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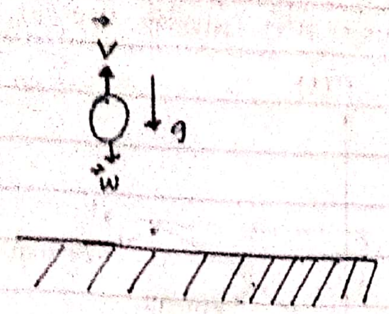
Gerak vertikal ke atas



Berdasarkan hukum 2 Newton

$\Sigma \vec{F} = m.\vec{a}$

$\vec{w} + \vec{F} = -m.\vec{a}$

$(mg + bv)\hat{y} = -ma\hat{y}$

$mg + bv = -ma$ → dikali $\frac{1}{m}$

$g + \frac{bv}{m} = -a$

$g + \frac{bv}{m} = \frac{d^2y(t)}{dt^2}$

$g + \frac{b}{m}v = -\frac{dv}{dt}$

$-\int_{V_0}^{V(t)} \frac{dv}{g + \frac{b}{m}v} = \int_0^t dt$

Misalkan $g + \frac{b}{m}v = u$

$\frac{b}{m}dv = du$

$dv = \frac{m}{b}du$

$-\int_{V_0}^{V(t)} \frac{1}{u}\, dv = \int_0^t dt$

$-\int_{V_0}^{V(t)} \frac{1}{g + \frac{b}{m}v}\, dv = \int_0^t dt$

$-\frac{m}{b} \ln\left|g + \frac{b}{m}v\right|_{V_0}^{V(t)} = t$

$-\frac{m}{b}\left(\ln\left|g + \frac{b}{m}v\right| - \ln\left|g + \frac{b}{m}(V_0)\right|\right) = t$

$\ln\left|\frac{g + \frac{b}{m}V_t}{g + \frac{b}{m}V_0}\right| = \frac{-bt}{m}$

$\exp\left(\ln\left|\frac{g + \frac{b}{m}V_t}{g + \frac{b}{m}V_0}\right|\right) = e^{\frac{-bt}{m}}$

$\frac{g + \frac{b}{m}V_t}{g + \frac{b}{m}V_0} = e^{-\frac{bt}{m}}$

$g + \frac{b}{m}V_t = \left(g + \frac{b}{m}V_0\right)e^{-\frac{bt}{m}}$

$\frac{b}{m}V_t = \left(g + \frac{b}{m}V_0\right)e^{\frac{-bt}{m}} - g$ (dikali $-\frac{m}{b}$)

$V_t = \frac{m}{b}\left(g + \frac{b}{m}V_0\right)e^{\frac{-bt}{m}} - \frac{mg}{b}$

$V_t = \left(\frac{mg}{b} + V_0\right)e^{\frac{-bt}{m}} - \frac{mg}{b}$

Karena, $F = w$, $bv = mg$, $V_T = \frac{mg}{b}$

$$\boxed{V_t = (V_T + V_0)e^{\frac{-bt}{m}} - V_T}$$

$V(t) = (V_T + V_0)e^{\frac{-bt}{m}} - V_T$

$= -V_T + (V_T + V_0)\left(1 - \frac{bt}{m}\right)$

$= -V_T + V_T - \frac{b}{m}V_T t + V_0 - \frac{b}{m}V_0 t$

$= V_0 - \frac{b}{m}\left(\frac{mg}{b}\right)t - \frac{b}{m}V_0 t$

$$\boxed{V(t) = V_0 - gt - \frac{b}{m}V_0 t}$$

Untuk $b = 0$ (tanpa gesekan udara)

$V(t) = V_0 - gt$

$a(t) = \frac{dv(t)}{dt}$

$= \frac{d}{dt}\left((V_T - V_0)e^{\frac{-bt}{m}} - V_T\right)$

$= (V_T + V_0)\left(-\frac{b}{m}\right)e^{\frac{-bt}{m}}$

$= -\frac{b}{m}(V_T + V_0)e^{\frac{-bt}{m}}$

$= \left(-\frac{b}{m} \cdot \frac{mg}{b} - \frac{b}{m}V_0\right)e^{\frac{-bt}{m}}$

$$\boxed{a(t) = \left(-g - \frac{b}{m}V_0\right)e^{\frac{-bt}{m}}}$$

$V(t) = \frac{dy(t)}{dt}$

$\int_0^{y(t)} dy(t) = \int_0^t V(t)\, dt$

$y(t) = \int_0^t \left((V_T + V_0)e^{\frac{-bt}{m}}\right)$

$$y(t) = \left[(v_f + v_0)\left(-\frac{m}{b}\right)e^{-\frac{bt}{m}} - v_f \cdot t\right]_0^t$$

$$y(t) = \left[(v_f + v_0)\left(-\frac{m}{b}\right)e^{-\frac{bt}{m}} - v_f \cdot t\right] - \left[(v_f + v_0)\left(-\frac{m}{b}\right)\right]$$

$$y(t) = -\frac{m}{b}(v_f + v_0)\left(e^{-\frac{bt}{m}}\right) - v_f \cdot t$$

$$\boxed{y(t) = \frac{m}{b}(v_f + v_0)\left(1 - e^{-\frac{bt}{m}}\right) - v_f \cdot t}$$

Aproksimasi

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots$$

$$e^{-\frac{bt}{m}} = 1 - \frac{bt}{m} + \frac{1}{2}\left(\frac{bt}{m}\right)^2 + \ldots$$

$$y(t) = -v_f t + \frac{m}{b}(v_f + v_0)\left(1 - e^{-\frac{bt}{m}}\right)$$

$$= -v_f t + \frac{m}{b}(v_f + v_0)\left(1 - 1 + \frac{bt}{m} - \frac{1}{2}\left(\frac{bt}{m}\right)^2\right)$$

$$= -v_f t + (v_f + v_0)\left(t - \frac{1}{2}\frac{b}{m}t^2\right)$$

$$= -v_f t + v_f t - \frac{1}{2}\frac{b}{m}v_f t^2 + v_0 t - \frac{1}{2}\frac{b}{m}v_0 t^2$$

$$= v_0 t - \frac{1}{2}\frac{b}{m}\left(\frac{mg}{b}\right)t^2 - \frac{1}{2}\frac{b}{m}v_0 t^2$$

$$= v_0 t - \frac{1}{2}gt^2 - \frac{1}{2}\frac{b}{m}v_0 t^2$$

$$\boxed{y(t) = v_0 t - \frac{1}{2}gt^2 - \frac{1}{2}\frac{b}{m}v_0 t^2}$$

Jika tanpa gesekan udara $(b=0)$

$$y(t) = v_0 t - \frac{1}{2}gt^2$$

$$a(t) = \left(-g - \frac{b}{m}v_0\right)e^{-bt/m}$$

$$= \left(-g - \frac{b}{m}v_0\right)\left(1 - \frac{bt}{m}\right)$$

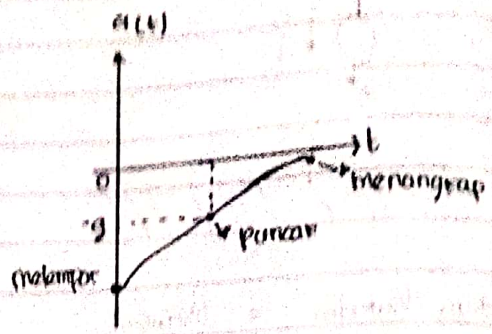
$$= -g + g\frac{bt}{m} - \frac{b}{m}v_0 + \frac{b^2}{m^2}v_0 t$$

$$\boxed{a(t) = -g - \frac{b}{m}v_0 + \left(\frac{b^2}{m^2}v_0 + \frac{gb}{m}\right)t}$$

Jika tanpa gesekan udara $(b=0)$

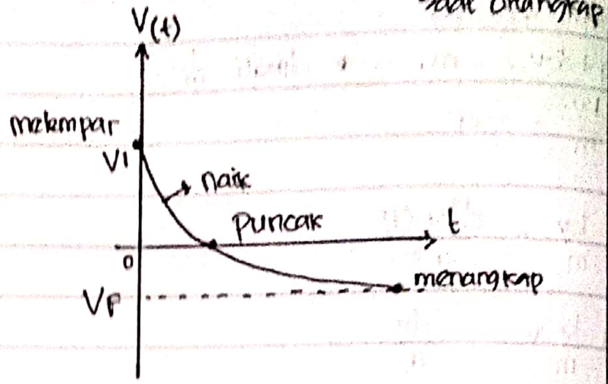
$$a(t) = -g$$

Grafik fungsi a(t) terhadap waktu



Grafik fungsi V(t) terhadap waktu

→ V_i : kecepatan awal, V_f : kecepatan saat ditangkap



Grafik fungsi y(t) terhadap waktu