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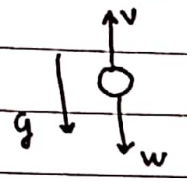
2013022056

Diskusi gerak vertikal yang dipengaruhi hambatan udara



Berdasarkan hk 2 Newton

$$\Sigma F = m \cdot \vec{a}$$

$$\vec{w} + \vec{F} = -m \cdot \vec{a}$$

$$(mg + bv)\vec{y} = -ma\vec{y}$$

$$mg + bv = -ma$$

$$g + \frac{bv}{m} = -a$$

$$g + \frac{bv}{m} = \frac{d^2y(t)}{dt^2}$$

$$g + \frac{b}{m}v = -\frac{dv}{dt}$$

$$-\int_{v_0}^{v(t)} \frac{dv}{g + \frac{b}{m}v} = \int_0^t dt$$

Selanjutnya kita misalkan $g + \frac{b}{m}v = u$

$$\frac{b}{m}dv = du$$

$$dv = \frac{m}{b}du$$

$$-\int_{v_0}^{v(t)} \frac{1}{u}\,dv = \int_0^t dt$$

$$-\int_{v_0}^{v(t)} \frac{1}{g + \frac{b}{m}v}\,dv = \int_0^t dt$$

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$$-\frac{m}{b}\ln\left|g+\frac{b}{m}v\right|_{v_0}^{v(t)} = t$$

$$-\frac{m}{b}\left(\ln\left|g+\frac{b}{m}v\right| - \ln\left|g+\frac{b}{m}(v_0)\right|\right) = t$$

$$\ln\left|\frac{g+\frac{b}{m}v_t}{g+\frac{b}{m}v_0}\right| = e^{-\frac{bt}{m}}$$

$$\frac{g+\frac{b}{m}v_t}{g+\frac{b}{m}v_0} = e^{-\frac{bt}{m}}$$

$$g+\frac{b}{m}v_t = \left(g+\frac{b}{m}v_0\right)e^{-\frac{bt}{m}}$$

$$\frac{b}{m}v_t = \left(g+\frac{b}{m}v_0\right)e^{-\frac{bt}{m}} - g$$

$$v_t = \frac{m}{b}\left(g+\frac{b}{m}v_0\right)e^{-\frac{bt}{m}} - \frac{mg}{b}$$

$$v_t = \left(\frac{mg}{b}+v_0\right)e^{-\frac{bt}{m}} - \frac{mg}{b}$$

Because $f = w$, $bv = mg$, $v_T = \frac{mg}{b}$

$$v_b = \left(v_T + v_0\right)e^{-\frac{bt}{m}} - v_T\right)$$

$$v(b) = \left(v_T + v_0\right)e^{-\frac{bt}{m}} - v_T$$

$$= -v_T + \left(v_T + v_0\right)\left(1-\frac{bt}{m}\right)$$

$$= -v_T + v_T - \frac{b}{m}v_T t + v_0 - \frac{b}{m}v_0 t$$

$$= v_0 - \frac{b}{m}\left(\frac{mg}{b}\right)t - \frac{b}{m}v_0 t$$

$$v(t) = v_0 - gt - \frac{b}{m}v_0 t$$

For $b=0$ (tanpa gesekan udara)

$$v(t) = v_0 - gt$$

$$a(t) = \frac{dv(t)}{dt}$$

$$= \frac{d}{dt}\left(\left(v_T - v_0\right)e^{-\frac{bt}{m}} - v_T\right)$$

$= (v_T + v_0)\left(-\frac{b}{m}\right)e^{-\frac{bt}{m}}$

$= -\frac{b}{m}(v_T + v_0)e^{-\frac{bt}{m}}$

$= \left(-\frac{b}{m}\cdot\frac{mg}{b} - \frac{b}{m}v_0\right)e^{-\frac{bt}{m}}$

$a_{(t)} = \left(-g - \frac{b}{m}v_0\right)e^{-\frac{bt}{m}}$

* $v_{(t)} = \frac{dy(t)}{dt}$

$\int_0^{y(t)} dy(t) = \int_0^t v(t)\, dt$

$y_{(t)} = \int_0^t \left((v_T + v_0)e^{-\frac{bt}{m}} - v_T\right)$

$y_{(t)} = \left[(v_T + v_0)\left(-\frac{m}{b}\right)e^{-\frac{bt}{m}} - v_T.t\right] - \left[v_T + v_0 - \left(-\frac{m}{b}\right)\right]$

$y_{(t)} = -\frac{m}{b}(v_T + v_0)(e^{-\frac{bt}{m}}) - v_T.t$

$y_{(t)} = -\frac{m}{b}(v_T + v_0)(1 - e^{-\frac{bt}{m}}) - v_T.t$

Aprokmasi

$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$

$e^{-\frac{bt}{m}} = 1 - \frac{bt}{m} + \frac{1}{2}\left(\frac{bt}{m}\right)^2 + \frac{1}{3}\left(\frac{bt}{m}\right)^3 + \dots$

$y_{(t)} = -v_T t + \frac{m}{b}(v_T + v_0)(1 - e^{-\frac{bt}{m}})$

$= -v_T t + \frac{m}{b}(v_T + v_0)\left(1 - 1 + \frac{bt}{m} - \frac{1}{2}\left(\frac{bt}{m}\right)^2\right)$

$= -v_T t + (v_T + v_0)\left(t - \frac{1}{2}\frac{b}{m}t^2\right)$

$= -v_T t + v_T t - \frac{1}{2}\frac{b}{m}v_T t^2 + v_0 t - \frac{1}{2}\frac{b}{m}v_0 t^2$

$= v_0 t - \frac{1}{2}\frac{b}{m}\left(\frac{mg}{b}\right)t^2 - \frac{1}{2}\frac{b}{m}v_0 t^2$

$y_{(t)} = v_0 t - \frac{1}{2}gt^2 - \frac{1}{2}\frac{b}{m}v_0 t^2$

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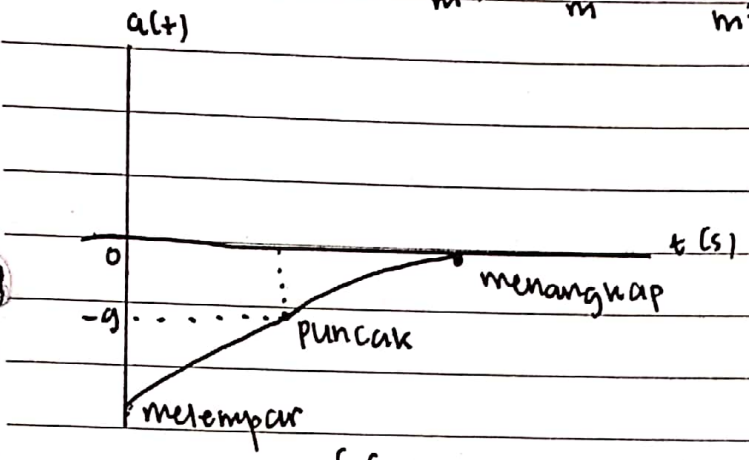
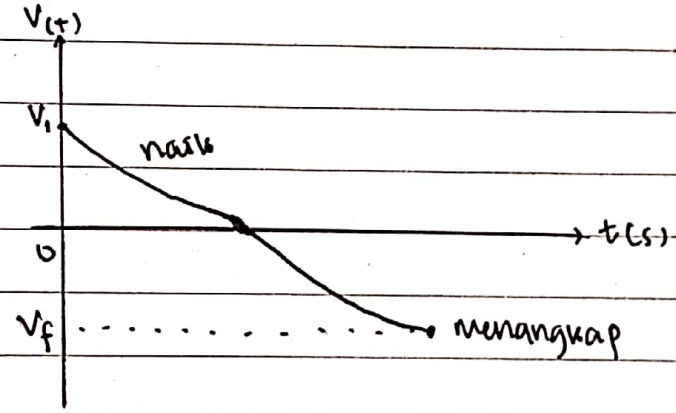
* $a_{(t)} = (-g - \frac{b}{m} V_0) e^{-\frac{bt}{m}}$

$= (-g - \frac{b}{m} V_0)(1 - \frac{bt}{m})$

$= (-g - g\frac{bt}{m}) - \frac{b}{m} V_0 + \frac{b^2}{m^2} V_0 t$



(Grafik $a_{(t)} - t$)



(Grafik $V_t - t$)