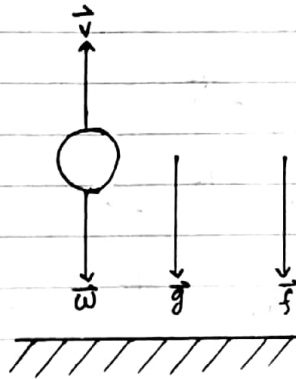


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Gerak Vertikal Yang Dipengaruhi Hambatan Udara



Berdasarkan Hukum 2 Newton

$$\sum \vec{F} = m \cdot \vec{a}$$

$$\vec{w} + \vec{F} = -m \cdot \vec{a}$$

$$(mg + bv) \hat{y} = -m \cdot a \cdot \hat{y}$$

$$mg + bv = -m \cdot a$$

$$g + \frac{bv}{m} = -a$$

$$g + \frac{bv}{m} = \frac{d^2 y(t)}{dt^2}$$

$$g + \frac{b}{m} v = -\frac{dv}{dt}$$

$$\int_{v_0}^{v(t)} \frac{dv}{g + \frac{b}{m} v} = \int_0^t dt$$

misalkan $g + \frac{b}{m} v = u$

$$\frac{b}{m} dv = du$$

$$dv = \frac{m}{b} du$$

$$-\int_{v_0}^{v(t)} \frac{1}{u} du = \int_0^t dt$$

$$-\int_{v_0}^{v(t)} \frac{1}{g + \frac{b}{m} v} dv = \int_0^t dt$$

$$-\frac{m}{b} \ln \left| g + \frac{b}{m} v \right|_{v_0}^{v(t)} = t$$

$$-\frac{m}{b} \left(\ln \left| g + \frac{b}{m} v \right| - \ln \left| g + \frac{b}{m} (V_0) \right| \right) = t$$

$$\ln \left| \frac{g + \frac{b}{m} v_t}{g + \frac{b}{m} V_0} \right| = -\frac{bt}{m}$$

$$\exp \left(\ln \left| \frac{g + \frac{b}{m} v_t}{g + \frac{b}{m} V_0} \right| \right) = e^{-\frac{bt}{m}}$$

$$\frac{g + \frac{b}{m} v_t}{g + \frac{b}{m} V_0} = e^{-\frac{bt}{m}}$$

$$g + \frac{b}{m} v_t = \left(g + \frac{b}{m} V_0 \right) e^{-\frac{bt}{m}}$$

$$\frac{b}{m} v_t = \left(g + \frac{b}{m} V_0 \right) e^{-\frac{bt}{m}} - g$$

$$v_t = \frac{m}{b} \left(g + \frac{b}{m} V_0 \right) e^{-\frac{bt}{m}} - \frac{mg}{b}$$

$$v_t = \left(\frac{mg}{b} + V_0 \right) e^{-\frac{bt}{m}} - \frac{mg}{b}$$

Karena, $F = \omega$, $bv = mg$, $V_T = \frac{mg}{b}$

$$v_t = (V_T + V_0) e^{-\frac{bt}{m}} - V_T$$

$$\begin{aligned} V(t) &= (V_T + V_0) e^{-\frac{bt}{m}} - V_T \\ &= -V_T + (V_T + V_0) \left(1 - \frac{bt}{m} \right) \\ &= -V_T + V_T - \frac{b}{m} V_T t + V_0 - \frac{b}{m} V_0 t \\ &= V_0 - \frac{b}{m} \left(\frac{mg}{b} \right) t - \frac{b}{m} V_0 t \end{aligned}$$

$$V(t) = V_0 - gt - \frac{b}{m} V_0 t$$

Untuk $b = 0$ (tanpa gesekan udara)

$$v(t) = V_0 - gt$$

$$a(t) = \frac{dv(t)}{dt}$$

$$= \frac{d}{dt} \left((V_T + V_0) e^{-\frac{bt}{m}} - V_T \right)$$

$$= -\frac{b}{m} (V_T + V_0) e^{-\frac{bt}{m}}$$

$$= \left(-\frac{b}{m} \cdot \frac{mg}{b} - \frac{b}{m} V_0 \right) e^{-\frac{bt}{m}}$$

$$a(t) = \left(-g - \frac{b}{m} V_0 \right) e^{-\frac{bt}{m}}$$

$$v_t = \frac{dy(t)}{dt}$$

$$\int_0^{y(t)} dy(t) = \int_0^t v(t) dt$$

$$y(t) = \int_0^t (v_T + v_0) e^{-\frac{bt}{m}} v_T dt$$

$$y(t) = \left[(v_T + v_0) \left(-\frac{m}{b}\right) e^{-\frac{bt}{m}} - v_T \cdot t \right]_0^t$$

$$y(t) = \left[(v_T + v_0) \left(-\frac{m}{b}\right) e^{-\frac{bt}{m}} - v_T \cdot t \right] - \left[v_T + v_0 \left(-\frac{m}{b}\right) \right]$$

$$y(t) = -\frac{m}{b} (v_T + v_0) \left(e^{-\frac{bt}{m}}\right) - v_T \cdot t$$

$$y(t) = \frac{m}{b} (v_T + v_0) \left(1 - e^{-\frac{bt}{m}}\right) - v_T \cdot t$$

$$y(t) = \frac{m}{b} (v_T + v_0) \left(1 - e^{-\frac{bt}{m}}\right) - v_T \cdot t$$

Aproksimasi

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots$$

$$e^{-\frac{bt}{m}} = 1 - \frac{bt}{m} + \frac{1}{2} \left(\frac{bt}{m}\right)^2 + \dots$$

$$y(t) = -v_T \cdot t + \frac{m}{b} (v_T + v_0) \left(1 - e^{-\frac{bt}{m}}\right)$$

$$= -v_T \cdot t + \frac{m}{b} (v_T + v_0) \left(1 - 1 + \frac{bt}{m} - \frac{1}{2} \left(\frac{bt}{m}\right)^2\right)$$

$$= -v_T \cdot t + (v_T + v_0) \left(t - \frac{1}{2} \frac{b}{m} t^2\right)$$

$$= -v_T t + v_T t - \frac{1}{2} \frac{b}{m} v_T t^2 + v_0 t - \frac{1}{2} \frac{b}{m} v_0 t^2$$

$$= v_0 t - \frac{1}{2} \frac{b}{m} \left(\frac{mg}{b}\right) t^2 - \frac{1}{2} \frac{b}{m} v_0 t^2$$

$$= v_0 t + \frac{1}{2} g t^2 - \frac{1}{2} \frac{b}{m} v_0 t^2$$

$$y(t) = v_0 t - \frac{1}{2} g t^2 - \frac{1}{2} \frac{b}{m} v_0 t^2$$

Jika tanpa gesekan udara ($b = 0$)

$$y(t) = v_0 t - \frac{1}{2} g t^2$$

$$a(t) = \left(-g - \frac{b}{m} v_0\right) e^{-\frac{bt}{m}}$$

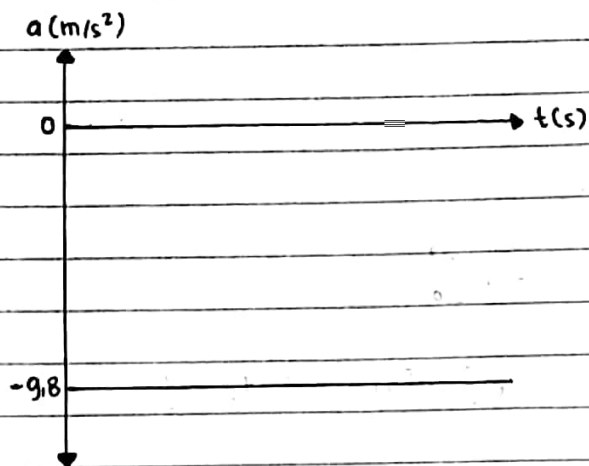
$$= \left(-g - \frac{b}{m} v_0\right) \left(1 - \frac{bt}{m}\right)$$

$$= -g + g \frac{bt}{m} - \frac{b}{m} v_0 + \frac{b^2}{m^2} v_0 t$$

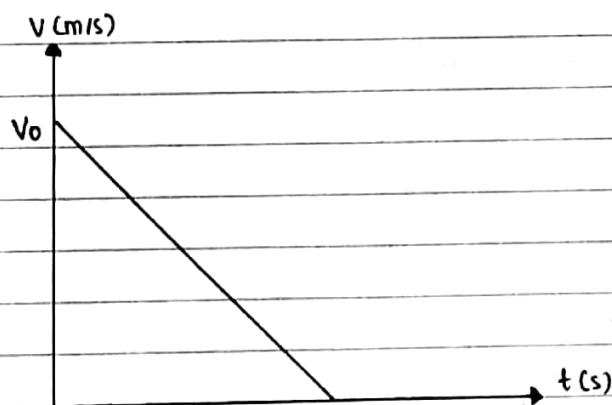
$$a(t) = -g - \frac{b}{m} v_0 + \left(\frac{b^2}{m^2} v_0 + \frac{b}{m}\right) t$$

maka, $a(t) = -g$

Grafik fungsi $a(t)$ terhadap waktu pada gerak vertikal ke atas



Grafik fungsi $v(t)$ terhadap waktu pada gerak vertikal ke atas



Grafik Fungsi $y(t)$ terhadap waktu pada gerak vertikal ke atas

