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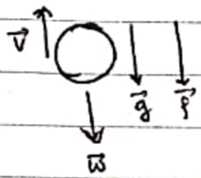
Prodi : Pendidikan Fisika (B)

MK : Mekanika

Diskusi : Gerak Vertikal yang dipengaruhi hambatan udara (Air Resistance)

$\Rightarrow$ Berdasarkan titik II Newton



$$\Sigma \vec{F} = m\vec{a}$$

$$\vec{w} + \vec{F} = -m\vec{a}$$

$$(mg + bv)\hat{y} = -ma\hat{y}$$

$$(mg + bv) = -m\vec{a} \left(\times \frac{1}{m}\right)$$

$$g + \frac{bv}{m} = -a$$

• Subtitusikan nilai $a \Rightarrow g + \frac{bv}{m} = \frac{-d^2 . y(t)}{dt}$

$$g + \frac{b}{m} v = \frac{-dv}{dt}$$

$$\frac{-dv}{g + \frac{b}{m} v} = dt$$

$$\Rightarrow \int_{v_0}^{v(t)} \frac{dv}{g + \frac{b}{m} v} = \int_0^t dt$$

misal $\rightarrow g + \frac{b}{m} v = u$

$$\frac{b}{m} dv = du$$

$$du = \frac{m}{b} dv$$

Maka, $\int_{v_0}^{v(t)} \frac{1}{u} dv = \int_0^t dt$

$$\int_{v_0}^{v(t)} \frac{1}{u} \left(\frac{m}{b} du = \int_0^t dt\right.$$

$$-\frac{m}{b} \ln |u| = t$$

$$-\frac{m}{b} \ln \left| g + \frac{b}{m} v \right|_{v_0}^{v(t)} = t$$

$$-\frac{m}{b} \left(\ln \left| g + \frac{b}{m} (v(t)) \right| - \ln \left| g + \frac{b}{m} (v_0) \right| \right)$$

Karena : $\ln a - \ln b = \ln \frac{a}{b}$, maka :

$$\ln \left| \frac{g + \frac{b}{m}(v_t)}{g + \frac{b}{m}(v_0)} \right| = \frac{-bt}{m}$$

$$\exp\left(\ln \left| \frac{g + \frac{b}{m} v(t)}{g + \frac{b}{m} v_0} \right| \right) = e^{\frac{-bt}{m}}$$

$$\Rightarrow \frac{g + \frac{b}{m}(v(t))}{g + \frac{b}{m} V_0} = e^{-\frac{bt}{m}}$$

$$\Rightarrow g + \frac{b}{m} v(t) = \left(g + \frac{b}{m} V_0 \right) e^{\frac{-bt}{m}}$$

$$\Rightarrow \frac{b}{m} \cdot v(t) = \left(g + \frac{b}{m} V_0 \right) e^{\frac{-bt}{m}} - g\left(x - \frac{m}{b} \right)$$

$$\Rightarrow V(t) = \frac{m}{b} \left(g + \frac{b}{m} V_0 \right) e^{\frac{-bt}{m}} - \frac{mg}{b}$$

$$\Rightarrow V(t)^2 = \left(\frac{mg}{b} + V_0 \right) e^{\frac{-bt}{m}} - \frac{mg}{b}$$

Karena : $f = w$

$bv = mg$

$v_T = \frac{mg}{b}$, maka :

$$\Rightarrow V_{(t)} = (V_T + V_0)\, e^{-\frac{bt}{m}} - V_T$$

* Untuk $a(t)$

$$a(t) = \frac{dv(t)}{dt}$$

$$a(t) = (V_T + V_0)\left(-\frac{b}{m}\right) e^{\frac{-bt}{m}}$$

$$a(t) = \frac{-b}{m}(V_T + V_0)\, e^{\frac{-bt}{m}}$$

$$a(t) = \left(-\frac{\cancel{b}}{\cancel{m}} \cdot \frac{\cancel{m} g}{\cancel{b}} - \frac{b}{m} V_0 \right) e^{\frac{-bt}{m}}$$

$$a(t) = \left(-g \cdot \frac{-b}{m} V_0 \right) e^{\frac{-bt}{m}}$$

* Untuk $y(t)$

$$V(t) = \frac{dy(t)}{dt}$$

$$dy(t) = v(t) - dt$$

$$\int_0^{y(t)} dy(t) = \int_0^t v(t)\, dt$$

$$y(t) = \int_0^t \left((V_T + V_0)\, e^{\frac{-bt}{m}} - V_T \right)$$

$$y(t) = \left[(V_T + V_0)\left(-\frac{m}{b}\right) e^{-bt/m} - V_T \cdot t \right]_0^x$$

$$y(t) = \left[(V_T + V_0)\left(-\frac{m}{b}\right) e^{\frac{-bt}{m}} - V_T t \right] - \left[(V_T + V_0)\left(-\frac{m}{b}\right) \right]$$

$$y(t) = -\frac{m}{b}(V_T + V_0)\left(e^{\frac{-bt}{m}} - 1\right) - V_T \cdot t$$

$$y(t) = \frac{-m}{b}(V_T + V_0)\left(1 - e^{\frac{-bt}{m}}\right) - V_T \cdot t$$

Aproksimasi

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$e^{-\frac{bt}{m}} = 1 - \frac{bt}{m} + \frac{1}{2}\left(\frac{bt}{m}\right)^2 + \dots$$

$$y(t) = -VT + \frac{m}{b}(VT + V_0)\left(1 - e^{-\frac{bt}{m}}\right)$$

$$y(t) = -VT.t + \frac{m}{b}(VT + V_0)\left(1 - 1 + \frac{bt}{m} - \frac{1}{2}\left(\frac{bt}{m}\right)^2\right)$$

$$y(t) = V_0 t = \frac{1}{2} g t^2 - \frac{1}{2} \cdot \frac{b}{m} V_0 t^2$$

$$y(t) = V_0 t - \frac{1}{2} g t^2 - \frac{1}{2} \frac{b}{m} V_0 t^2$$

* Untuk $b = 0$, maka : $y(t) = V_0 t - \frac{1}{2} g t^2$

$$\Rightarrow V(t) = (VT + V_0) e^{-\frac{bt}{m}} - VT$$

$$V(t) = -VT + (VT + V_0)\left(1 - \frac{bt}{m}\right)$$

$$V(t) = -VT + VT - \frac{b}{m} VT.t + V_0 . \frac{-b}{m} V_0.t$$

$$V(t) = V_0 - \frac{\cancel{b}}{\cancel{m}}\left(\frac{\cancel{m}g}{\cancel{b}}\right) t - \frac{b}{m} V_0 t$$

$$V(t) = V_0 - gt - \frac{b}{m} V_0.t$$

* Untuk $b = 0$, maka $\rightarrow V(t) = V_0 - gt$

$$\Rightarrow a(t) = \left(-g - \frac{b}{m} V_0\right) e^{-\frac{bt}{m}}$$

$$a(t) = \left(-g - \frac{b}{m} V_0\right)\left(1 - \frac{bt}{m}\right)$$

$$a(t) = -g + g\frac{bt}{m} - \frac{b}{m} V_0 + \frac{b^2}{m^2} V_0.t$$

$$a(t) = -g. - \frac{b}{m} . V_0 + \left(\frac{b^2}{m^2} . V_0 + \frac{gb}{m}\right)$$

Jadi, $b = 0 \rightarrow a(t) = -g.$