

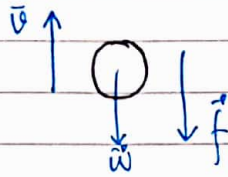
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Kelas : B.

Prodi : Pendidikan Fisika

Menurunkan Rumus Gerak Vertikal ke Atas dipengaruhi gaya gesekan udara.



dengan $f = -bv$.

Berdasarkan Hk. II Newton.

$$\sum F = m \cdot a.$$

$$-w - f = m \cdot a.$$

$$-mg - bv = m \cdot a \quad | : m$$

$$-g - \frac{b}{m} v = a$$

$$-g - \frac{b}{m} v = \frac{dv}{dt}$$

$$\int_{v_0}^{v(t)} \frac{dv}{-g - \frac{b}{m} v} = \int_0^t dt \quad \dots (i)$$

$$\Rightarrow \text{Misalkan } -g - \frac{b}{m} v = u.$$

$$-\frac{b}{m} dv = du.$$

$$dv = -\frac{m}{b} du.$$

$\Rightarrow$ Substitusikan $dv = -\frac{m}{b} du$ ke pers. (i). dan u ke pers (i).

$$\int_{v_0}^{v(t)} \frac{1}{u} dv = \int_{v_0}^{v(t)} \frac{1}{u} \cdot \frac{-m}{b} du.$$

$$= -\frac{m}{b} \int_{v_0}^{v(t)} \frac{1}{u} du.$$

$$= -\frac{m}{b} \ln u \quad \dots (ii).$$

$\Rightarrow$ Substitusikan $\int_0^{vt} \frac{dv}{u} = -\frac{m}{b} \ln u$ ke pers (i).

$$\int_{v_0}^{vt} \frac{dv}{\left(-g - \frac{b \cdot v}{m}\right)} = \int_0^t dt.$$

$$-\frac{m}{b} \ln \left[-g - \frac{b \cdot v}{m} \right] \Big|_{v_0}^{vt} = t.$$

$$-\frac{m}{b} \left[\ln \left(-g - \frac{b}{m} vt \right) - \ln \left(-g - \frac{b}{m} v_0 \right) \right] = t.$$

Ingat sifat logaritma. $\ln a - \ln b = \ln \frac{a}{b}$.

$$\ln \left(\frac{-g - \frac{b}{m} vt}{-g - \frac{b}{m} v_0} \right) = -\frac{b}{m} t.$$

$$\exp \left(\ln \left| \frac{-g - \frac{b}{m} vt}{-g - \frac{b}{m} v_0} \right| \right) = e^{-\frac{b}{m} t}.$$

Sifat $e^{\ln x} = x$.

$$\frac{-g - \frac{b}{m} vt}{-g - \frac{b}{m} v_0} = e^{-\frac{b}{m} t}.$$

$$-g - \frac{b}{m} vt = \left(-g - \frac{b \cdot v_0}{m} \right) e^{-\frac{b}{m} t}.$$

$$-\frac{b}{m} vt = g + \left(-g - \frac{b \cdot v_0}{m} \right) e^{-\frac{b}{m} t}.$$

$$vt = \left(\frac{mg}{b} + v_0 \right) e^{-\frac{bt}{m}} - \frac{mg}{b}.$$

Karena $f = -w$.

$$f = -mg.$$

$$bv = -mg$$

$$V_T = \frac{-mg}{b}.$$

•> Persamaan $V(t)$ dapat ditulis.

$$V(t) = (-V_T + V_0) e^{-\frac{bt}{m}} + V_T.$$

$$\boxed{V(t) = (V_0 - V_T) e^{-\frac{bt}{m}} + V_T} \leadsto \text{Persamaan } V(t) \text{ sebelum Aproksimasi.}$$

•> Persamaan $a(t)$ dapat dicari dg:

$$a(t) = \frac{dV(t)}{dt}$$

$$= \frac{d}{dt} (V_0 - V_T) e^{-\frac{bt}{m}} + V_T.$$

$$= -\frac{b}{m} (V_0 - V_T) e^{-\frac{bt}{m}} + 0.$$

$$= \left(\frac{b \cdot V_T}{m} - \frac{b \cdot V_0}{m} \right) e^{-\frac{bt}{m}}.$$

$$= \left(\frac{b}{m} \left(\frac{-mg}{-b} \right) - \frac{b \cdot V_0}{m} \right) e^{-\frac{bt}{m}}.$$

$$\boxed{a(t) = \left(-g - \frac{b \cdot V_0}{m} \right) e^{-\frac{bt}{m}}} \leadsto \text{Persamaan } a(t) \text{ sebelum Aproksimasi.}$$

•> Persamaan $y(t)$ dapat dicari dg.

$$V(t) = \frac{dy(t)}{dt}.$$

$$\int_0^{y(t)} dy(t) = \int V(t) dt.$$

$$y(t) = \int_0^t (V_0 - V_T) e^{-\frac{bt}{m}} + V_T dt.$$

$$= V_T t - \frac{m}{b} (V_0 - V_T) e^{-\frac{bt}{m}} \Big|_0^t.$$

$$y(t) = \left(V_T t - \frac{m \cdot (V_0 - V_T)}{b} e^{-\frac{bt}{m}} \right) - \left(-\frac{m \cdot (V_0 - V_T)}{b} \right)$$

$$y(t) = V_T t + \frac{m \cdot (V_0 - V_T)}{b} (1 - e^{-\frac{bt}{m}})$$

Persamaan $y(t)$ sebelum Aproksimasi.

~> Tahap Aproksimasi.

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

maka $e^{-\frac{bt}{m}} = 1 + \left(-\frac{bt}{m}\right) + \frac{1}{2} \left(\frac{bt}{m}\right)^2 + \dots$

•> Substitusikan pd pers. $v(t) = (V_0 - V_T) e^{-\frac{bt}{m}} + V_T$.

$$V_t = V_T + (V_0 - V_T) \left(1 - \frac{bt}{m}\right)$$

$$= \cancel{V_T} + V_0 - V_0 \frac{b}{m} t - \cancel{V_T} + \frac{b}{m} V_T t$$

$$= V_0 - V_0 \frac{b}{m} t + \frac{b}{m} V_T t$$

$$= V_0 - gt - V_0 \frac{b}{m} t$$

$$V(t) = V_0 - gt - \frac{b}{m} V_0 t$$

Persamaan $v(t)$ gerak vertikal ke atas dg pengaruh gesekan udara.

Jika kita substitusikan $b=0$ maka akan diperoleh persamaan $V(t) = V_0 - gt$ (rumus gerak vertikal ke atas).

•> Substitusikan $e^{-\frac{bt}{m}} = 1 + \left(-\frac{bt}{m}\right) + \frac{1}{2} \left(\frac{bt}{m}\right)^2$ pada persamaan

$$y(t) = V_T t + \frac{m \cdot (V_0 - V_T)}{b} (1 - e^{-\frac{bt}{m}})$$

$$y(t) = V_T t + \frac{m \cdot (V_0 - V_T)}{b} \left(1 - \left(1 - \frac{bt}{m} + \frac{1}{2} \left(\frac{bt}{m}\right)^2\right)\right)$$

$$= V_T t + \frac{m \cdot (V_0 - V_T)}{b} \left(\frac{bt}{m} - \frac{1}{2} \left(\frac{bt}{m}\right)^2\right)$$

$$y(t) = V_T t + (V_0 - V_T) \left(t - \frac{1}{2} \frac{b}{m} t^2 \right)$$

$$= \cancel{V_T t} + V_0 t - \cancel{V_T t} - \frac{1}{2} \frac{b}{m} V_0 t^2 + \frac{1}{2} \frac{b}{m} V_T t^2$$

$$= V_0 t - \frac{1}{2} \frac{b}{m} V_0 t^2 + \frac{1}{2} \frac{b}{m} \left(-\frac{mg}{b} \right) t^2$$

$$= V_0 t - \frac{1}{2} g t^2 - \frac{1}{2} \frac{b}{m} V_0 t^2$$

Jadi persamaan $y(t)$ untuk gerak vertikal ke atas dg pengaruh gaya gesek udara adalah

$$y(t) = V_0 t - \frac{1}{2} g t^2 - \frac{1}{2} \frac{b}{m} V_0 t^2$$

Jika di substitusikan $b=0$ maka akan diperoleh

$$y(t) = V_0 t - \frac{1}{2} g t^2 \quad (\text{rumus } y(t) \text{ gerak vertikal ke atas}).$$

→ Substitusikan $e^{-bt/m} = 1 - \frac{bt}{m}$ pada persamaan $a(t) = \left(-g - \frac{b}{m} V_0 \right) e^{-bt/m}$

$$a(t) = \left(-g - \frac{b}{m} V_0 \right) \left(1 - \frac{bt}{m} \right)$$

$$= -g + \frac{b}{m} g t - \frac{b}{m} V_0 + \frac{b^2}{m^2} V_0 t$$

$$a(t) = -g - \frac{b}{m} V_0 + \left(\frac{b^2}{m^2} V_0 + \frac{b}{m} g \right) t$$

Jadi persamaan $a(t)$ untuk gerak vertikal ke atas dg pengaruh gaya gesek udara adalah

$$a(t) = -g - \frac{b}{m} V_0 + \left(\frac{b^2}{m^2} V_0 + \frac{b}{m} g \right) t$$

Jika disubstitusikan $b=0$ maka akan diperoleh $a(t) = -g$ sama seperti gerak vertikal ke atas.

Kesimpulan, Rumus ($y(t)$, $v(t)$, dan $a(t)$) gerak Vertikal ke Atas dipengaruhi gaya gesekan adalah . . .

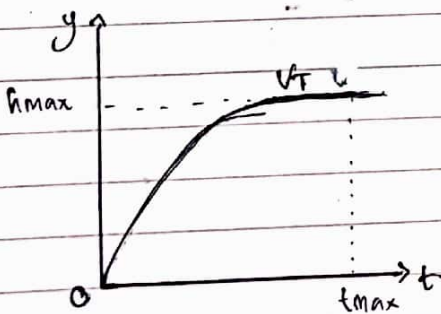
$$y(t) = V_0 t - \frac{1}{2} g t^2 - \frac{1}{2} \frac{b}{m} V_0 t^2$$

$$v(t) = V_0 - g t - \frac{b V_0 t}{m}$$

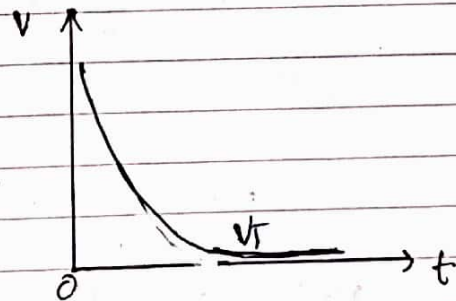
$$a(t) = -g - \frac{b \cdot V_0}{m} + \left(\frac{b^2 V_0}{m^2} + \frac{b \cdot g}{m} \right) t$$

~ Grafikny~

1). Grafik y terhadap t :



2). Grafik v terhadap t :



3). Grafik a terhadap t :

