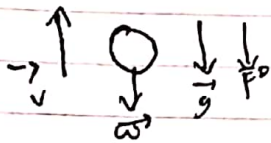


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Prodi : Pendidikan Fisika (B)



Berdasarkan titik II Newton

$$\sum \vec{F} = m\vec{a}$$

$$\vec{w} + \vec{F} = -m\vec{a}$$

$$(mg + bv)\hat{y} = -m\hat{a}_y$$

$$(mg + bv) = -ma \quad \left(\times \frac{1}{m} \right)$$

$$g + \frac{bv}{m} = -a$$

Substitusi nilai a

$$g + \frac{b}{m} v = -\frac{d^2y(t)}{dt^2}$$

$$g + \frac{b}{m} v = -\frac{dv}{dt}$$

$$\frac{-dv}{g + \frac{b}{m} v} = dt$$

$$= \int_{v_0}^{v(t)} \frac{dv}{g + \frac{b}{m} v} = \int_0^t dt$$

$$\text{misal} = g + \frac{b}{m} \cdot v = u$$

$$= \frac{b}{m} dv = du$$

$$du = \frac{m}{b} du$$

$$\text{maka} : \int_{v_0}^{v(t)} \frac{1}{u} du = \int_0^t dt$$

$$\int_{v_0}^{v(t)} \frac{1}{u} \left(\frac{m}{b} du \right) = \int_0^t dt$$

$$-\frac{m}{b} \ln |u| = t$$

$$-\frac{m}{b} \ln \left| g + \frac{b}{m} v \right|_{v_0}^{v(t)} = t$$

$$-\frac{m}{b} \left(\ln \left| g + \frac{b}{m} (v(t)) \right| - \ln \left| g + \frac{b}{m} (v_0) \right| \right)$$

$$\text{karena} : \ln a - \ln b = \ln \frac{a}{b}$$

maka :

$$\ln \left| g + \frac{b}{m} (v(t)) \right| = -\frac{bt}{m}$$

$$\exp \left(\ln \left| g + \frac{b}{m} v(t) \right| \right) = e^{-\frac{bt}{m}}$$

$$g + \frac{b}{m} v(t) = e^{-\frac{bt}{m}}$$

$$g + \frac{b}{m} v_0$$

$$g + \frac{b}{m} v(t) = \left(g + \frac{b}{m} v_0 \right) e^{-\frac{bt}{m}}$$

$$\frac{b}{m} v(t) = \left(g + \frac{b}{m} v_0 \right) e^{-\frac{bt}{m}} - g$$

$$v(t) = \frac{m}{b} \left(g + \frac{b}{m} v_0 \right) e^{-\frac{bt}{m}} - \frac{mg}{b}$$

$$v(t)^2 = \left(\frac{mg}{b} + v_0 \right) e^{-\frac{bt}{m}} - \frac{mg}{b}$$

$$\text{karena} : f = a$$

$$bv = mg$$

$$v_T = \frac{mg}{b}, \text{ maka}$$

$$v(t) = (v_T + v_0) e^{-\frac{bt}{m}} - v_T$$

untuk $a(t)$

$$a(t) = \frac{dv(t)}{dt}$$

$$a(t) = (v_T + v_0) \left(-\frac{b}{m}\right) e^{-\frac{bt}{m}}$$

$$= -\frac{b}{m} (v_T + v_0) e^{-\frac{bt}{m}}$$

$$= \left(-\frac{b}{m} \cdot \frac{mg}{b} - \frac{b}{m} v_0\right) e^{-\frac{bt}{m}}$$

$$a(t) = \left(-g - \frac{b}{m} v_0\right) e^{-\frac{bt}{m}}$$

untuk $y(t)$

$$v(t) = \frac{dy(t)}{dt}$$

$$dy(t) = v(t) \cdot dt$$

$$\int_0^t dy(t) = \int_0^t v(t) dt$$

$$y(t) = \int_0^t (v_T + v_0) e^{-\frac{bt}{m}} - v_T dt$$

$$y(t) = \left[(v_T + v_0) \left(-\frac{m}{b}\right) e^{-\frac{bt}{m}} - v_T \cdot t \right]_0^t$$

$$y(t) = \left[(v_T + v_0) \left(-\frac{m}{b}\right) e^{-\frac{bt}{m}} - v_T t \right] - \left[(v_T + v_0) \left(-\frac{m}{b}\right) \right]$$

$$y(t) = -\frac{m}{b} (v_T + v_0) (e^{-\frac{bt}{m}} - 1) - v_T \cdot t$$

aproksiasi

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$e^{-\frac{bt}{m}} = 1 - \frac{bt}{m} + \frac{1}{2} \left(\frac{bt}{m}\right)^2 + \dots$$

$$y(t) = -v_T t + \frac{m}{b} (v_T + v_0) (1 - e^{-\frac{bt}{m}})$$

$$= -v_T t + \frac{m}{b} (v_T + v_0) \left(1 - 1 + \frac{bt}{m} - \frac{1}{2} \left(\frac{bt}{m}\right)^2\right)$$

$$= v_0 t + \frac{1}{2} g t^2 - \frac{1}{2} \frac{b}{m} v_0 t^2$$

$$y(t) = v_0 t - \frac{1}{2} g t^2 - \frac{1}{2} \frac{b}{m} v_0 t^2$$

untuk $b=0$, maka

$$y(t) = v_0 t - \frac{1}{2} g t^2$$

$$v(t)$$

$$\begin{aligned} v(t) &= (v_T + v_0) e^{-bt/m} - v_T \\ &= -v_T + (v_T + v_0) (1 - bt/m) \\ &= -v_T + v_T - \frac{b}{m} v_T t + \dots + v_0 - \frac{b}{m} v_0 t \\ &= v_0 - \frac{b}{m} \left(\frac{mg}{b} \right) t - \frac{b}{m} v_0 t \end{aligned}$$

$$v(t) = v_0 - g t - \frac{b}{m} v_0 t$$

untuk $b=0$ maka,

$$v(t) = v_0 - g t$$

$$\begin{aligned} a(t) &= (-g - b/m v_0) e^{-bt/m} \\ &= (-g - b/m v_0) (1 - bt/m) \\ &= -g + g \frac{bt}{m} - \frac{b}{m} v_0 + \frac{b^2}{m^2} v_0 t \end{aligned}$$

$$a(t) = -g - \frac{b}{m} v_0 + \left(\frac{b^2}{m^2} v_0 + \frac{gb}{m} \right) t$$

maka $b=0$

$$a(t) = -g$$