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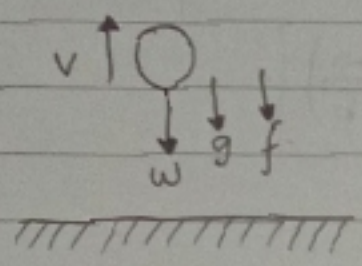
Mata kuliah : Mekanika

Gerak Vertikal yang dipengaruhi Hambatan Udara (Air Resistance)



Berdasarkan Hukum II Newton

$\Sigma \bar{F} = m \cdot a$

$\bar{w} + \bar{f} = -m \cdot \bar{a}$

$(mg + bv)\hat{y} = -m\bar{a}\hat{y}$

$(mg + bv) = -ma\left(x\frac{1}{m}\right)$

$g + \frac{b}{m}v = -a$ (substitusikan nilai a)

$g + \frac{b}{m}v = -\frac{d^2y(t)}{dt^2}$

$g + \frac{b}{m}v = -\frac{dv}{dt}$

$\frac{-dv}{g + \frac{b}{m}v} = dt$

$-\int_{V_0}^{V(t)} \frac{dv}{g + \frac{b}{m}v} = \int_0^t dt$

Misalkan $\Rightarrow g + \frac{b}{m}v = u$

$\Rightarrow \frac{b}{m}dv = du$

$dv = \frac{m}{b}du$

$-\int_{V_0}^{V(t)} \frac{1}{u}du = \int_0^t dt$

$-\int_{V_0}^{V(t)} \frac{1}{u}\left(\frac{m}{b}du\right) = \int_0^t dt$

$-\frac{m}{b}\int_{V_0}^{V(t)} \frac{1}{u}du = t$

$-\frac{m}{b}\left|\ln|u|\right| = t$

$-\frac{m}{b}\left|\ln\left|g + \frac{b}{m}v\right|\right|_{V_0}^{V(t)} = t$

$-\frac{m}{b}\left(\left|\ln\left|g + \frac{b}{m}(V_t)\right| - \ln\left|g + \frac{b}{m}(V_0)\right|\right|\right) = t$

Karena $\ln a - \ln b = \ln\frac{a}{b}$, maka

$\left|\ln\frac{g + \frac{b}{m}(V_t)}{g + \frac{b}{m}(V_0)}\right| = -\frac{bt}{m}$

Karena $f = w$

$bv = mg$

$V_T = \frac{mg}{b}$

$\boxed{V_t = (V_T + V_0)e^{-\frac{bt}{m}} - V_T}$

Untuk a_t

$a_t = \frac{dv(t)}{dt}$

$a_t = \frac{d}{dt}\left((V_T + V_0)e^{-\frac{bt}{m}} - V_T\right)$

$a_t = (V_T + V_0)\left(-\frac{b}{m}\right)e^{\frac{-bt}{m}}$

$= -\frac{b}{m}(V_T + V_0)e^{-\frac{bt}{m}}$

$= \left(-\frac{b}{m}\cdot\frac{mg}{b} - \frac{b}{m}V_0\right)e^{-\frac{bt}{m}}$

$\boxed{a_t = \left(-g - \frac{b}{m}V_0\right)e^{-\frac{bt}{m}}}$

Untuk y_t

$V_t = \frac{dy(t}{dt}$

$$dy_t = V(t)\cdot dt$$

$$\int_0^{y_t} dy(t) = \int_0^t V(t)\,dt$$

$$y(t) = \int_0^t \left(V_T + V_0\right)e^{-\frac{bt}{m}} - V_T)$$

$$y(t) = \left[(V_T + V_0)\left(-\frac{m}{b}\right)e^{-\frac{bt}{m}} - V_T.t\right]_0^t$$

$$y(t) = \left[(V_T + V_0)\left(-\frac{m}{b}\right)e^{-\frac{bt}{m}} - V_T t\right] - \left[(V_T + V_0)\left(-\frac{m}{b}\right)\right]$$

$$y(t) = -\frac{m}{b}(V_T + V_0)\left(e^{-\frac{bt}{m}} - 1\right) - V_T.t$$

$$\boxed{y(t) = \frac{m}{b}(V_T + V_0)\cdot\left(1 - e^{-\frac{bt}{m}}\right) - V_T.t}$$

Aproksimasi

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$e^{-\frac{bt}{m}} = 1 - \frac{bt}{m} + \frac{1}{2}\left(\frac{bt}{m}\right)^2 + \cdots$$

$$\boxed{y(t) = V_0 t - \frac{1}{2}gt^2 - \frac{1}{2}\frac{b}{m}V_0 t^2}$$

Untuk $b = 0$

$$\boxed{y(t) = V_0 t - \frac{1}{2}gt^2}$$

$$\boxed{V(t) = V_0 - gt - \frac{b}{m}}$$

$b = 0$

$$\boxed{V(t) = V_0 - gt}$$

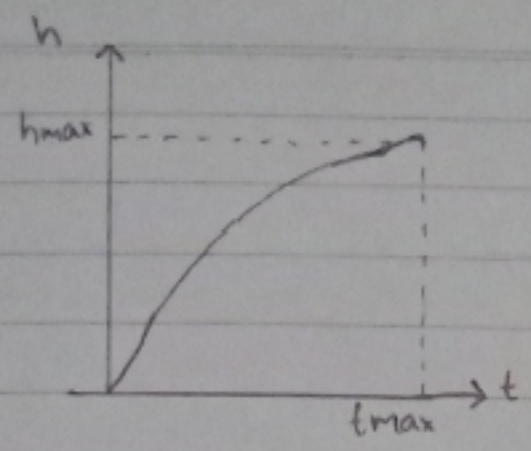
$$a_t = \left(-g - \frac{b}{m}V_0\right)e^{-\frac{bt}{m}}$$
$$= \left(-g - \frac{b}{m}V_0\right)\left(1 - \frac{bt}{m}\right)$$
$$= -g + g\frac{bt}{m} - \frac{b}{m}V_0 + \frac{b^2}{m^2}V_0 t$$

$$\boxed{a_t = -g - \frac{b}{m}V_0 + \left(\frac{b^2}{m^2}V_0 + \frac{gb}{m}\right)t}$$

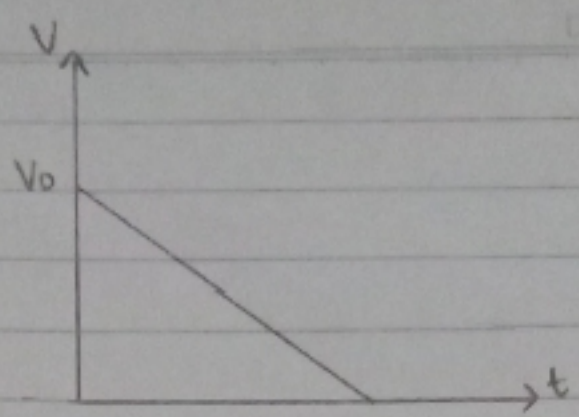
$b = 0$

$$\boxed{a_t = -g}$$

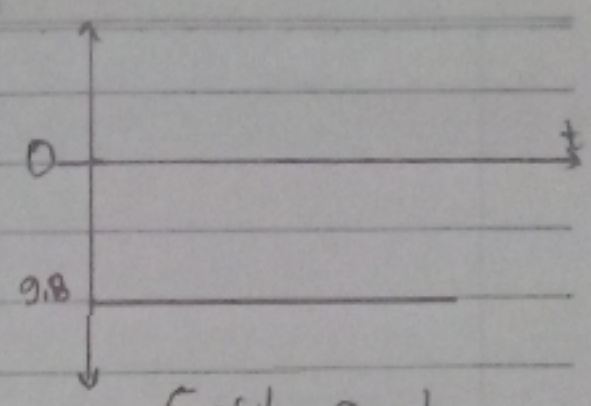
OKEY



Grafik y - t



Grafik V - t



Grafik a - t