No.

Date :

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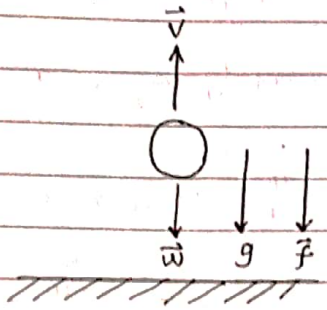
Kelas : B

Gerak Vertikal yang dipengaruhi hambatan udara



Berdasarkan hukum 2 Newton

$$\Sigma \vec{F} = m.\vec{a}$$

$$\vec{w} + \vec{f} = -m.\vec{a}$$

$$(mg + bv)\hat{y} = -m.a.\hat{y}$$

$$mg + bv = -m.a$$

$$g + \frac{bv}{m} = -a$$

$$g + \frac{bv}{m} = \frac{d^2y(t)}{dt^2}$$

$$g + \frac{b}{m}v = -\frac{dv}{dt}$$

$$\int_{v_0}^{v(t)} \frac{dv}{g + \frac{b}{m}v} = \int_0^t dt$$

misalkan $g + \frac{b}{m}v = u$

$$\frac{b}{m}dv = du$$

$$dv = \frac{m}{b}du$$

$$-\int_{v_0}^{v(t)} \frac{1}{u}\,dv = \int_0^t dt$$

$$-\int_{v_0}^{v(t)} \frac{1}{g + \frac{b}{m}v}\,dv = \int_0^t dt$$

$$-\frac{m}{b}\ln\left|g + \frac{b}{m}v\right|_{v(0)}^{v(t)} = t$$

$$-\frac{m}{b}\left(\ln\left|g + \frac{b}{m}v\right| - \ln\left|g + \frac{b}{m}(v_0)\right|\right) = t$$

$$\ln\left|\frac{g + \frac{b}{m}vt}{g + \frac{b}{m}v_0}\right| = -\frac{bt}{m}$$

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$$\exp\left(\ln\left|\frac{g+\frac{b}{m}v_t}{g+\frac{b}{m}v_0}\right|\right) = e^{-\frac{bt}{m}}$$

$$\frac{g+\frac{b}{m}v_t}{g+\frac{b}{m}v_0} = e^{-\frac{bt}{m}}$$

$$g+\frac{b}{m}v_t = \left(g+\frac{b}{m}v_0\right)e^{-\frac{bt}{m}}$$

$$\frac{b}{m}v_t = \left(g+\frac{b}{m}v_0\right)e^{-\frac{bt}{m}} - g$$

$$v_t = \frac{m}{b}\left(g+\frac{b}{m}v_0\right)e^{-\frac{bt}{m}} - \frac{mg}{b}$$

$$v_t = \left(\frac{mg}{b}+v_0\right)e^{-\frac{bt}{m}} - \frac{mg}{b}$$

Karena, $F = w$, $bv = mg$, $v_T = \frac{mg}{b}$

$$v_t = (v_T + v_0)\,e^{-\frac{bt}{m}} - v_T$$

$$v(t) = (v_T+v_0)\,e^{-\frac{bt}{m}} - v_T$$

$$= -v_T + (v_T+v_0)\left(1-\frac{bt}{m}\right)$$

$$= -v_T + v_T - \frac{b}{m}v_T t + v_0 - \frac{b}{m}v_0 t$$

$$= v_0 - \frac{b}{m}\left(\frac{mg}{b}\right)t - \frac{b}{m}v_0 t$$

$$v(t) = v_0 - gt - \frac{b}{m}v_0 t$$

untuk $b = 0$ (tanpa gesekan udara)

$$v(t) = v_0 - gt$$

$$a(t) = \frac{dv(t)}{dt}$$

$$= \frac{d}{dt}\left((v_T - v_0)\,e^{-\frac{bt}{m}} - v_T\right)$$

$$= (v_T+v_0)\left(-\frac{b}{m}\right)e^{-\frac{bt}{m}}$$

$$= -\frac{b}{m}(v_T+v_0)\,e^{-\frac{bt}{m}}$$

$$= \left(-\frac{b}{m}\cdot\frac{mg}{b} - \frac{b}{m}v_0\right)e^{-\frac{bt}{m}}$$

$$a(t) = \left(-g - \frac{b}{m}v_0\right)e^{-\frac{bt}{m}}$$

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$$v_t = \frac{dy(t)}{dt}$$

$$\int_0^{y(t)} dy(t) = \int_0^t v(t)\, dt$$

$$y(t) = \int_0^t \left((v_T + v_0)\, e^{-\frac{bt}{m}} v_T \right)$$

$$y(t) = \left[(v_T + v_0)\left(-\frac{m}{b}\right) e^{-\frac{bt}{m}} - v_T \cdot t \right]_0^t$$

$$y(t) = \left[(v_T + v_0)\left(-\frac{m}{b}\right) e^{-\frac{bt}{m}} - v_T \cdot t \right] - \left[v_T + v_0 \left(-\frac{m}{b}\right) \right]$$

$$y(t) = -\frac{m}{b} (v_T + v_0)\left(e^{-\frac{bt}{m}}\right) - v_T \cdot t$$

$$y(t) = \frac{m}{b} (v_T + v_0)\left(1 - e^{-\frac{bt}{m}}\right) - v_T \cdot t$$

$$y(t) = \frac{m}{b} (v_T + v_0)\left(1 - e^{-\frac{bt}{m}}\right) - v_T \cdot t$$

Aproksimasi

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots$$

$$e^{-\frac{bt}{m}} = 1 - \frac{bt}{m} + \frac{1}{2}\left(\frac{bt}{m}\right)^2 + \cdots$$

$$y(t) = -v_T \cdot t + \frac{m}{b} (v_T + v_0)\left(1 - e^{-bt}\right)$$

$$= -v_T \cdot t + \frac{m}{b} (v_T + v_0)\left(1 - 1 + \frac{bt}{m} - \frac{1}{2}\left(\frac{bt}{m}\right)^2\right)$$

$$= -v_T \cdot t + (v_T + v_0)\left(t - \frac{1}{2}\frac{b}{m} t^2\right)$$

$$= -v_T t + v_T t - \frac{1}{2}\frac{b}{m} v_T t^2 + v_0 t - \frac{1}{2}\frac{b}{m} v_0 t^2$$

$$= v_0 t - \frac{1}{2}\frac{b}{m}\left(\frac{mg}{b}\right) t^2 - \frac{1}{2}\frac{b}{m} v_0 t^2$$

$$= v_0 t + \frac{1}{2} g t^2 - \frac{1}{2}\frac{b}{m} v_0 t^2$$

$$y(t) = v_0 t - \frac{1}{2} g t^2 - \frac{1}{2}\frac{b}{m} v_0 t^2$$

Jika tanpa gesekan udara ($b = 0$)

$$y(t) = v_0 t - \frac{1}{2} g t^2$$

$$a(t) = \left(-g - \frac{b}{m} v_0\right) e^{-\frac{bt}{m}}$$

$$= \left(-g - \frac{b}{m} v_0\right)\left(1 - \frac{bt}{m}\right)$$

$$= -g + g\frac{bt}{m} - \frac{b}{m} v_0 + \frac{b^2}{m^2} v_0 t$$

$$a(t) = -g - \frac{b}{m} v_0 + \left(\frac{b^2}{m^2} v_0 + \frac{gb}{m}\right) t$$

maka, $a(t) = -g$

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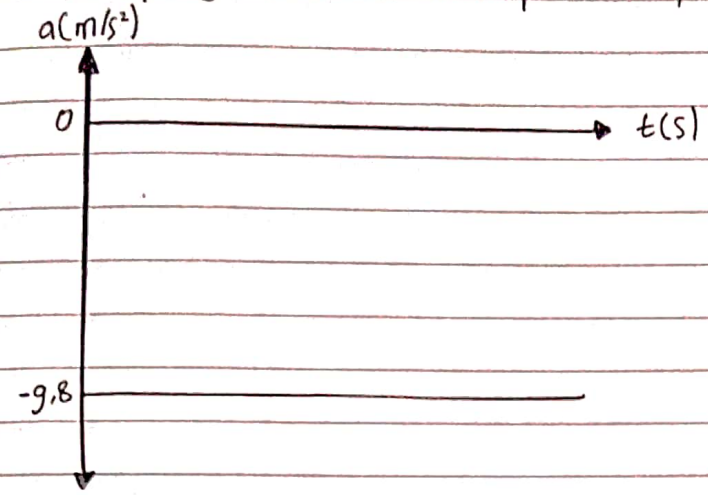
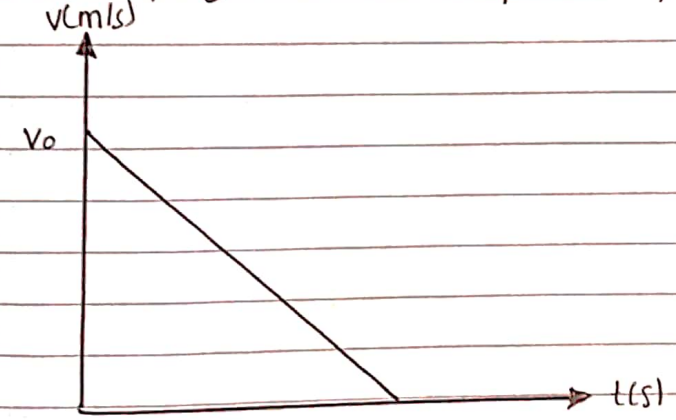
grafik fungsi a(t) terhadap waktu pada gerak vertikal keatas



Grafik fungsi v(t) terhadap waktu pada gerak vertikal keatas



Grafik fungsi y(t) terhadap waktu pada gerak vertikal keatas