No 6VA
Date

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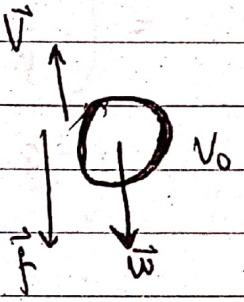
Gerak Vertikal ke Atas + hambatan udara



Gunakan Hk. 2 Newton

$$\Sigma \vec{F} = m \cdot \vec{a}$$

$$-\vec{w} - \vec{f} = m \cdot \vec{a}$$

$$-mg - bv\,\hat{y} = m.a\,\hat{y}$$

$$-mg - bv = m.a \quad \left(\text{dikali } \frac{1}{m}\right)$$

$$-g - \frac{bv}{m} = a$$

$$-\left(g + \frac{b}{m}v\right) = \frac{dv}{dt}$$

$$-\frac{dv}{g + \frac{b}{m}v} = dt$$

$$\int_{v_0}^{v} -\frac{dv}{g + \frac{b}{m}v} = \int_0^t dt$$

Misalkan : $g + \frac{b}{m}v = u$

$$\frac{b}{m}dv = du$$

$$dv = \frac{m}{b}du$$

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$v(t)$

*) $\int_{v_0}^{v(t)} \frac{1}{u}\left(\frac{m}{b}\,du\right) = \int_0^t dt$

$-\frac{m}{b}\int_{v_0}^{v(t)} \frac{1}{u}\,du = t$

$-\frac{m}{b}\ln|u| = t$

$-\frac{m}{b}\ln\left|g+\frac{b}{m}v\right|_{v_0}^{v(t)} = t$

$-\frac{m}{b}\left(\ln\left|g+\frac{b}{m}(v(t))\right| - \ln\left|g+\frac{b}{m}(v_0)\right|\right) = t$

$\ln a - \ln b = \ln\frac{a}{b}$, maka:

$\ln\left|\frac{g+\frac{b}{m}(v_t)}{g+\frac{b}{m}(v_0)}\right| = -\frac{b}{m}t$

$\exp\left(\ln\left|\frac{g+\frac{b}{m}(v_t)}{g+\frac{b}{m}(v_0)}\right|\right) = e^{-b/m\,t}$

$g+\frac{b}{m}(v(t)) = \left(g+\frac{b}{m}v_0\right)e^{-bt/m}$

$\frac{b}{m}v(t) = \left(g+\frac{b}{m}v_0\right)e^{-bt/m} + g\left(\frac{x-m}{b}\right)$

$v(t) = \frac{m}{b}\left(g+\frac{b}{m}v_0\right)e^{-bt/m} - \frac{mg}{b}$

$v(t)^2 = \left(\frac{mg}{b}+v_0\right)e^{-bt/m} - \frac{mg}{b}$

~ $f = w$

~ $bv = mg$

~ $v_T = \frac{mg}{b}$, jadi:

$\boxed{v(t) = (v_T+v_0)e^{-bt/m} - v_T)}$

*) $a(t) = \frac{dv(t)}{dt}$

$a(t) = \frac{d}{dt}\left((v_T+v_0)e^{-bt/m} - v_T\right)$

$= (v_T+v_0)\left(-\frac{b}{m}\right)e^{-bt/m}$

$= -b/m\,(v_T+v_0)e^{-bt/m}$

$= \left(-\frac{b}{m}\cdot\frac{mg}{b} - \frac{b}{m}\cdot v_0\right)e^{-bt/m}$

$\boxed{a(t) = \left(-g-\frac{b}{m}v_0\right)e^{-bt/m}}$

*) Untuk $y(t)$

$v(t) = \frac{dy(t)}{dt}$

$\int_0^{y(t)} dy(t) = \int_0^t v(t)\,dt$

$y(t) = \int_0^t \left((v_T+v_0)e^{-\frac{bt}{m}} - v_T\right)$

$= \left[(v_T+v_0)\left(-\frac{m}{b}\right)e^{-bt/m} - v_T.t\right]_0^t$

$= \left[(v_T+v_0)\left(-\frac{m}{b}\right)e^{-bt/m} - v_T.t\right]$

$-\left[v_T+v_0\left(-\frac{m}{b}\right)\right]$

$y(t) = -\frac{m}{b}(v_T+v_0)\left(e^{-bt/m}\right) - v_T.t$

$\boxed{y(t) = \frac{m}{b}(v_T+v_0)(1-e^{-bt/m}) - v_T.t}$

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*) Aproksimasi

$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots$

$e^{-\frac{bt}{m}} = 1 - \frac{bt}{m} + \frac{1}{2}\left(\frac{bt}{m}\right)^2 + \cdots$

$y(t) = -v_T.t + \frac{m}{b}(v_T + v_0)\left(1 - e^{-bt/m}\right)$

$= -v_T.t + \frac{m}{b}(v_T + v_0)\left(1 - 1 + \frac{bt}{m} - \frac{1}{2}\left(\frac{bt}{m}\right)^2\right)$

$= -v_T.t + (v_T + v_0)\left(t - \frac{1}{2}\frac{b}{m}t^2\right)$

$= -v_T.t + v_T t - \frac{1}{2}\frac{b}{m}v_T t^2 + v_0 t - \frac{1}{2}\frac{b}{m}v_0 t^2$

$= v_0 t - \frac{1}{2}\frac{b}{m}\left(\frac{mg}{b}\right)t^2 - \frac{1}{2}\frac{b}{m}v_0 t^2$

$= v_0 t - \frac{1}{2}gt^2 - \frac{1}{2}\frac{b}{m}v_0 t^2$

$\boxed{y(t) = v_0 t - \frac{1}{2}gt^2 - \frac{1}{2}\frac{b}{m}v_0 t^2}$

Jika, $b = 0$ (tanpa gesekan udara)

$y(t) = v_0 t - \frac{1}{2}gt^2$

*) $v(t)$

$v(t) = (v_T + v_0)e^{-bt/m} - v_T$

$= -v_T + (v_T + v_0)\left(1 - \frac{bt}{m}\right)$

$= -\frac{b}{m}v_T + v_0 - \frac{b}{m}v_0 t$

$= v_0 - \frac{b}{m}\left(\frac{mg}{b}\right)t - \frac{b}{m}v_0 t$

$\boxed{v(t) = v_0 - gt - \frac{b}{m}v_0 t}$

Untuk $b = 0$,

$v(t) = v_0 - gt$

*) Untuk $a(t)$

$a(t) = \left(-g - \frac{b}{m}v_0\right)e^{-bt/m}$

$= \left(-g - \frac{b}{m}v_0\right)\left(1 - bt/m\right)$

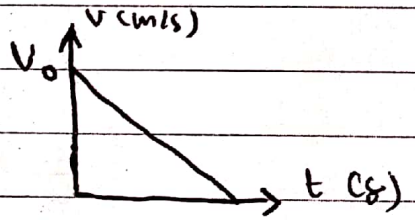
$= -g + g.\frac{bt}{m} - \frac{b}{m}v_0 + \frac{b^2}{m^2}v_0 t$

$\boxed{a(t) = -g - \frac{b}{m}v_0 + \left(\frac{b^2}{m^2}v_0 + \frac{gb}{m}\right)t}$

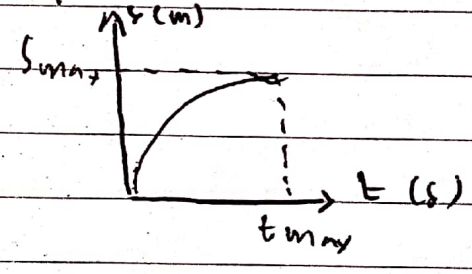
Untuk, $b = 0$

$a(t) = -g$

*) Grafik $v - t$ GVA



*) Grafik $y - t$ GVA



*) Grafik $a - t$ GVA