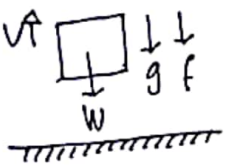
Nama : Tri Lestari
NPM : 2013022058
Kelas : B
Matkul : Mekanika

Gerak Vertikal keatas dengan $f=-bv$



$\Sigma f = ma$

$w + f = m \cdot a$

$(mg + bv)\hat{y} = (-ma)\hat{y}$

$g + \frac{bv}{m} = -a$

$g + \frac{bv}{m} = \frac{-dv}{dt}$

$\frac{-dv}{g+\frac{b}{m}v} = dt$

$-\int_{v_0}^{v(t)} \frac{dv}{g+\frac{b}{m}v} = \int_0^t dt$

Misal : $g + \frac{b}{m}v = u$ maka, $\frac{b}{m}dv = du$

$dv = \frac{m}{b}du$

Maka : $-\int_{v_0}^{v(t)} \frac{1}{u}dv = \int_0^t dt$

$-\int_{v_0}^{v(t)} \frac{1}{u}\left(\frac{m}{b}du\right) = \int_0^t dt$

$-\frac{m}{b}\int_{v_0}^{v(t)} \frac{1}{u}du = t$

$-\frac{m}{b}\int \ln|u| = t$

$-\frac{m}{b}\left(\ln\left|g+\frac{b}{m}(v(t))\right| - \ln\left|g+\frac{b}{m}(v_0)\right|\right) = t$

Karena $\ln a - \ln b = \ln \frac{a}{b}$

maka, $\ln\left|\frac{g+\frac{b}{m}(vt)}{g+\frac{b}{m}(v_0)}\right| = -\frac{bt}{m}$

$\frac{g+\frac{b}{m}v(t)}{g+\frac{b}{m}v_0} = e^{-\frac{bt}{m}}$

$g+\frac{b}{m}(vt) - (g+\frac{b}{m}v_0)e^{-\frac{bt}{m}}$

$\frac{b}{m}v(t) = g+\frac{b}{m}v_0 e^{-\frac{bt}{m}} - g(\frac{m}{b})$

$v(t) = \frac{m}{b}(g+\frac{b}{m}v_0)e^{-\frac{bt}{m}} - x - \frac{m}{b}$

$v(t)^2 = (\frac{mg}{b}+v_0)e^{-\frac{bt}{m}} - \frac{mg}{b}$

karena : $f = w$

$bv = mg$

$vT = \frac{mg}{b}$

Maka : $v(t) = (vT+v_0)e^{-\frac{bt}{m}} - vT$

=> untuk $a(t)$

$a(t) = \frac{dv(t)}{dt}$

$a(t) = \frac{d}{dt}((vT+v_0)e^{-\frac{bt}{m}} - vT)$

$a(t) = (vT+v_0)(-\frac{b}{m})e^{-\frac{bt}{m}}$

$= -\frac{b}{m}(vT+v_0)e^{-\frac{bt}{m}}$

$= (-\frac{b}{m}\cdot\frac{mg}{b} - \frac{b}{m}v_0)e^{-\frac{bt}{m}}$

$a(t) = (-g - \frac{b}{m}v_0)e^{-\frac{bt}{m}}$

Untuk $y(t)$

$v(t) = \frac{dy}{dt}(t)$

$\frac{dy(t)}{y(t)} = v(t)\,dt$

$\int_0^{y(t)} dy(t) = \int_0^t v(t)\,dt$

$y(t) = \int_0^t \left((VT + V_0)e^{-\frac{bt}{M}} - VT\right)$

$= \left[(VT + V_0)(-\frac{M}{b})e^{-\frac{bt}{M}} - VT\right]_0^t$

$y(t) = \left[(VT + V_0)(-\frac{M}{b})e^{-\frac{bt}{M}} - VT.t\right] - \left[VT + V_0)(-\frac{M}{b})\right]$

$y(t) = -\frac{M}{g}(VT + V_0)(e^{-\frac{bt}{M}}) - VT.t$

$y(t) = -\frac{M}{b}(VT - V_0)(1 - e^{-\frac{bt}{M}}) - VT.b$

Untuk $b = 0$

Maka, $(y(t) = V_0 t - \frac{1}{2}gt^2$

$v(t) = (VT + V_0)e^{-\frac{bt}{M}} - VT)$

$= -VT + (VT + V_0)(1 - \frac{bt}{M})$

$v(t) = -VT + VT - \frac{b}{M}VT\,t + V_0 - \frac{b}{M}V_0.t$

$v(t) = V_0 - gt - \frac{b}{M}V_0 t$

Aproksimasi

$e^x = 1 + \frac{x}{11} + \frac{x^2}{21} + \frac{x^3}{31} + \cdots$

$e^{-\frac{bt}{M}} = 1 - \frac{bt}{M} + \frac{1}{2}(\frac{bt}{M})^2 + \cdots$

$y(t) = -VT + \frac{M}{b}(VT + V_0)(1 - e^{-\frac{bt}{M}})$

$= -VT.t + \frac{M}{b}(VT + V_0)(1 - 1 + \frac{bt}{M} - \frac{1}{2}(\frac{bt}{M})^2)$

$= V_0 t - \frac{1}{2}gt^2 - \frac{1}{2}\frac{b}{M}V_0 t^2$

$y(t) = V_0 t - \frac{1}{2}gt^2 - \frac{1}{2}\frac{b}{m}V_0 t^2$

Jadi, $a(t) = -g\frac{b}{m}V_0(\frac{b^2}{m^2}V_0 + \frac{gb}{M}) + (b = 0)$

Maka, $a(t) = -g$

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