

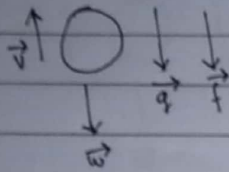
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Kelas : B

Prodi : Pendidikan Fisika

Duf

Mekanika

Berdasarkan titik II Newton :

$$\sum \vec{F} = m \vec{a}$$

$$\vec{w} + \vec{F} = -m \vec{a}$$

$$(mg + bv) \hat{y} = -m \hat{a} \hat{y}$$

$$(mg + bv) = -m \hat{a} \left(\times \frac{1}{m} \right)$$

$$g + \frac{bv}{m} = -a$$

Substitusi nilai a

$$g + \frac{b}{m} v = - \frac{d^2 y(t)}{dt^2}$$

$$g + \frac{b}{m} v = \frac{-dv}{dt}$$

$$\frac{-dv}{g + \frac{b}{m} v} = dt$$

$$= \int_{v_0}^{v(t)} \frac{dv}{g + \frac{b}{m} v} = \int_0^t dt$$

$$\text{misal} = g + \frac{b}{m} v = u$$

$$= \frac{b}{m} dv = du$$

$$du = \frac{m}{b} du$$

Maka :

$$= \int_{v_0}^{v(t)} \frac{1}{u} du = \int_0^t dt$$

$$= \int_{v_0}^{v(t)} \frac{1}{u} \left(\frac{m}{b} du \right) = \int_0^t dt$$

$$= - \frac{m}{b} \ln |u| = t$$

$$- \frac{m}{b} \ln \left| g + \frac{b}{m} v \right|_{v_0}^{v(t)} = t$$

$$- \frac{m}{b} \left(\ln \left| g + \frac{b}{m} v(t) \right| - \ln \left| g + \frac{b}{m} v_0 \right| \right)$$

Karena : $\ln a - \ln b = \ln \frac{a}{b}$, maka

$$\ln \left| \frac{g + \frac{b}{m} v(t)}{g + \frac{b}{m} v_0} \right| = - \frac{bt}{m}$$

$$\exp \left(\ln \left| \frac{g + \frac{b}{m} v(t)}{g + \frac{b}{m} v_0} \right| \right) = e^{-\frac{bt}{m}}$$

$$g + \frac{b}{m} v(t) = e^{-\frac{bt}{m}}$$

$$g + \frac{b}{m} v_0$$

$$g + \frac{b}{m} v(t) = \left(g + \frac{b}{m} v_0 \right) e^{-\frac{bt}{m}}$$

$$\frac{b}{m} v(t) = \left(g + \frac{b}{m} v_0 \right) e^{-\frac{bt}{m}} - g \left(\times \frac{m}{b} \right)$$

$$v(t) = \frac{m}{b} \left(g + \frac{b}{m} v_0 \right) e^{-\frac{bt}{m}} - \frac{mg}{b}$$

$$V(t) = \left(\frac{mg}{b} + V_0 \right) e^{-\frac{bt}{m}} - \frac{mg}{b}$$

Karena : $f = w$

$$bv = mg$$

$$V_T = \frac{mg}{b}, \text{ maka :}$$

$$v(t) = (V_T + V_0) e^{-\frac{bt}{m}} - V_T$$

untuk $a(t)$

$$a(t) = \frac{dv(t)}{dt}$$

$$a(t) = (V_T + V_0) \left(-\frac{b}{m} \right) e^{-\frac{bt}{m}}$$

$$= -\frac{b}{m} (V_T + V_0) e^{-\frac{bt}{m}}$$

$$= \left(-\frac{b}{m} \cdot \frac{mg}{b} - \frac{b}{m} V_0 \right) e^{-\frac{bt}{m}}$$

$$a(t) = \left(-g - \frac{b}{m} V_0 \right) e^{-\frac{bt}{m}}$$

Untuk $y(t)$

$$v(t) = \frac{dy(t)}{dt}$$

$$dy(t) = v(t) \cdot dt$$

$$\int_0^{y(t)} dy(t) = \int_0^t v(t) dt$$

$$y(t) = \int_0^t (V_T + V_0) e^{-\frac{bt}{m}} - V_T dt$$

$$y(t) = \left[(V_T + V_0) \left(-\frac{m}{b} \right) e^{-\frac{bt}{m}} - V_T \cdot t \right]_0^t$$

$$y(t) = \left[(V_T + V_0) \left(-\frac{m}{b} \right) e^{-\frac{bt}{m}} - V_T \cdot t \right] - \left[(V_T + V_0) \left(-\frac{m}{b} \right) \right]$$

$$y(t) = -\frac{m}{b} (V_T + V_0) (e^{-\frac{bt}{m}} - 1) - V_T \cdot t$$

$$y(t) = -\frac{m}{b} (V_T + V_0) (1 - e^{-\frac{bt}{m}}) - V_T \cdot t$$

Aproksimasi :

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$e^{-\frac{bt}{m}} = \left(-\frac{bt}{m} + \frac{1}{m} \left(\frac{bt}{m} \right)^2 + \dots \right)$$

$$y(t) = -V_T t + \frac{m}{b} (V_T + V_0) (1 - e^{-\frac{bt}{m}})$$

$$= -V_T t + \frac{m}{b} (V_T + V_0) \left(1 - 1 + \frac{bt}{m} - \frac{1}{2} \left(\frac{bt}{m} \right)^2 + \dots \right)$$

$$= V_0 t - \frac{1}{2} g t^2 - \frac{1}{2} \frac{b}{m} V_0 t^2$$

$$y(t) = V_0 t - \frac{1}{2} g t^2 - \frac{1}{2} \frac{b}{m} V_0 t^2$$

Untuk $b = 0$, maka :

$$y(t) = V_0 t - \frac{1}{2} g t^2$$

$V(t)$

$$V(t) = (V_T + V_0) e^{-bt/m} - V_T$$

$$= -V_T + (V_T + V_0) (1 + bt/m)$$

$$= -V_T + V_T + \frac{b}{m} V_T t + V_0 + \frac{b}{m} V_0 t$$

$$= V_0 - \frac{b}{m} \left(\frac{mg}{b} \right) t + \frac{b}{m} V_0 t$$

$$V(t) = V_0 - gt - \frac{b}{m} V_0 t$$

Untuk $b = 0$, maka

$$V(t) = V_0 - gt$$

$$a(t) = \left(-g - \frac{b}{m} V_0 \right) e^{-bt/m}$$

$$= \left(-g - \frac{b}{m} V_0 \right) (1 - bt/m)$$

$$= -g + g \frac{bt}{m} - \frac{b}{m} V_0 + \frac{b^2}{m^2} V_0 t$$

$$a(t) = -g - \frac{b}{m} V_0 + \left(\frac{b^2}{m^2} V_0 + \frac{gb}{m} \right) t$$

maka $b = 0$

$$a(t) = -g$$