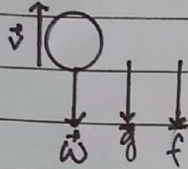


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Kelas B



Berdasarkan HK II Newton

$$\sum \vec{F} = m\vec{a}$$

$$\vec{w} + \vec{f} = -m\vec{a}$$

$$(mg + bv)\hat{y} = -m\vec{a}\hat{y}$$

$$(mg + bv) = -ma \left(\times \frac{1}{a} \right)$$

$$g + \frac{bv}{m} = -a$$

Substitusikan nilai a

$$\Rightarrow g + \frac{b}{m}v = -\frac{d^2y(t)}{dt^2}$$

$$g + \frac{b}{m}v = -\frac{dv}{dt}$$

$$-dv = dt$$

$$-\int_{v_0}^{v(t)} \frac{dv}{g + \frac{b}{m}v} = \int_0^t dt$$

$$\Rightarrow \text{Misalkan } g + \frac{b}{m}v = u,$$

$$= \frac{b}{m}dv = du$$

$$du = \frac{m}{b} du$$

maka,

$$\int_{v_0}^{v(t)} \frac{1}{u} dv = \int_0^t dt$$

$$-\int_{v_0}^{v(t)} \frac{1}{u} \left(\frac{m}{b} du \right) = \int_0^t dt$$

$$-\frac{m}{b} \int_{v_0}^{v(t)} \frac{1}{u} du = t$$

$$-\frac{m}{b} \ln |u| = t$$

$$-\frac{m}{b} \ln \left| g + \frac{b}{m}v \right|_{v_0}^{v(t)} = t$$

$$-\frac{m}{b} \left(\ln \left| g + \frac{b}{m}v(t) \right| - \ln \left| g + \frac{b}{m}v_0 \right| \right) = t$$

Karena: $\ln a - \ln b = \ln \frac{a}{b}$, maka

$$\ln \left| \frac{g + \frac{b}{m}v(t)}{g + \frac{b}{m}v_0} \right| = -\frac{bt}{m}$$

$$\exp \left(\ln \left| \frac{g + \frac{b}{m}v(t)}{g + \frac{b}{m}v_0} \right| \right) = e^{-\frac{bt}{m}}$$

$$\Rightarrow \frac{g + \frac{b}{m}v(t)}{g + \frac{b}{m}v_0} = e^{-\frac{bt}{m}}$$

$$g + \frac{b}{m}v(t) = \left(g + \frac{b}{m}v_0 \right) e^{-\frac{bt}{m}}$$

$$\frac{b}{m}v(t) = \left(g + \frac{b}{m}v_0 \right) e^{-\frac{bt}{m}} - g \left(\times \frac{m}{b} \right)$$

$$v(t) = \frac{m}{b} \left(g + \frac{b}{m}v_0 \right) e^{-\frac{bt}{m}} - \frac{mg}{b}$$

$$v(t) = \left(\frac{mg}{b} + v_0 \right) e^{-\frac{bt}{m}} - \frac{mg}{b}$$

Karena: $f = \omega$

$$bv = mg$$

$$v_T = \frac{mg}{b} \text{ maka,}$$

$$v(t) = (v_T + v_0) e^{-\frac{bt}{m}} - v_T$$

untuk $a(t)$

$$\Rightarrow a(t) = \frac{dv(t)}{dt}$$

$$a(t) = \frac{d}{dt} \left((v_T + v_0) e^{-\frac{bt}{m}} - v_T \right)$$

$$a(t) = (v_T + v_0) \left(-\frac{b}{m} \right) e^{-\frac{bt}{m}} \\ = -\frac{b}{m} (v_T + v_0) e^{-\frac{bt}{m}}$$

$$= \left(-\frac{b}{m} \cdot \frac{mg}{b} - \frac{b}{m} v_0 \right) e^{-\frac{bt}{m}}$$

$$\Rightarrow a(t) = \left(-g - \frac{b}{m} v_0 \right) e^{-\frac{bt}{m}}$$

$\Rightarrow$ untuk $y(t)$

$$v(t) = \frac{dy(t)}{dt}$$

$$dy(t) = v(t) \cdot dt$$

$$\int_0^{y(t)} dy(t) = \int_0^t v(t) dt$$

$$y(t) = \int_0^t (v_T + v_0) e^{-\frac{bt}{m}} - v_T dt$$

$$y(t) = \left[(v_T + v_0) \left(-\frac{m}{b} \right) e^{-\frac{bt}{m}} - v_T t \right]_0^t$$

$$y(t) = \left[(v_T + v_0) \left(-\frac{m}{b} \right) e^{-\frac{bt}{m}} - v_T t \right] - \left[(v_T + v_0) \left(-\frac{m}{b} \right) \right]$$

$$y(t) = -\frac{m}{b} (v_T + v_0) (e^{-\frac{bt}{m}} - 1) - v_T \cdot t$$

$$\Rightarrow y(t) = -\frac{m}{b} (v_T + v_0) (1 - e^{-\frac{bt}{m}}) - v_T \cdot t$$

$\Rightarrow$ Aproksimasi

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$e^{-\frac{bt}{m}} = 1 - \frac{bt}{m} + \frac{1}{2} \left(\frac{bt}{m} \right)^2 + \dots$$

$$\Rightarrow y(t) = -v_T t + \frac{m}{b} (v_T + v_0) (1 - e^{-\frac{bt}{m}})$$

$$= -v_T t + \frac{m}{b} (v_T + v_0) \left(1 - 1 + \frac{bt}{m} - \frac{1}{2} \left(\frac{bt}{m} \right)^2 \right)$$

$$= V_0 t - \frac{1}{2} g t^2 - \frac{1}{2} \frac{b}{m} V_0 t^2$$

$$\Rightarrow y(t) = V_0 t - \frac{1}{2} g t^2 - \frac{1}{2} \frac{b}{m} V_0 t^2$$

Untuk $b=0$ maka,

$$y(t) = V_0 t - \frac{1}{2} g t^2$$

$\rightarrow V(t)$

$$V(t) = (V_T + V_0) e^{-bt/m} - V_T$$

$$= -V_T + (V_T + V_0) (1 - bt/m)$$

$$= \cancel{-V_T} + \cancel{V_T} - \frac{b}{m} V_T t + V_0 - \frac{b}{m} V_0 t$$

$$= V_0 - \frac{b}{m} \left(\frac{mg}{b} \right) t - \frac{b}{m} V_0 t$$

$$V(t) = V_0 - g t - \frac{b}{m} V_0 t$$

Untuk $b=0$ maka,

$$V(t) = V_0 - g t$$

$\Rightarrow a(t)$

$$a(t) = (-g - b/m V_0) e^{-bt/m}$$

$$= (-g - b/m V_0) (1 - bt/m)$$

$$= -g + g \frac{bt}{m} - \frac{b}{m} V_0 + \frac{b^2}{m^2} V_0 t$$

$$a(t) = -g - \frac{b}{m} V_0 + \left(\frac{b^2}{m^2} V_0 t + \frac{gb}{m} \right) \cancel{\frac{1}{m}} \cancel{(1 - bt/m)}$$

maka, $b=0$

$$a(t) = -g$$