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Date

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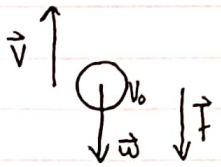
Diskusi Gerak Vertikal ke atas yang dipengaruhi Hambatan Udara



$f = -b\vec{v}$

• Berdasarkan Hukum II Newton

$\Sigma \vec{F} = m.\vec{a}$

$-\vec{w} - \vec{f} = m.\vec{a}$

$-mg - bv = m.a \quad | \times \frac{1}{m}$

$-g - \frac{b}{m} v = a$

$-g - \frac{b}{m} v = \frac{dv}{dt}$

$\int_{v_0}^{v(t)} \frac{dv}{-g - \frac{b}{m} v} = \int_0^t dt$

• misalkan : $-g - \frac{b}{m} v = u$

$-\frac{b}{m} dv = du$

$dv = -\frac{m}{b} du$

$\int \frac{1}{u} dv = \int \frac{1}{u} -\frac{m}{b} du$

$\int \frac{1}{u} \left(-\frac{m}{b} du\right) = -\frac{m}{b} \int \frac{1}{u} du$

$= -\frac{m}{b} \ln |u|$

$$\int_{V_0}^{V(t)} \frac{1}{-g-\frac{b}{m}V}\,dV = \int_0^t dt$$

$$-\frac{m}{b}\ln|u|\Big]_{V_0}^{V(t)} = t$$

$$-\frac{m}{b}\ln\left|-g-\frac{b}{m}V\right|\Big]_{V_0}^{V(t)} = t$$

$$-\frac{m}{b}\left(\ln\left|-g-\frac{b}{m}V(t)\right| - \ln\left|-g-\frac{b}{m}V_0\right|\right) = t$$

$$\ln a - \ln b = \ln\frac{a}{b}$$

$$\ln\left|\frac{-g-\frac{b}{m}V(t)}{-g-\frac{b}{m}V_0}\right| = -\frac{b}{m}t$$

$$\exp\left(\ln\left|\frac{-g-\frac{b}{m}V(t)}{-g-\frac{b}{m}V_0}\right|\right) = e^{-\frac{b}{m}t}$$

$$e^{\ln x} = x$$

$$\frac{-g-\frac{b}{m}V(t)}{-g-\frac{b}{m}V_0} = e^{-\frac{b}{m}t}$$

$$-g-\frac{b}{m}V(t) = \left(-g-\frac{b}{m}V_0\right)e^{-\frac{b}{m}t}$$

$$-\frac{b}{m}V(t) = g+\left(-g-\frac{b}{m}V_0\right)e^{-\frac{b}{m}t} \quad \Big| \times -\frac{m}{b}$$

$$V(t) = -\frac{m}{b}g-\frac{m}{b}\left(-g-\frac{b}{m}V_0\right)e^{-\frac{b}{m}t}$$

$$V(t) = \left(\frac{mg}{b}+V_0\right)e^{-\frac{b}{m}t} - \frac{mg}{b}$$

- Karena, $f = -w$
  $f = -mg$
  $bv = -mg$
  $v_t = -\frac{mg}{b}$

$$v(t) = (-V_T + V_0)\, e^{-\frac{b}{m}t} + V_T$$

$$\boxed{v(t) = (V_0 - V_T)\, e^{-\frac{b}{m}t} + V_T}$$ → Persamaan v(t) sebelum Aproksimasi

• Persamaan a(t) :

$$a(t) = \frac{dv(t)}{dt}$$

$$= \frac{d}{dt}\left((V_0 - V_T)\, e^{-\frac{b}{m}t} + V_T\right)$$

$$= -\frac{b}{m} \cdot (V_0 - V_T)\, e^{-\frac{b}{m}t} + 0$$

$$= \left(\frac{b}{m} V_T - \frac{b}{m} V_0\right) e^{-\frac{b}{m}t}$$

$$= \left(\frac{b}{m}\left(-\frac{m}{b} g\right) - \frac{b}{m} V_0\right) e^{-\frac{b}{m}t}$$

$$\boxed{a(t) = \left(-g - \frac{b}{m} V_0\right) e^{-\frac{b}{m}t}}$$ → Persamaan a(t) sebelum Aproksimasi

• Persamaan y(t) :

$$v(t) = \frac{dy(t)}{dt}$$

$$\int_0^{y(t)} dy(t) = \int_0^t v(t)\, dt$$

$$y(t) = \int_0^t \left((V_0 - V_T)\, e^{-\frac{b}{m}t} + V_T\right) dt$$

$$y(t) = V_T t - \frac{m}{b}(V_0 - V_T) \cdot e^{-\frac{bt}{m}} \Big]_0^t$$

$$y(t) = \left[V_T t - \frac{m}{b}(V_0 - V_T)\, e^{-\frac{b}{m}t}\right] - \left[-\frac{m}{b}(V_0 - V_T)\right]$$

$$\boxed{y(t) = V_T t + \frac{m}{b}(V_0 - V_T)\left(1 - e^{-\frac{b}{m}t}\right)}$$ → Persamaan y(t) sebelum Aproksimasi

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- Aproksimasi

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$e^{-\frac{bt}{m}} = 1 + \left(\frac{bt}{m}\right) + \frac{1}{2}\left(\frac{bt}{m}\right)^2 + \dots$$

⇒ $v(t) = (v_0 - v_T)\, e^{-\frac{bt}{m}} + v_T$

$v(t) = v_T + (v_0 - v_T)\left(1 - \frac{bt}{m}\right)$

$v(t) = v_T + v_0 - v_0 \frac{b}{m} t - v_T + \frac{b}{m} v_T t$

$v(t) = v_0 - v_0 t \frac{b}{m} + \frac{b}{m}\left(-\frac{mg}{b}\right) t$

$v(t) = v_0 - gt - v_0 t \frac{b}{m}$

$$\boxed{v(t) = v_0 - gt - \frac{b}{m} v_0 t}$$

Persamaan v(t) gerak vertikal ke atas dengan pengaruh gesekan udara

jika mensubstitusikan $b=0$ maka akan didapat persamaan:
$v(t) = v_0 - gt$ (rumus gerak vertikal ke atas).

- $a(t) = \left(-g - \frac{b}{m} v_0\right) e^{-\frac{bt}{m}}$

$a(t) = \left(-g - \frac{b}{m} v_0\right)\left(1 - \frac{bt}{m}\right)$

$a(t) = -g + \frac{b}{m} gt - \frac{b}{m} v_0 + \frac{b^2}{m^2} v_0 t$

$a(t) = -g - \frac{b}{m} v_0 + \left(\frac{b^2}{m^2} v_0 + \frac{b}{m} g\right) t$

$$\boxed{a(t) = -g - \frac{b}{m} v_0 + \left(\frac{b^2}{m^2} v_0 + \frac{b}{m} g\right) t}$$

Persamaan a(t) gerak vertikal ke atas dengan pengaruh gaya gesek udara.

jika mensubstitusikan $b=0$ maka akan didapat:
$a(t) = -g$ (rumus gerak vertikal ke atas)

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$$y(t) = V_T t + \frac{m}{b}(V_0 - V_T)\left(1 - e^{-\frac{bt}{m}}\right)$$

$$y(t) = V_T t + \frac{m}{b}(V_0 - V_T)\left(1 - 1 - \frac{bt}{m} + \frac{1}{2}\left(\frac{bt}{m}\right)^2\right)$$

$$y(t) = V_T t + \frac{m}{b}(V_0 - V_T)\left(\frac{bt}{m} - \frac{1}{2}\left(\frac{bt}{m}\right)^2\right)$$

$$y(t) = V_T t + (V_0 - V_T)\left(t - \frac{1}{2}\frac{b}{m}t^2\right)$$

$$y(t) = \cancel{V_T t} + V_0 t - \cancel{V_T t} - \frac{1}{2}\frac{b}{m}V_0 t^2 + \frac{1}{2}\frac{b}{m}V_T t^2$$

$$y(t) = V_0 t - \frac{1}{2}\frac{b}{m}V_0 t^2 + \frac{1}{2}\frac{b}{m}\left(-\frac{mg}{b}\right)t^2$$

$$y(t) = V_0 t - \frac{1}{2}gt^2 - \frac{1}{2}\frac{b}{m}V_0 t^2$$

$$\boxed{y(t) = V_0 t - \frac{1}{2}gt^2 - \frac{1}{2}\frac{b}{m}V_0 t^2}$$

Persamaan $y(t)$ untuk gerak vertikal keatas dengan pengaruh gaya gesek udara.

jika mensubstitusikan $b = 0$ maka akan didapat:

$y(t) = V_0 t - \frac{1}{2}gt^2$ (rumus gerak vertikal ke atas)