T + (V_0 - V_T)\left(1 - \frac{bt}{m}\right)$$

$$= V_T + V_0 - V_0 \frac{bt}{m} - V_t + \frac{b}{m} V_T \cdot t$$

$$= V_0 - V_0 \frac{bt}{m} + \frac{b}{m}\left(\frac{-mg}{b}\right) +$$

$$= V_0 - gt - V_0 + \frac{b}{m}$$

$$V_t = V_0 - gt - \frac{b}{m} V_0 t$$ → Persamaan V(t) gerak vertikal keatas dg pengaruh gesekan udara.

jika disubstitusikan $b = 0$ maka, akan diperoleh pers. $V(t) = V_0 - gt$ → Rumus gerak vertikal atas.

·) substitusikan $e^{-\frac{bt}{m}} = 1 + \left(\frac{-bt}{m}\right) + \frac{1}{2}\left(\frac{-bt}{m}\right)^2$ pd pers.

$$y(t) = V_T t + \frac{m}{b}(V_0 - V_T)\left(1 - e^{-\frac{bt}{m}}\right)$$

$$= V_T t + \frac{m}{b}(V_0 - V_T)\left(1 - \left(1 - \frac{bt}{m} + \frac{1}{2}\left(\frac{bt}{m}\right)^2\right)\right)$$

$$= V_T t + \frac{m}{b}(V_0 - V_T)\left(\frac{bt}{m} - \frac{1}{2}\left(\frac{bt}{m}\right)^2\right)$$

$$= V_T t + (V_0 - V_T)\left(t - \frac{1}{2}\frac{b}{m}t^2\right)$$

$$= V_T t + V_0 t - V_T t - \frac{1}{2}\frac{b}{m} V_0 t^2 + \frac{1}{2} b m V_T t^2.$$

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$$y(t) = v_0 t - \frac{1}{2}\frac{b}{m} v_0 t^2 + \frac{1}{2}\frac{b}{m}\left(\frac{-mg}{b}\right)t^2$$

$$= v_0 t - \frac{1}{2} g t^2 - \frac{1}{2}\frac{b}{m} v_0 t^2 .$$

Jadi, persamaan $y(t)$ untuk gerak vertikal keatas dg pengaruh gaya gesek udara adalah

$$\boxed{y(t) = v_0 t - \frac{1}{2} g t^2 - \frac{1}{2}\frac{b}{m} v_0 t^2}$$

jika substitusi $b = 0$

$y(t) = v_0 t - \frac{1}{2} g t^2$ → (Rumus $y(t)$ gerak vertikal keatas)

•) substitusikan $e^{-\frac{bt}{m}} = 1 - \frac{bt}{m}$ pd persamaan $a(t) = \left(-g - \frac{b}{m} v_0\right) e^{-\frac{bt}{m}}$

$$a(t) = \left(-g - \frac{b}{m} v_0\right)\left(1 - \frac{bt}{m}\right)$$

$$= -g + \frac{b}{m} gt - \frac{b}{m} v_0 + \frac{b^2 v_0 t}{m^2}$$

$$a(t) = -g - \frac{b}{m} v_0 t + \left(\frac{b^2}{m^2} v_0 + \frac{b}{m} g\right) t$$

Jadi, persamaan $a(t)$ u/ gerak vertikal keatas :

$$\boxed{a(t) = -g - \frac{b}{m} v_0 + \left(\frac{b^2}{m^2} v_0 + \frac{b}{m} g\right) t .}$$

jika substitusi $b = 0$

$$a(t) = -g - \frac{b}{m} v_0 + \left(\frac{b^2}{m^2} v_0 + \frac{b}{m} g\right) t$$

$$a(t) = -g . \text{ (gerak vertikal keatas).}$$

Jadi, dapat disimpulkan bahwa :

$$y(t) = v_0 t - \frac{1}{2} g t^2 - \frac{1}{2}\frac{b}{m} v_0 t^2$$

$$v(t) = v_0 - gt - \frac{b}{m} v_0 t$$

$$a(t) = -g - \frac{b}{m} v_0 + \left(\frac{b^2}{m^2} v_0 + \frac{b}{m} g\right) t$$

adalah rumus $y(t), a(t), v(t)$ gerak vertikal keatas dipengaruhi gaya gesekan.

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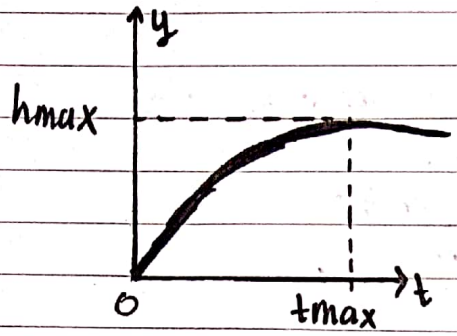
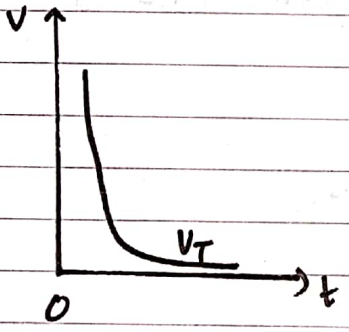
↳ Cerafik :

1) y(t)



2) v(t)



3) a(t)