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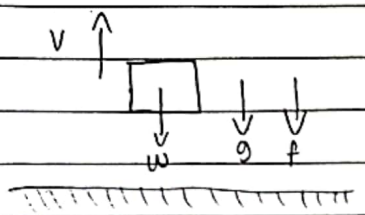
Gerak Vertikal ke Atas dengan $f = -bv$



$\sum F = ma$

$w + F = -ma$

$(m.g + bv)\hat{y} = (-ma)\hat{y}$

$g + \frac{bv}{m} = -a$

$g + \frac{b}{m}v = -\frac{dv}{dt}$

$\frac{-dv}{g + \frac{b}{m}v} = dt$

$-\int_{v_0}^{v(t)} \frac{dv}{g + \frac{b}{m}v} = \int_0^t dt$

↳ misal : $g + \frac{b}{m}v = u$

$\Rightarrow \frac{b}{m}dv = du$

$dv = \frac{m}{b}du$

↳ maka, $-\int_{v_0}^{v(t)} \frac{1}{u}dv = \int_0^t dt$

$-\int_{v_0}^{v(t)} \frac{1}{u}\left(\frac{m}{b}du\right) = \int_0^t dt$

$-\frac{m}{b}\int_{v_0}^{v(t)} \frac{1}{u}du = t$

$-\frac{m}{b}\ln|u| = t$

$-\frac{m}{b}\left(\ln\left|g + \frac{b}{m}(v(t))\right| - \ln\left|g + \frac{b}{m}(v_0)\right|\right) = t$

karena $\ln a - \ln b = \ln \frac{a}{b}$,

↳ maka, $\ln\left|\frac{g + \frac{b}{m}(v_t)}{g + \frac{b}{m}(v_0)}\right| = -\frac{bt}{m}$

$\exp\left(\ln\left|\frac{g + \frac{b}{m}v(t)}{g + \frac{b}{m}v_0}\right|\right) = e^{\frac{-bt}{m}}$

$\frac{g + \frac{b}{m}v(t)}{g + \frac{b}{m}v_0} = e^{\frac{-bt}{m}}$

$g + \frac{b}{m}v(t) = \left(g + \frac{b}{m}v_0\right)e^{\frac{-bt}{m}}$

$\frac{b}{m}v(t) = g + \frac{b}{m}v_0 e^{\frac{-bt}{m}} - g\left(x - \frac{m}{b}\right)$

$v_t = \frac{m}{b}\left(g + \frac{b}{m}v_0\right)e^{\frac{-bt}{m}} - \frac{x - m}{b}$

$v(t)^2 = \left(\frac{mg}{b} + v_0\right)e^{\frac{-bt}{m}} - \frac{mg}{b}$

karena, $f = w$

$bv = mg$

$vT = \frac{mg}{b}$

maka, $\boxed{v(t) = (vT + v_0)\ e^{\frac{-bt}{m}} - vT}$

untuk $a(t)$

$a(t) = \frac{dv(t)}{dt}$

$a(t) = \frac{d}{dt}\left((VT + V_0)\, e^{\frac{-bt}{m}} - VT\right)$

$a(t) = (VT + V_0)\left(\frac{-b}{m}\right) e^{\frac{-bt}{m}}$

$= \frac{-b}{m}\,(VT + V_0)\, e^{\frac{-bt}{m}}$

$= \left(-\frac{b}{m} \cdot \frac{mg}{b} - \frac{b}{m} V_0\right) e^{-bt/m}$

$\boxed{a(t) = \left(-g - \frac{b}{m} V_0\right) e^{\frac{-bt}{m}}}$

untuk $b = 0$

maka, $\boxed{y(t) = V_0 t - \frac{1}{2} g t^2}$

$v(t) = (VT + V_0)\, e^{\frac{-bt}{m}} - VT$

$= -VT + (VT + V_0)\left(1 - \frac{bt}{m}\right)$

$= -VT + VT - \frac{b}{m} VT\, t + V_0 - \frac{b}{m} V_0 . t$

$= V_0 - \frac{mg}{b}\, t - \frac{b}{m} V_0 t$

$v(t) = V_0 - gt - \frac{b}{m} V_0 t$

untuk $b = 0$

maka, $\boxed{v(t) = V_0 - gt}$

Untuk $y(t)$

$v(t) = \frac{dy(t)}{dt}$

$dy(t) = v(t)\, dt$

$\int_0^{y(t)} dy(t) = \int_0^t v(t)\, dt$

$y(t) = \int_0^t \left((VT + V_0)\, e^{\frac{-bt}{m}} - VT\right)$

$= \left[(VT + V_0)\left(\frac{-m}{b}\right) e^{\frac{-bt}{m}} - VT\right]_0^t$

$= \left[(VT + V_0)\left(\frac{-m}{b}\right) e^{\frac{-bt}{m}} - VT.t\right] - \left[(VT + V_0)\left(\frac{-m}{b}\right)\right]$

$= -\frac{m}{b}\,(VT + V_0)\,(e^{\frac{-bt}{m}} - 1) - VT.t$

$\boxed{y(t) = \frac{-m}{b}(VT + V_0)\left(1 - e^{\frac{-bt}{m}}\right) - VT.t}$

$a(t) = \left(-g - \frac{b}{m} V_0\right) e^{\frac{-bt}{m}}$

$= \left(-g - \frac{b}{m} V_0\right)\left(1 - \frac{bt}{m}\right)$

$= -g + g\frac{bt}{m} - \frac{b}{m} V_0 + \frac{b^2}{m^2} V_0 t$

$= -g - \frac{b}{m} V_0 + \left(\frac{b^2}{m^2} V_0 t + g\frac{b}{m}\right)$

maka, $b = 0$

$\boxed{a(t) = -g}$

Aproksimasi

$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$

$e^{\frac{-bt}{m}} = 1 - \frac{bt}{m} + \frac{1}{2}\left(\frac{bt}{m}\right)^2 + \ldots$

$y(t) = -VT + \frac{m}{b}(VT + V_0)\left(1 - e^{\frac{-bt}{m}}\right)$

$= -VT.t + \frac{m}{b}(V_T + V_0)\left(1 - 1 + \frac{bt}{m} - \frac{1}{2}\left(\frac{bt}{m}\right)^2\right)$

$= V_0 t - \frac{1}{2} g t^2 - \frac{1}{2} \frac{b}{m} V_0 t^2$

$y(t) = V_0 t - \frac{1}{2} g t^2 - \frac{1}{2} \frac{b}{m} V_0 t^2$