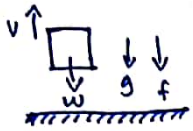
Nama : Latia Merinda
NPM : 2013022026
Prodi : Pendidikan Fisika (B)

Gerak Vertikal keatas dengan $f = -bv$



$\Sigma F = ma$

$w + F = -m \cdot a$

$(m.g + bv)\hat{y} = (-ma)\hat{y}$

$g + \frac{bv}{m} = -a$

$g + \frac{bv}{m} = \frac{-dv}{dt}$

$\frac{-dv}{g + \frac{b}{m}v} = dt$

$-\int_{v_0}^{v(t)} \frac{dv}{g + \frac{b}{m}v} \cdot \int_0^t dt$

↳ Misal : $g + \frac{b}{m}v = u$ Maka, $\frac{b}{m}dv = du$

$dv = \frac{m}{b}du$

↳ Maka : $-\int_{v_0}^{v(t)} \frac{1}{u}dv = \int_0^t dt$

$-\int_{v_0}^{v(t)} \frac{1}{u}\left(\frac{m}{b}du\right) = \int_0^t dt$

$-\frac{m}{b}\int_{v_0}^{v(t)} \frac{1}{u}du = t$

$-\frac{m}{b}\ln|u| = t$

$-\frac{m}{b}\left(\ln\left|g + \frac{b}{m}(v(t))\right| - \ln\left|g + \frac{b}{m}(v_0)\right|\right) = t$

↳ Karena $\ln a - \ln b = \ln \frac{a}{b}$,

↳ Maka, $\ln\left|\frac{g + \frac{b}{m}(V_t)}{g + \frac{b}{m}(V_0)}\right| = -\frac{bt}{m}$

$\exp\left(\ln\left|\frac{g + \frac{b}{m}(V_t)}{g + \frac{b}{m}V_0}\right|\right) = e^{-\frac{bt}{m}}$

$\frac{g + \frac{b}{m}V(t)}{g + \frac{b}{m}V_0} = e^{-\frac{bt}{m}}$

$g + \frac{b}{m}(V_t) = \left(g + \frac{b}{m}V_0\right)e^{-\frac{bt}{m}}$

$\frac{b}{m}V(t) = g + \frac{b}{m}V_0 e^{-\frac{bt}{m}} - g\left(x - \frac{m}{b}\right)$

$V(t) = \frac{m}{b}\left(g + \frac{b}{m}V_0\right)e^{-\frac{bt}{m}} - x - \frac{m}{b}$

$V(t)^2 = \left(\frac{mg}{b} + V_0\right)e^{-\frac{bt}{m}} - \frac{mg}{b}$

Karena : $f = w$

$bv = mg$

$V_T = \frac{mg}{b}$

Maka: $V(t) = (V_T + V_0)e^{-\frac{bt}{m}} - V_T$

* Untuk $a(t)$

$a(t) = \frac{dv(t)}{dt}$

$a(t) = \frac{d}{dt}\left((V_T + V_0)e^{-\frac{bt}{m}} - V_T\right)$

$a(t) = (V_T + V_0)\left(-\frac{b}{m}\right)e^{-\frac{bt}{m}}$

$= -\frac{b}{m}(V_T + V_0)e^{-\frac{bt}{m}}$

$= \left(-\frac{b}{m}\cdot\frac{mg}{b} - \frac{b}{m}V_0\right)e^{-\frac{bt}{m}}$

$a(t) = \left(-g - \frac{b}{m}V_0\right)e^{-\frac{bt}{m}}$

* Untuk $y(t)$

$V(t) = \frac{dy}{dt}(t)$

$\frac{dy(t)}{y(t)} = V(t) \cdot dt$

$\int_0^{y(t)} dy(t) = \int_0^t V(t)\,dt$

$y(t) = \int_0^t \left((V_T + V_0)e^{-\frac{bt}{m}} - V_T\right)$

$= \left[(V_T + V_0)\left(-\frac{m}{b}\right)e^{-\frac{bt}{m}} - V_T\right]_0^t$

$y(t) = \left[(V_T + V_0)\left(-\frac{m}{b}\right)e^{-\frac{bt}{m}} - V_T.t\right] - \left[V_T + V_0)\left(-\frac{m}{b}\right)\right]$

$y(t) = -\frac{m}{g}(V_T + V_0)\left(e^{-\frac{bt}{m}}\right) - V_T.t$

$y(t) = -\frac{m}{b}(V_T - V_0)\left(1 - e^{-\frac{bt}{m}}\right) - V_T.t$

Untuk $b=0$

Maka, $y(t) = V_0 t - \frac{1}{2} g t^2$

$$V(t) = (V_T + V_0) e^{-\frac{bt}{m}} - V_T$$

$$= -V_T + (V_T + V_0)\left(1 - \frac{bt}{m}\right)$$

$$= -\cancel{V_T} + \cancel{V_T} - \frac{b}{m} V_T t + V_0 - \frac{b}{m} V_0 . t$$

$$= V_0 - \frac{b}{m}\left(\frac{mg}{\cancel{b}}\right) t - \frac{b}{m} V_0 t$$

$$V(t) = V_0 - gt - \frac{b}{m} V_0 t$$

Aproksimasi

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

$$e^{-\frac{bt}{m}} = 1 - \frac{bt}{m} + \frac{1}{2}\left(\frac{bt}{m}\right)^2 + \ldots$$

$$y(t) = -V_T + \frac{m}{b}(V_T + V_0)\left(1 - e^{-\frac{bt}{m}}\right)$$

$$= -V_T . t + \frac{m}{b}(V_T + V_0)\left(1 - 1 + \frac{bt}{m} - \frac{1}{2}\left(\frac{bt}{m}\right)^2\right)$$

$$= V_0 t - \frac{1}{2} g t^2 - \frac{1}{2}\frac{b}{m} V_0 t^2$$

$$y(t) = V_0 t - \frac{1}{2} g t^2 - \frac{1}{2}\frac{b}{m} V_0 t^2$$