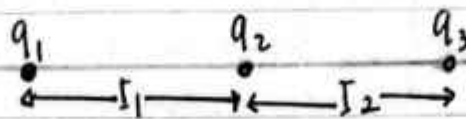


tugas II

No 3 Oktober 2020
Date Jumat

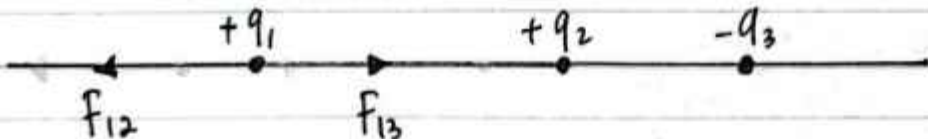
- 1) Tiga buah muatan titik q_1, q_2, q_3 yg masing-masing bermuatan $2 \cdot 10^{-6}$ C (positif), $3 \cdot 10^{-6}$ C (positif), dan $5 \cdot 10^{-6}$ C (Negatif) terletak pd sebuah garis lurus. Jika jarak q_1 dan q_2 adalah 3m, dan q_2 dan q_3 adalah 2m. hitung gaya elektrostatika yg dialami oleh q_1 ?

Jawab:



q_1 mengalami gaya tolak menolak dari q_2 karena muatannya sejenis.

q_1 mengalami gaya tarik menarik dari q_3 karena muatannya berlawanan jenis.



Besar gaya antara q_1 dan q_2

$$\begin{aligned} F_{12} &= \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{l_1^2} = \frac{9 \cdot 10^9 \times 2 \cdot 10^{-6} \times 3 \cdot 10^{-6}}{3^2} \\ &= \frac{9 \cdot 10^9 \times 2 \cdot 10^{-6} \times 3 \cdot 10^{-6}}{9} \\ &= 6 \cdot 10^{9+(-12)} = 6 \cdot 10^{-3} \text{ N} \end{aligned}$$

$$\begin{aligned} F_{13} &= \frac{1}{4\pi\epsilon_0} \frac{q_1 q_3}{(l_1 + l_2)^2} = \frac{9 \cdot 10^9 \times 2 \cdot 10^{-6} \times 5 \cdot 10^{-6}}{(5)^2} \\ &= \frac{9 \cdot 10^9 \times 2 \cdot 10^{-6} \times 5 \cdot 10^{-6}}{25} \\ &= \frac{18 \cdot 10^{-3}}{5} = 3,6 \cdot 10^{-3} \text{ N} \end{aligned}$$

Resultan gaya dialami oleh q_1 besarnya

$$= (6 - 3,6) \cdot 10^{-3} \text{ N}$$

$$= 2,4 \cdot 10^{-3} \text{ N}, \text{ arahnya searah dengan } F_{12} \text{ (ke arah kiri).}$$

2. Dua buah partikel bermuatan positif dan sama besar terpisah pada jarak $2,60 \cdot 10^{-2} \text{ m}$ dilepaskan dari keadaan diam. sesaat setelah dilepas, partikel 1 yang bermassa $6,0 \cdot 10^{-6} \text{ kg}$ bergerak dengan percepatan $4,6 \cdot 10^3 \text{ ms}^{-2}$, sedangkan partikel 2 bergerak dengan percepatan $8,5 \cdot 10^3 \text{ ms}^{-2}$. Tentukan:
- Besar muatan masing-masing partikel.
 - Massa partikel 2.

Jawab:

Dik:

$$r = 2,60 \cdot 10^{-2} \text{ m.}$$

$$m_1 = 6,0 \cdot 10^{-6} \text{ kg.}$$

$$a_1 = 4,6 \cdot 10^3 \text{ ms}^{-2}$$

$$a_2 = 8,5 \cdot 10^3 \text{ ms}^{-2}$$

- a) Besar muatan masing-masing partikel.

$$q_1 = q_2 = q$$

$$F = m_1 \times a_1$$

$$= 6,0 \cdot 10^{-6} \times 4,6 \cdot 10^3$$

$$= 2,76 \cdot 10^{-2} \text{ N}$$

$$F = k \cdot \frac{q_1 q_2}{r^2}$$

$$F = k \cdot \frac{q^2}{r^2}$$

$$2,76 \cdot 10^{-2} = 9 \cdot 10^9 \cdot \frac{q^2}{(2,60 \cdot 10^{-2})^2}$$

$$2,76 \cdot 10^{-2} = 9 \cdot 10^9 \cdot \frac{q^2}{6,76 \cdot 10^{-4}}$$

$$\begin{aligned} 2,76 \cdot 10^{-2} \times 6,76 \cdot 10^{-4} &= 9 \cdot 10^9 \times q^2 \\ 18,6576 \cdot 10^{-6} &= 9 \cdot 10^9 \times q^2 \\ q^2 &= \frac{18,6576 \cdot 10^{-6}}{9 \cdot 10^9} \end{aligned}$$

$$q^2 = 20,7 \cdot 10^{-6-9}$$

$$q^2 = 20,7 \cdot 10^{-15}$$

$$q = \sqrt{20,7 \cdot 10^{-15}}$$

$$q = 4,55 \cdot 10^{-8} \text{ C.}$$

$$q_1 = q_2 = 4,55 \cdot 10^{-8} \text{ C.}$$

b) Massa Partikel 2.

$$\text{HK. Newton II} = F = m \cdot a$$

$$F = M_2 \cdot a_2$$

$$M_2 = \frac{F}{a_2}$$

$$M_2 = \frac{2,76 \cdot 10^{-2}}{8,5 \cdot 10^3}$$

$$M_2 = 3,25 \cdot 10^{-6} \text{ kg.}$$

$$M_2 = 3,2 \cdot 10^{-6} \text{ kg.}$$

Jadi, massa partikel 2 adalah $3,2 \cdot 10^{-6} \text{ kg.}$

3) Tentukan gaya yang dialami q_1 , jika :

$$q_1 (-4,0,0) = 5 \text{ C}$$

$$q_2 (4,0,0) = 10 \text{ C}$$

$$q_3 (0,5,0) = 6 \text{ C}$$

Jawab:

Gaya total yang dialami oleh q_1 adalah resultan vektor dari gaya yang diberikan oleh q_2 pada q_1 (F_{12}) dan gaya yang diberikan oleh q_3 pada q_1 (F_{13})

$$\begin{aligned} r_{12} &= \sqrt{(4 - (-4))^2 + (0 - 0)^2 + (0 - 0)^2} \\ &= \sqrt{8^2} \\ &= 8 \text{ m.} \end{aligned}$$

$$\begin{aligned} r_{13} &= \sqrt{(0 - (-4))^2 + (5 - 0)^2 + (0 - 0)^2} \\ &= \sqrt{4^2 + 5^2} \\ &= \sqrt{16 + 25} \\ &= \sqrt{41} \text{ m.} \end{aligned}$$

$$\begin{aligned} F_{12} &= k \frac{q_1 q_2}{r_{12}^2} \\ &= 9 \cdot 10^9 \frac{5 \cdot 10}{8^2} \\ &= 9 \cdot 10^9 \frac{50}{64} \\ &= 7,03125 \cdot 10^9 \text{ N.} \end{aligned}$$

$$\begin{aligned} F_{13} &= k \cdot \frac{q_1 q_3}{r_{13}^2} \\ &= 9 \cdot 10^9 \frac{5 \cdot 6}{(\sqrt{41})^2} \\ &= 9 \cdot 10^9 \frac{30}{41} \\ &= 6,585 \cdot 10^9 \text{ N.} \end{aligned}$$

Karena q_1 dan q_2 positif, F_{12} gayanya tolak-menolak, sehingga arahnya menjauhi q_2 dan searah sumbu x negatif.

Vektor F_{12} adalah $\vec{F}_{12} = (-7,03125 \cdot 10^9) \hat{i}$

vektor satuan q_1 dan q_3 adalah $\hat{r}_{13} = \frac{(0-(-4))\hat{i} + (5-0)\hat{j}}{\sqrt{41}}$
 $= \frac{4\hat{i} + 5\hat{j}}{\sqrt{41}}$

Arah gaya F_{13} adalah berlawanan dengan $\hat{r}_{13}$, sehingga:

$$\vec{F}_{13} = -F_{13} \hat{r}_{13} = -(6,585 \cdot 10^9) \frac{4\hat{i} + 5\hat{j}}{\sqrt{41}}$$

$$F_{13x} = -(6,585 \cdot 10^9) \frac{4}{\sqrt{41}} = -4,118 \cdot 10^9 \text{ N.}$$

$$F_{13y} = -(6,585 \cdot 10^9) \frac{5}{\sqrt{41}} = -5,147 \cdot 10^9 \text{ N.}$$

Resultan gaya pada q_1 adalah:

$$|\vec{F}_1| = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{((-7,03125 \cdot 10^9) + (-4,118 \cdot 10^9))^2 + ((-5,147 \cdot 10^9) + 0)^2}$$

$$= \sqrt{(-11,14925 \cdot 10^9)^2 + (-5,147 \cdot 10^9)^2}$$

$$= \sqrt{-124,30578 \cdot 10^{18} + (-26,491609 \cdot 10^{18})}$$

$$= \sqrt{-150,7974 \cdot 10^{18}} \cdot 10^9$$

$$= 12,27 \cdot 10^9$$

$$= 1,227 \cdot 10^{10} \text{ N}$$

Jadi gaya yang dialami q_1 sebesar $1,22 \cdot 10^{10} \text{ N}$.

4) Tiga partikel bermuatan terdistribusi sebagai berikut :

$$q_1 (0, 5, 0) = 10 \text{ C}$$

$$q_2 (5, 5, 0) = 5 \text{ C}$$

$$q_3 (0, 5, 5) = -10 \text{ C}$$

Tentukan gaya yang dialami q_3 !

Jawab:

Dik:

$$q_1 = 10 \text{ C} \rightarrow (0, 5, 0)$$

$$q_2 = 5 \text{ C} \rightarrow (5, 5, 0)$$

$$q_3 = -10 \text{ C} \rightarrow (0, 5, 5)$$

$$k = 9 \cdot 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2$$

↑ z

• $q_3 = -10 \text{ C}$

$q_1 = 10 \text{ C}$
↑ y

Jarak antara q_3 dan q_1 (r_{13})

$$\vec{r}_{13} = \vec{r}_3 - \vec{r}_1$$

$$= (0, 5, 5) - (0, 5, 0)$$

$$= (0, 0, 5) \text{ m}$$

$$r_{13} = \sqrt{0^2 + 0^2 + 5^2}$$

$$= \sqrt{25} = 5 \text{ m}$$

Vektor satuan $\hat{r}_{13}$

$$= (0, 0, 1) \hat{k}$$

jarak antara q_3 dan q_2 (r_{23})

$$\begin{aligned} r_{23} &= r_3 - r_2 \\ &= (0, 5, 5) - (5, 5, 0) \\ &= (-5, 0, 5) \text{ m} \end{aligned}$$

$$\begin{aligned} r_{23} &= \sqrt{(-5)^2 + 0^2 + 5^2} \\ &= \sqrt{25 + 25} \\ &= \sqrt{50} \text{ m.} \end{aligned}$$

$$\text{vektor satuan } \hat{r}_{23} = \left(-\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2} \right)$$

$F_{13} \Rightarrow -q_3$ dan $+q_1$ akan tarik-menarik.

$$\begin{aligned} F_{13} &= k \frac{|q_1 q_3|}{r_{13}^2} \\ &= (9 \cdot 10^9) \frac{|(10) \cdot (10)|}{5^2} \\ &= 9 \cdot 10^9 \frac{100}{25} \end{aligned}$$

$$F_{13} = 9 \cdot 10^9 \cdot 4 = 36 \cdot 10^9 \text{ N}$$

$$\text{vektor gaya } f_{13} = F_{13} \cdot (-\hat{e}) = -36 \cdot 10^9 \hat{e}$$

$F_{23} \Rightarrow -q_3$ dan $+q_2$ akan tarik-menarik.

$$\begin{aligned} F_{23} &= k \frac{q_2 q_3}{r_{23}^2} \\ &= 9 \cdot 10^9 \frac{5 \cdot 10}{(\sqrt{50})^2} \\ &= 9 \cdot 10^9 \frac{50}{50} \end{aligned}$$

$$F_{23} = 9 \cdot 10^9 \text{ N}$$

$$\begin{aligned} \text{vektor } F_{23} &= F_{23} \cdot (-\hat{r}_{23}) \\ &= 9 \cdot 10^9 \cdot \left(-\frac{\sqrt{2}}{2} \hat{i} - \frac{\sqrt{2}}{2} \hat{k} \right) \end{aligned}$$

$$= 9 \cdot 10^9 \cdot (0,707 \hat{i} - 0,707 \hat{k})$$

$$F_{23} = 6,363 \cdot 10^9 \hat{i} - 6,363 \cdot 10^9 \hat{k} \text{ N.}$$

Gaya total F_3

$$F_3 = F_{13} + F_{23}$$

$$= (0\hat{i} + 0\hat{j} - 36 \cdot 10^9 \hat{k}) + (6,363 \cdot 10^9 \hat{i} + 0\hat{j} - 6,363 \cdot 10^9 \hat{k})$$

$$= 6,363 \cdot 10^9 \hat{i} + 0\hat{j} + (-36 \times 6,363) 10^9 \hat{k}$$
$$= 6,363 \cdot 10^9 \hat{i} - 42,363 \cdot 10^9 \hat{k} \text{ N.}$$

$$F_3 = \sqrt{F_{3x}^2 + F_{3y}^2}$$

$$= \sqrt{(6,363 \cdot 10^9)^2 + (-42,363 \cdot 10^9)^2}$$

$$= \sqrt{(40,49 + 1794,6) 10^{18}}$$

$$= \sqrt{1835,09 \cdot 10^{18}}$$

$$F_3 = 42,84 \cdot 10^9 \text{ N}$$

5) Berapa gaya yang dialami muatan $q = 10 \text{ C}$ di $(0,0,0)$

karena pengaruh muatan:

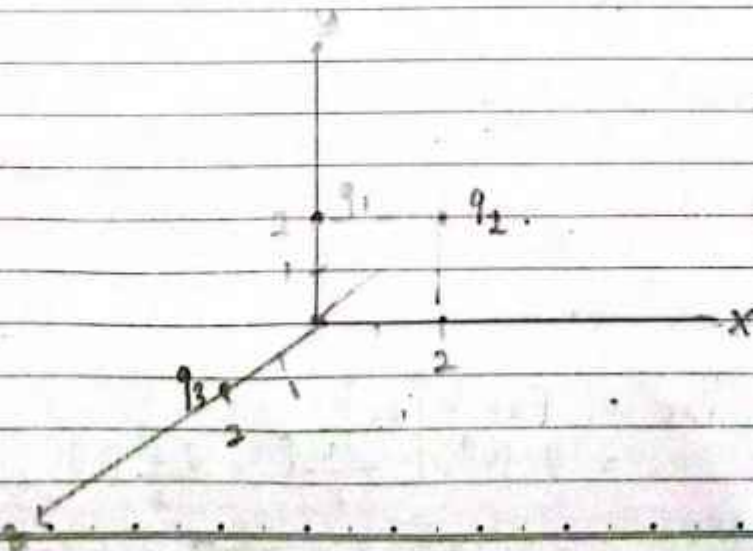
$$q_1 = 2 \text{ C di } (0,2,0)$$

$$q_2 = 4 \text{ C di } (2,2,0)$$

$$q_3 = 5 \text{ C di } (0,2,2)$$

$$q_4 = -9 \text{ C di } (2,2,2)$$

Jawab =



• Dari $q_1 = 2 \text{ C}$ di $(0, 2, 0)$ ke q di $(0, 0, 0)$

$$\begin{aligned} \vec{r}_1 &= (0, 0, 0) - (0, 2, 0) \\ &= (0, -2, 0) \text{ m.} = 2 \text{ m.} \\ \hat{r}_1 &= (0, -1, 0) \end{aligned}$$

$$\begin{aligned} F_1 &= k \cdot \frac{q \cdot q_1}{r_1^2} \\ &= 9 \cdot 10^9 \frac{10 \cdot 2}{2^2} \\ &= 9 \cdot 10^9 \frac{20}{4} \\ &= 9 \cdot 10^9 \times 5 \\ &= 45 \cdot 10^9 \text{ N} \end{aligned}$$

$$\begin{aligned} \text{vektor gaya } \vec{F}_1 &= F_1 \cdot (\hat{r}_1) \\ &= 45 \cdot 10^9 \cdot (0, -1, 0) \\ &= (0, -45 \cdot 10^9, 0) \text{ N.} \end{aligned}$$

• Dari $q_2 = 4 \text{ C}$ di $(2, 2, 0)$ ke q di $(0, 0, 0)$

$$\begin{aligned} \vec{r}_2 &= (0, 0, 0) - (2, 2, 0) \\ &= (-2, -2, 0) \text{ m} \\ r_2 &= \sqrt{(-2)^2 + (-2)^2 + 0^2} \\ &= \sqrt{4 + 4} \\ &= \sqrt{8} = 2\sqrt{2} \text{ m.} \\ \hat{r}_2 &= \left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0 \right) \end{aligned}$$

$$\begin{aligned} \vec{F}_2 &= k \cdot \frac{q \cdot q_2}{r_2^2} \\ &= 9 \cdot 10^9 \frac{10 \cdot 4}{(\sqrt{8})^2} \\ &= 9 \cdot 10^9 \frac{40}{8} \\ &= 45 \cdot 10^9 \text{ N} \end{aligned}$$

$$\begin{aligned} \text{vektor gaya } \vec{F}_2 &= F_2 \cdot (\hat{r}_2) \\ &= 45 \cdot 10^9 \left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0 \right) \\ &= \frac{-45 \cdot 10^9}{\sqrt{2}}, \frac{-45 \cdot 10^9}{\sqrt{2}}, 0 \\ &= (-31,82 \cdot 10^9, -31,82 \cdot 10^9, 0) \text{ N.} \end{aligned}$$

- Dari $q_3 = 5C$ di $(0, 2, 2)$ ke q di $(0, 0, 0)$

$$\vec{r}_3 = (0, 0, 0) - (0, 2, 2)$$

$$= (0, -2, -2)$$

$$r_3 = \sqrt{0^2 + (-2)^2 + (-2)^2}$$

$$= \sqrt{4 + 4}$$

$$= \sqrt{8} = 2\sqrt{2} \text{ m.}$$

$$\hat{r}_3 = (0, \frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}})$$

$$F_3 = k \frac{q \cdot q_3}{r_3^2}$$

$$= 9 \cdot 10^9 \frac{10 \cdot 5}{(\sqrt{8})^2}$$

$$= 9 \cdot 10^9 \frac{50}{8}$$

$$= 56,25 \cdot 10^9 \text{ N}$$

$$\text{vektor gaya } \vec{F}_3 = F_3 \cdot (\hat{r}_3)$$

$$= 56,25 \cdot 10^9 (0, \frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}})$$

$$= (0, \frac{-56,25 \cdot 10^9}{\sqrt{2}}, \frac{-56,25 \cdot 10^9}{\sqrt{2}})$$

$$= (0, -39,77 \cdot 10^9, -39,77 \cdot 10^9) \text{ N.}$$

- Dari $q_4 = -5C$ di $(2, 2, 2)$ ke q di $(0, 0, 0)$

$$\vec{r}_4 = (0, 0, 0) - (2, 2, 2)$$

$$= (-2, -2, -2)$$

$$r_4 = \sqrt{(-2)^2 + (-2)^2 + (-2)^2}$$

$$= \sqrt{4 + 4 + 4}$$

$$= \sqrt{12} = 2\sqrt{3} \text{ m.}$$

$$\hat{r}_4 = (-\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}})$$

$$F_4 = k \cdot \frac{q \cdot q_4}{r_4^2}$$

$$\begin{aligned}F_4 &= 9 \cdot 10^9 \frac{10 \cdot (-5)}{(\sqrt{12})^2} \\&= 9 \cdot 10^9 \frac{-50}{12} \\&= 37,5 \cdot 10^9 \text{ N}\end{aligned}$$

vektor gaya $\vec{F}_4 = F_4 \cdot (\hat{r}_4)$

$$\begin{aligned}&= 37,5 \cdot 10^9 \cdot \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right) \\&= \left(\frac{37,5 \cdot 10^9}{\sqrt{3}}, \frac{37,5 \cdot 10^9}{\sqrt{3}}, \frac{37,5 \cdot 10^9}{\sqrt{3}} \right) \\&= (21,65 \cdot 10^9, 21,65 \cdot 10^9, 21,65 \cdot 10^9) : \text{N}\end{aligned}$$

$$F_{\text{total}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$F_x = -31,82 \cdot 10^9 + 21,65 \cdot 10^9 = -10,17 \cdot 10^9 \text{ N}$$

$$\begin{aligned}F_y &= -45 \cdot 10^9 - 31,82 \cdot 10^9 - 39,77 \cdot 10^9 + 21,65 \cdot 10^9 \\&= -94,94 \cdot 10^9 \text{ N}\end{aligned}$$

$$F_z = -39,77 \cdot 10^9 + 21,65 \cdot 10^9 = -18,12 \cdot 10^9 \text{ N}$$

$$F_{\text{total}} = -10,17 \cdot 10^9 \hat{i} - 94,94 \cdot 10^9 \hat{j} - 18,12 \cdot 10^9 \hat{k}$$

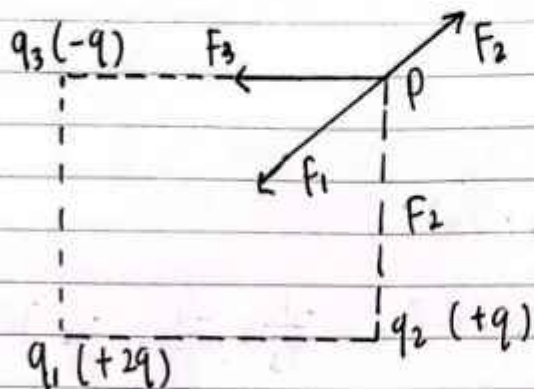
$$\begin{aligned}\text{Besar gaya } \vec{F} &= \sqrt{(F_x^2 + F_y^2 + F_z^2)} \\&= \sqrt{(-10,17 \cdot 10^9)^2 + (-94,94 \cdot 10^9)^2 + (-18,12 \cdot 10^9)^2} \\&= 96,9 \cdot 10^9 \text{ N}\end{aligned}$$

6.) a. Gambar gaya-gaya pada muatan $3q$.

$F_1 (+2q)$ = tolak-menolak, arah diagonal ke kiri bawah.

$F_2 (+q)$ = tolak-menolak, arah ke atas $(+\hat{j})$

$F_3 (-q)$ = tarik-menarik, arah ke kiri $(-\hat{i})$



b. Hitung besar gaya

- Hitung vektor gaya

F_2 (jarak s , arah $+\hat{j}$)

$$F_2 = k \cdot \frac{3q \cdot q}{s^2}$$

$$= 3k'$$

$$= 3k'\hat{j}$$

F_3 (jarak s , arah $-\hat{i}$)

$$F_3 = k \cdot \frac{(3q) \cdot (-q)}{s^2}$$

$$= -3k' = -3k'\hat{i}$$

F_1 (jarak $s\sqrt{2}$, arah $-\hat{i}, -\hat{j}$)

$$F_1 = k \cdot \frac{3q \cdot 2q}{(s\sqrt{2})^2}$$

$$= k \cdot \frac{6q^2}{2s^2}$$

$$= 3k'$$

$$= 3k' \cdot \frac{1}{\sqrt{2}} (-\hat{i}, -\hat{j})$$

$$F_1 = -\frac{3k'}{\sqrt{2}}\hat{i}, -\frac{3k'}{\sqrt{2}}\hat{j}$$

- Besar gaya total

$$F_{\text{total}} = \sqrt{F_x^2 + F_y^2}$$

$$= 3k' \sqrt{\left(-1 - \frac{1}{\sqrt{2}}\right)^2 + \left(1 - \frac{1}{\sqrt{2}}\right)^2}$$

$$= 3k' \sqrt{1 + 1 + \frac{1}{2} + \frac{1}{2}}$$

$$F = 3k'\sqrt{3} = 3\sqrt{3} \cdot k \frac{q_2}{s^2} //$$

c. Hitung medan listrik.

$$\begin{aligned}\vec{E} &= \frac{F_{\text{total}}}{3q} \\ &= \frac{3k' \frac{q^2}{s^2} \left[\left(-1 - \frac{1}{2}\right) \hat{i} + \left(1 - \frac{1}{\sqrt{2}}\right) \hat{j} \right]}{3q} \\ &= k \frac{q}{s^2} \left[\left(-1 - \frac{1}{2}\right) \hat{i} + \left(1 - \frac{1}{\sqrt{2}}\right) \hat{j} \right] \quad //\end{aligned}$$

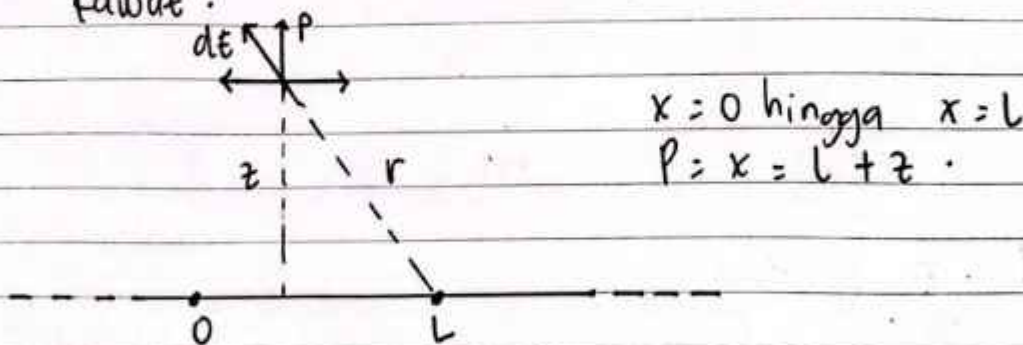
d. $E = \frac{13kq}{4r^2}$ arah ke kiri //

$$\begin{aligned}e. V &= k \sum \frac{q_i}{r_i} \\ &= k \left(\frac{-q}{r} + \frac{2q}{r} + \frac{q}{2r} \right) \\ &= k \left(\frac{-q + 2q + 0,5q}{2} \right) \\ &= k \frac{1,5q}{r} = \frac{3kq}{2r} \quad //\end{aligned}$$

7.) Sebuah batang panjangnya L , rapat muatan persatuan panjang adalah λ .

Hitung:

a. kuat medan listrik di titik P yg berjarak z diatas ujung kawat.



$$d\vec{E} = k \frac{dq}{r^2} = k \frac{\lambda dx'}{(L+z-x')^2}$$

$$\vec{E} = \int_0^L k\lambda \frac{dx'}{(l+z-x')^2}$$

gunakan $u = (l+z) - x'$

maka, $du = -dx'$

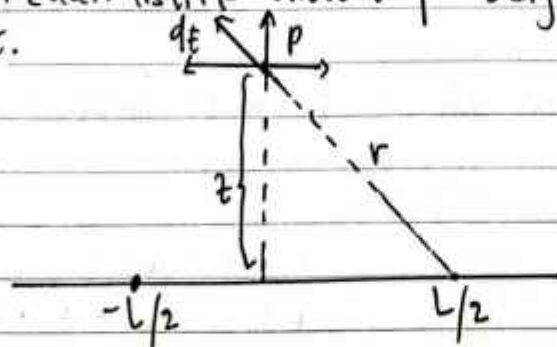
untuk $x' = 0$, $u = l+z$

untuk $x' = l$, $u = z$

$$\begin{aligned}\vec{E} &= k\lambda \int_{l+z}^z \frac{-du}{u^2} \\ &= -k\lambda \left[-\frac{1}{u} \right]_{l+z}^z \\ &= k\lambda \left[\frac{1}{u} \right]_{l+z}^z \\ &= k\lambda \left(\frac{1}{z} - \frac{1}{l+z} \right) \\ &= k\lambda \left(\frac{(l+z) - z}{z(l+z)} \right)\end{aligned}$$

$$\vec{E} = k\lambda \frac{l}{z(l+z)}$$

b. Kuat medan listrik di titik P berjarak z ditengah-tengah kawat.



$$d\vec{E} = k \cdot \frac{\lambda dx'}{(z-x')^2}$$

$$\vec{E} = \int_{-L/2}^{L/2} k\lambda \frac{dx}{(z-x')^2}$$

gunakan substitusi $u = z - x'$, maka $du = -dx'$

$$\text{Untuk } x' = -\frac{L}{2}, \quad u = z + L/2$$

$$\text{Untuk } x' = \frac{L}{2}, \quad u = z - \frac{L}{2}$$

$$\vec{E} = k\lambda \int_{z+L/2}^{z-L/2} \frac{-du}{u^2}$$

$$= -k\lambda \left[-\frac{1}{u} \right]_{z+L/2}^{z-L/2}$$

$$= k\lambda \left[\frac{1}{u} \right]_{z+L/2}^{z-L/2}$$

$$= k\lambda \left(\frac{1}{z-L/2} - \frac{1}{z+L/2} \right)$$

$$= k\lambda \left(\frac{(z+L/2) - (z-L/2)}{(z-L/2)(z+L/2)} \right)$$

$$\vec{E} = k\lambda \frac{L}{z^2 - (L/2)^2} //$$

3) Diketahui: σ = rapat muatan permukaan (C/m^2)

R = jari-jari bola konduktor (m)

$$Q = \sigma \cdot \text{luas permukaan} = \sigma \cdot 4\pi R^2$$

Ditanya = $\vec{E}(r)$? (medan listrik didalam dan diluar bola konduktor?)

$$\text{Jawab: } \oint_{\text{Gauss}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

• Untuk daerah didalam bola konduktor ($r < R$)
 $Q_{\text{enc}} = 0$

$$\vec{E}(r) \cdot 4\pi r^2 = \frac{Q_{\text{enc}}}{\epsilon_0} = 0 \Rightarrow \vec{E}(r) = 0 \quad (r < R)$$

artinya didalam bola konduktor tidak ada medan listrik.

- Untuk daerah diluar bola konduktor ($r > R$)

$$Q = 4\pi R^2 \sigma$$

menggunakan hukum Gauss

$$E(r) \cdot 4\pi r^2 \epsilon_0 = \frac{Q}{\epsilon_0}$$

$$\vec{E}(r) = \frac{Q}{4\pi \epsilon_0 r^2}$$

$$\vec{E}(r) = \frac{4\pi R^2 \sigma}{4\pi \epsilon_0 r^2}$$

$$= \frac{\sigma R^2}{\epsilon_0 r^2} \quad (r > R)$$

Arah medan listrik radial ke luar jika $\sigma > 0$

- Tepat dipermukaan bola $r = R$

$$E_{\text{luar}} - E_{\text{dalam}} = \frac{\sigma}{\epsilon_0}$$

$$E_{\text{dalam}} = 0$$

$$E_{\text{permukaan}} = \frac{\sigma}{\epsilon_0}$$

jadi,

$$\vec{E}(r) = \begin{cases} 0 & r < R \\ \frac{\sigma R^2}{\epsilon_0 r^2} & r \geq R \end{cases}$$

$$\text{dan } Q = 4\pi R^2 \sigma$$

- 9.) Bila kuat medan listrik $\vec{E} = Kr^3 \hat{r}_0$ dalam koordinat bola, dengan $K = \text{konstan}$.

a. hitung rapat muatannya.

b. Cari muatan total yg terdapat dalam suatu bola dengan jari-jari R Berpusat di O .

Jawab =

Diketahui : $E = Kr^3 \hat{r}_0$ dalam koordinat bola.

$K = \text{konstanta}$

Ditanya: a.) ρ ?
b.) Q_{total} ?

Jawab: a) hukum Gauss (divergensi) $E_r = kr^3$

$$\begin{aligned}\vec{\nabla} \cdot \vec{E} &= \frac{\rho}{\epsilon_0} \\ &= \frac{1}{r^2} \cdot \frac{d}{dr} (r^2 E_r) \\ &= \frac{1}{r^2} \cdot \frac{d}{dr} (r^2 \cdot kr^3) \\ &= \frac{1}{r^2} \cdot \frac{d}{dr} (kr^5) \\ &= \frac{1}{r^2} (5kr^4) \\ &= 5kr^2\end{aligned}$$

$$\text{maka, } \frac{\rho}{\epsilon_0} = 5kr^2$$

$$\rho = 5\epsilon_0 kr^2 //$$

b) muatan total dalam bola jari R .

$$Q = \int_V \rho \, dv$$

dalam koordinat bola

$$dv = 4\pi r^2 dr$$

$$\begin{aligned}Q &= \int_0^R \rho \, 4\pi r^2 dr \\ &= \int_0^R (5\epsilon_0 kr^2) (4\pi r^2) dr \\ &= 20\pi\epsilon_0 k \int_0^R r^4 dr \\ &= 20\pi\epsilon_0 k \left[\frac{r^5}{5} \right]_0^R\end{aligned}$$

$$Q_{\text{total}} = 4\pi\epsilon_0 k R^5 //$$