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Gerak vertikal keatas dengan pengaruh gaya gesekan $f = -bv$

$\vec{v}\uparrow$ $\vec{w}\downarrow$ $\vec{f}\downarrow$

$\Sigma F = m.a$

$-\vec{w} - \vec{f} = m.a$

$-mg - bv^2 = m.a$

$$\int_{v_0}^{v_t} \frac{dv}{-g - \frac{bv^2}{m}} = \int_0^t dt$$

$$\int_{v_0}^{v_t} \frac{f}{v} \frac{dv}{-g - bv/m} = \int_0^t dt$$

misal $-g - \frac{bv}{m} = u$

$$-\frac{b}{m} dv = du$$

$$dv = \frac{m}{b} . du$$

$$\int 1/q \left(-\frac{m}{b} - du\right) = -\frac{m}{b} \int 1/q \, du = \frac{m}{b} \ln . u$$

$$\int_{v_0}^{v_t} \frac{1}{-g - \frac{bv^2}{m}} dv = \int_0^t dt$$

$$-\frac{m}{bv} \ln . u = t$$

$$-\frac{m}{bv} \ln \left[-g - \frac{bv^2}{m}\right]_{v_0}^{v_t} = t$$

$$-\frac{m}{bv} \left(\ln \left| -g - \frac{bv^2}{m} \right|\right)^{v} = t$$

$$\frac{-g - \frac{bv_t}{m}}{-g - \frac{bv_0}{m}} = e^{-\frac{bv}{m}}$$

$$-g - \frac{b}{m} v_t = \left(-g - \frac{b}{m} v_0\right) e^{-\frac{bv^2}{m}}$$

$$v_t = \frac{-mg}{bv} - \left(-\frac{mg}{bv} - v_0\right) e^{-\frac{bvt}{m}}$$

$f = -w$, $bv = mg$, $v_t = mg/bv$

$$v(t) = v_t - (v_t - v_0) e^{-\frac{bvt}{m}}$$

$$a(t) = \frac{dv(t)}{dt}$$

$$= \frac{d}{dt} \left(v_T - (-v_0 + v_t) e^{-\frac{bvt}{m}}\right)$$

$$= 0 - (-v_0 + v_t)\left(\frac{-bv}{m}\right) e^{\frac{bvt}{m}}$$

$$= \frac{bv}{m} (-v_0 + v_t) e^{-\frac{bvt}{m}}$$

$$= -\frac{bv}{m} \left(v_0 + \frac{bv}{m} v_t\right) e^{-\frac{bvt}{m}}$$

$$= \left(\frac{bv}{m} - \frac{mg}{bv} - \frac{bv}{m} v_0\right) e^{-\frac{bvt}{m}}$$

$$a(t) = (-g - b/m \, v_0) e^{-\frac{bvt}{m}}$$

$$v(t) = \frac{dy(t)}{dt}$$

$$\int_{y_0}^{y_x} dy(t) = \int_0^t v(t) \, dt$$

$$y(t) = \int_0^t v_t - (-v_0 + v_t) e^{-\frac{bvt}{m}}$$

$$y(t) = \left[v_t - (v_0 + v_t)(-m/bv) e^{-\frac{bvt}{m}}\right]_0^t$$

$$y(t) = \left[v_t - (v_0 + v_t)(m/bv) . e^{-\frac{bvt}{m}}\right] - \left[(-v_0 - v_t)(-m/bv)\right]$$

$$y(t) = v_t - (-m/bv)(v_0 + v_t)(e^{-\frac{bvt}{m}}$$

$$y(t) = v_t - \frac{m}{bv}(-v_0 + v_t)\left(1 - e^{-\frac{bvt}{m}}\right)$$

$y(t)$, $v(t)$, $a(t)$, diaproksimasi

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$e^{-\frac{bvt}{m}} = 1 - \frac{bvt}{m} + \frac{1}{2}\left(\frac{bvt}{m}\right)^2 + \dots$$

$$y(t) = v_t - \frac{m}{bv}(-v_0 - v_t)\left(1 - e^{-\frac{bvt}{m}}\right)$$

$$y(t) = v_t - (-v_0 + v_t)\left(t - \tfrac{1}{2}\tfrac{bv}{m}t^2\right)$$

$$= v_0 t + \frac{1}{2}\frac{bv}{m}v_0 t^2 + \frac{1}{2}\frac{bv}{m}\left(\frac{-mg}{bv}\right)t^2$$

$b = 0$

$$y(t) = v_0 t - \frac{1}{2}gt^2$$

$$v(t) = v_t - (-v_0 + v_t)e^{-\frac{bvt}{m}}$$

$$= v_t - (-v_0 + v_t)\left(1 - \left(-\frac{bvt}{m}\right)\right)$$

$$= v_0 + \frac{bv}{m}v_0 t - gt$$

$b = 0$

$$v(t) = v_0 - gt$$

$$a(t) = \left(-g - \frac{bv}{m}v_0\right)e^{-\frac{bvt}{m}}$$

$$= \left(-g - \frac{bv}{m}v_0\right)\left(1 - \frac{bvt}{m}\right)$$

$$= -g + g\frac{bvt}{m} - \frac{bv}{m}v_0 + \frac{b^2v^2}{m^2}v_0 t$$

$b = 0$

$$a(t) = -g$$