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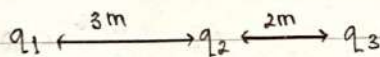
MK : Medan Elektromagnetik

1.) Tiga buah muatan titik q_1 , q_2 dan q_3 yang masing-masing bermuatan $2 \cdot 10^{-6} \text{ C}$ (Positif), $3 \cdot 10^{-6} \text{ C}$ (Positif) dan $5 \cdot 10^{-6} \text{ C}$ (negatif) terletak pada sebuah garis lurus. Jika jarak q_1 dan q_2 adalah 3m dan q_2 dan q_3 adalah 2m, hitung gaya elektrostatika yang dialami oleh q_1 !

→ Diket : • Muatan $q_1 : + 2 \cdot 10^{-6} \text{ C}$ • Jarak $r_{12} : 3 \text{ m}$
• Muatan $q_2 : + 3 \cdot 10^{-6} \text{ C}$ • Jarak $r_{23} : 2 \text{ m}$
• Muatan $q_3 : - 5 \cdot 10^{-6} \text{ C}$ • Konstanta Coulomb $k = 9 \times 10^9 \text{ N m}^2/\text{C}^2$

Ditanya : Gaya elektrostatika yang dialami oleh q_1

Jawab :



• Tentukan jarak r_{13}

$$\begin{aligned} r_{13} &= r_{12} + r_{23} \\ &= 3 \text{ m} + 2 \text{ m} \\ &= 5 \text{ m} \end{aligned}$$

• Hitung besar gaya F_{31}

Karena q_1 Positif dan q_3 negatif, gayanya tarik-menarik, arah ke Kanan.

$$F_{31} = k \frac{|q_1 q_3|}{r_{13}^2}$$

• Hitung besar gaya F_{21}

Karena q_1 dan q_2 sama-sama positif gayanya adalah tolak-mendak. Arah Kiri.

$$F_{21} = k \frac{|q_1 q_2|}{r_{12}^2}$$

$$F_{21} = (9 \times 10^9) \frac{(2 \cdot 10^{-6})(3 \cdot 10^{-6})}{(3)^2}$$

$$F_2 = (9 \times 10^9) \frac{6 \cdot 10^{-12}}{9}$$

$$F_2 = 6 \cdot 10^{-3} \text{ N (Negatif)}$$

$$F_{31} = (9 \times 10^9) \frac{(2 \cdot 10^{-6})(5 \cdot 10^{-6})}{(5)^2}$$

$$F_{31} = (9 \times 10^9) \frac{10 \cdot 10^{-12}}{25}$$

$$F_{31} = 9 \times 10^9 \times 0.4$$

$$F_{31} = 3.6 \times 10^{-3} \text{ N (Positif)}$$

• Hitung gaya total F_1

$$F_1 = -F_{21} + F_{31}$$

$$F_1 = (-6 \cdot 10^{-3}) + (3.6 \cdot 10^{-3})$$

$$F_1 = -2.4 \cdot 10^{-3} \text{ N}$$

2.) Dua buah Partikel bermuatan positif dan sama besar terpisah pada jarak $2.60 \cdot 10^{-2} \text{ m}$ dilepaskan dari keadaan diam. Sesaat setelah dilepas, Partikel 1 yang bermassa $6.0 \cdot 10^{-6} \text{ kg}$ bergerak dengan percepatan $4.6 \cdot 10^3 \text{ ms}^{-2}$, sedangkan Partikel 2 bergerak dengan percepatan $8.5 \cdot 10^3 \text{ ms}^{-2}$. Tentukan

a.) Besar muatan masing-masing Partikel

b. Massa Partikel 2

- Diketahui :
- q_1 (Positif)
 - q_2 (Positif)
 - $|q_1| = |q_2| = q$
 - $r = 2.60 \times 10^{-2} \text{ m}$
 - $m_1 = 6.0 \times 10^{-6} \text{ kg}$
 - $a_1 = 4.6 \times 10^3 \text{ m/s}^2$
 - $a_2 = 8.5 \times 10^3 \text{ m/s}^2$

Karena kedua muatan positif dan dilepaskan, gaya yang terjadi adalah tolak - menolak ($F_{12} = F_{21} = F$)

Jawab :

a.) Besar muatan masing - masing partikel

- Hitung Besar gaya F

$$F = m_1 a_1$$

$$F = (6.0 \times 10^{-6} \text{ kg}) \times (4.6 \times 10^3 \text{ m/s}^2)$$

$$F = (6.0 \times 4.6) \times 10^{(-6+3)} \text{ N}$$

$$F = 27.6 \times 10^{-3} \text{ N} = 0.0276 \text{ N}$$

- Hitung besar muatan q

$$F = k \frac{q^2}{r^2}$$

$$q^2 = \frac{F \cdot r^2}{k}$$

$$q^2 = \frac{(27.6 \times 10^{-3}) \cdot (2.60 \times 10^{-2})^2}{9 \times 10^9}$$

$$q^2 = \frac{(27.6 \times 10^{-3}) \cdot (6.67 \times 10^{-4})}{9 \times 10^9}$$

$$q^2 = \frac{186.576 \times 10^{-7}}{9 \times 10^9}$$

$$q^2 \approx 20.73 \times 10^{(-7-9)}$$

$$q^2 \approx 20.73 \times 10^{-16} \text{ C}^2$$

$$q = \sqrt{20.73 \times 10^{-16} \text{ C}^2}$$

$$q \approx 4.55 \times 10^{-8} \text{ C}$$

b.) Massa Partikel 2

$$F = m_2 a_2$$

$$m_2 = \frac{F}{a_2}$$

$$m_2 = \frac{27.6 \times 10^{-3} \text{ N}}{8.5 \times 10^3 \text{ m/s}^2}$$

$$m_2 = \frac{27.6}{8.5} \times 10^{(-3-3)} \text{ kg}$$

$$m_2 \approx 3.247 \times 10^{-6} \text{ kg}$$

3.) Tentukan gaya yang dialami q_1 , jika :

$$q_1 (-4,0,0) : 5 \text{ C}$$

$$q_2 (4,0,0) : 10 \text{ C}$$

$$q_3 (0,5,0) : 6 \text{ C}$$

- Diketahui :
- $q_1 = -5 \text{ C}$ Pada Posisi $r_1 = (-4,0,0) \text{ m}$
 - $q_2 = +10 \text{ C}$ Pada Posisi $r_2 = (4,0,0) \text{ m}$
 - $q_3 = +6 \text{ C}$ Pada Posisi $r_3 = (0,5,0) \text{ m}$

Ditanya : Gaya yang dialami q_1

$$F_1 = F_{21} + F_{31}$$

Jawab :

• Tentukan vektor jarak dan besar jarak

+ Jarak q_1 dan q_2 (r_{21})

$$\begin{aligned} r_{21} &= r_1 - r_2 \\ &= (-4, 0, 0) - (4, 0, 0) \\ &= (-8, 0, 0) \text{ m} \end{aligned}$$

$$r_{21} = \sqrt{(-8)^2 + 0^2 + 0^2}$$

$$r_{21} = 8 \text{ m}$$

+ Jarak q_1 dan q_3 (r_{31})

$$\begin{aligned} r_{31} &= r_1 - r_3 \\ &= (-4, 0, 0) - (0, 5, 0) \\ &= (-4, -5, 0) \text{ m} \end{aligned}$$

$$r_{31} = \sqrt{(-4)^2 + (-5)^2 + 0}$$

$$= \sqrt{16 + 25}$$

$$r_{31} = \sqrt{41} \text{ m}$$

• Hitung gaya F_{31}

+ q_1 dan q_3 akan tarik - menarik

+ Besar gaya F_{31}

$$F_{31} = k \frac{|q_1 q_3|}{r_{31}^2}$$

$$= (9 \times 10^9) \frac{(5)(6)}{(\sqrt{41})^2} = 9 \times 10^9 \frac{30}{41}$$

$$F_{31} \approx 9 \times 10^9 \times 0.7317 \approx 6.5853 \times 10^9 \text{ N}$$

+ Vektor F_{31}

$$F_{31} = F_{31} \cdot \hat{r}_{31}$$

$$= (6.5853 \times 10^9) \cdot \frac{(-4\hat{i} - 5\hat{j})}{\sqrt{41}}$$

$$F_{31} \approx 6.5853 \times 10^9 \cdot (-0.6246\hat{i} - 0.7809\hat{j})$$

$$F_{31} \approx (-4.113 \times 10^9 \hat{i}) + (-5.143 \times 10^9 \hat{j}) \text{ N}$$

• Hitung F_{21}

+ q_1 dan $-q_2$ akan tarik - menarik, arah

Sumbu $\times$ Positif

$$F_{21} = k \frac{|q_1 q_2|}{r_{21}^2}$$

$$= (9 \times 10^9) \frac{(5)(10)}{(8)^2}$$

$$= 9 \times 10^9 \frac{50}{64}$$

$$F_{21} = 9 \times 10^9 \times 0.78125$$

$$F_{21} = 7.03125 \times 10^9 \text{ N}$$

$$\rightarrow \text{Vektor } F_{21} = 7.03125 \times 10^9 \hat{i} \text{ N}$$

• Hitung gaya total F_1

$$F_1 = F_{21} + F_{31}$$

$$F_1 \approx (7.03125 \times 10^9 \hat{i}) + (-4.113 \times 10^9 \hat{i} - 5.143 \times 10^9 \hat{j})$$

$$\approx (7.03125 - 4.113) \times 10^9 \hat{i} - 5.143 \times 10^9 \hat{j}$$

$$F_1 \approx 2.918 \times 10^9 \hat{i} - 5.143 \times 10^9 \hat{j} \text{ N}$$

• Besar gaya total F_1

$$F_1 = \sqrt{F_{1x}^2 + F_{1y}^2}$$

$$= \sqrt{(2.918 \times 10^9)^2 + (-5.143 \times 10^9)^2}$$

$$= 10^9 \sqrt{8.51 + 26.45}$$

$$= 10^9 \sqrt{34.96}$$

$$F_1 = 5.91 \times 10^9 \text{ N}$$

4.) Tiga Partikel bermuatan terdistribusi sebagai berikut :

$$q_1 (0, 5, 0) : 10 \text{ C}$$

$$q_2 (5, 5, 0) : 5 \text{ C}$$

$$q_3 (0, 5, 5) : -10 \text{ C}$$

Tentukan gaya yang dialami oleh q_3

$\rightarrow$ Diket : $q_1 = +10 \text{ C}$ pada posisi $r_1 = (0, 5, 0) \text{ m}$

- $q_2 = +5\text{C}$ Pada Posisi $r_2 = (5, 5, 0)\text{ m}$
- $q_3 = -10\text{C}$ Pada Posisi $r_3 = (0, 5, 5)\text{ m}$

Ditanya : Gaya yang dialami oleh q_3
 $F_3 = F_{13} + F_{23}$

Jawab :

- Jarak antara q_3 dan q_1 (r_{13})

$$\begin{aligned} r_{13} &= r_3 - r_1 \\ &= (0, 5, 5) - (0, 5, 0) \\ &= (0, 0, 5)\text{ m} \end{aligned}$$

$$r_{13} = \sqrt{0^2 + 0^2 + 5^2} = 5\text{ m}$$

$$\text{Vektor satuan } \hat{r}_{13} = (0, 0, 1) = \hat{k}$$

- Jarak antara q_3 dan q_2 (r_{23})

$$\begin{aligned} r_{23} &= r_3 - r_2 \\ &= (0, 5, 5) - (5, 5, 0) = (-5, 0, 5)\text{ m} \end{aligned}$$

$$r_{23} = \sqrt{(-5)^2 + 0^2 + 5^2} = \sqrt{50}\text{ m}$$

$$\text{Vektor satuan } \hat{r}_{23} = \left(-\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}\right)$$

- Hitung gaya F_{13} (Gaya pada q_3 oleh q_1)

- q_3 dan $+q_1$ akan tarik - menarik

$$\begin{aligned} F_{13} &= k \frac{|q_1 q_3|}{r_{13}^2} \\ &= (9 \times 10^9) \frac{(10)(10)}{(5)^2} = 9 \times 10^9 \frac{100}{25} \end{aligned}$$

$$F_{13} = 9 \times 10^9 \times 4 = 36 \times 10^9\text{ N}$$

$$\text{Vektor gaya } F_{13} = F_{13} \cdot (-\hat{k}) = -36 \times 10^9 \hat{k}\text{ N}$$

- Hitung gaya F_{23} (gaya pada q_3 oleh q_2)

- q_3 dan $+q_2$ akan tarik - menarik

$$F_{23} = F_{23} \cdot (-\hat{r}_{23})$$

- Besar F_{23}

$$\begin{aligned} F_{23} &= k \frac{|q_2 q_3|}{r_{23}^2} \\ &= (9 \times 10^9) \frac{(5)(10)}{(\sqrt{50})^2} = 9 \times 10^9 \frac{50}{50} \end{aligned}$$

$$F_{23} = 9 \times 10^9\text{ N}$$

- Vektor F_{23}

$$\begin{aligned} F_{23} &= 9 \times 10^9 \cdot (-\hat{r}_{23}) = 9 \times 10^9 \cdot \left(\frac{\sqrt{2}}{2} \hat{i} - \frac{\sqrt{2}}{2} \hat{k}\right) \\ &\approx 9 \times 10^9 \cdot (0.707 \hat{i} - 0.707 \hat{k}) \end{aligned}$$

$$F_{23} \approx 6.363 \times 10^9 \hat{i} - 6.363 \times 10^9 \hat{k}\text{ N}$$

- Gaya total F_3

$$\begin{aligned} F_3 &= F_{13} + F_{23} \\ &\approx (0 \hat{i} + 0 \hat{j} - 36 \times 10^9 \hat{k}) + (6.363 \times 10^9 \hat{i} + 0 \hat{j} - 6.363 \times 10^9 \hat{k}) \end{aligned}$$

$$F_3 \approx 6.363 \times 10^9 \hat{i} + 0 \hat{j} + (-36 - 6.363) \times 10^9 \hat{k}$$

$$F_3 \approx 6.363 \times 10^9 \hat{i} - 42.363 \times 10^9 \hat{k}\text{ N}$$

- Besar gaya total F_3

$$F_3 = \sqrt{F_{3x}^2 + F_{3z}^2}$$

$$\begin{aligned} F_3 &= \sqrt{(6.363 \times 10^9)^2 + (-42.363 \times 10^9)^2} \\ &= 10^9 \sqrt{40.49 + 1794.6} \\ &= 10^9 \sqrt{1835.09} \end{aligned}$$

$$F_3 = 42.84 \times 10^9\text{ N}$$

5.) Berapa gaya yang dialami muatan $q = 10\text{C}$ di $(0, 0, 0)$ karena Pengaruh muatan

$$q_1 = 2\text{C di } (0, 2, 0)$$

$$q_2 = 4\text{C di } (2, 2, 0)$$

$$q_3 = 6\text{C di } (0, 2, 2)$$

$$q_4 = -5\text{C di } (2, 2, 2)$$

- Hitung Besar F_i

$$F_i = k \frac{|q q_i|}{r_i^2} \rightarrow k \cdot k \cdot q = (9 \times 10^9) \cdot 10 = 90 \times 10^9 \text{ N m}^2/\text{C}$$

$$\cdot F_1 = k \frac{q q_1}{r_1^2} = k \frac{2}{2^2} = 0.5 k$$

$$\cdot F_3 = k \frac{q q_3}{r_3^2} = k \frac{5}{(\sqrt{8})^2} = 0.625 k$$

$$\cdot F_2 = k \frac{q q_2}{r_2^2} = k \frac{4}{(\sqrt{8})^2} = 0.5 k$$

$$\cdot F_4 = k \frac{q q_4}{r_4^2} = k \frac{5}{(\sqrt{12})^2} = 0.4167 k$$

- Hitung vektor F_i

$$F_i = F_i \cdot \frac{r_i}{r_i}$$

- $+q, +q_1 \rightarrow$ tolak, arah $F_1 = -\hat{r}_1$

$$F_1 = 0.5 k \cdot (-\hat{j}) = -0.5 k \hat{j}$$

- $+q, +q_2 \rightarrow$ tolak, arah $F_2 = -\hat{r}_2$

$$F_2 = 0.5 k \cdot \left(\frac{1}{\sqrt{8}} (2, 2, 0) \right) = 0.5 k \cdot \left(\frac{2}{2\sqrt{2}} \hat{i} + \frac{2}{2\sqrt{2}} \hat{j} \right) = \frac{0.5 k}{\sqrt{2}} (\hat{i} + \hat{j})$$

- $+q, +q_3 \rightarrow$ tolak, arah $F_3 = -\hat{r}_3$

$$F_3 = 0.625 k \cdot \left(\frac{1}{\sqrt{8}} (0, 2, 2) \right) = 0.625 k \cdot \left(\frac{2}{2\sqrt{2}} \hat{j} + \frac{2}{2\sqrt{2}} \hat{k} \right) = \frac{0.625 k}{\sqrt{2}} (\hat{j} + \hat{k})$$

- $+q, -q_4 \rightarrow$ tarik, arah $F_4 = \hat{r}_4$

$$F_4 = 0.4167 k \cdot \left(\frac{1}{\sqrt{12}} (-2, -2, -2) \right) = 0.4167 k \cdot \left(-\frac{2}{\sqrt{12}} (\hat{i} + \hat{j} + \hat{k}) \right)$$

$$= -0.4167 k \cdot \frac{2}{2\sqrt{3}} (\hat{i} + \hat{j} + \hat{k}) = -\frac{0.4167 k}{\sqrt{3}} (\hat{i} + \hat{j} + \hat{k})$$

- Hitung gaya total F

$$F = F_1 + F_2 + F_3 + F_4$$

$$F = k \left[\left(\frac{0.5}{\sqrt{2}} - \frac{0.4167}{\sqrt{3}} \right) \hat{i} + \left(-0.5 + \frac{0.5}{\sqrt{2}} + \frac{0.625}{\sqrt{2}} - \frac{0.4167}{\sqrt{3}} \right) \hat{j} + \left(\frac{0.625}{\sqrt{2}} + -\frac{0.4167}{\sqrt{3}} \right) \hat{k} \right]$$

$$\text{dengan } \rightarrow \frac{1}{\sqrt{2}} = 0.7071 \text{ dan } \frac{1}{\sqrt{3}} = 0.5774$$

$$F = 90 \times 10^9 \left[(0.3536 - 0.2403) \hat{i} + (-0.5 + 0.5336 + 0.4419 - 0.2403) \hat{j} + (0.4419 - 0.2403) \hat{k} \right]$$

$$= 90 \times 10^9 \left[0.1133 \hat{i} + 0.0552 \hat{j} + 0.2016 \hat{k} \right] \text{ N}$$

$$F = (10.197 \hat{i} + 4.968 \hat{j} + 18.144 \hat{k}) \times 10^9 \text{ N}$$

- Besar gaya total yang dialami muatan q

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$= 10^9 \sqrt{(10.197)^2 + (4.968)^2 + (18.144)^2}$$

$$= 10^9 \sqrt{104.0 + 27.7 + 329.2}$$

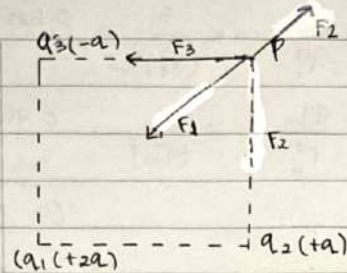
$$F = 10^9 \sqrt{457.9}$$

$$= 21.399 \times 10^9 \text{ N}$$

$$F = 21.4 \times 10^9 \text{ N}$$

6) a. Gambar Gaya - Gaya pada muatan $3q$

- $F_1 (+2q)$: tolak - menolak, arah diagonal ke kiri bawah
- $F_2 (+q)$: tolak - menolak, arah ke atas $(+\hat{j})$
- $F_3 (-q)$: tarik - menarik, arah ke kiri $(-\hat{i})$



b.) Hitung besar gaya

• Hitung vektor gaya

• F_2 (Jarak s , arah $+\hat{j}$)

$$F_2 = k \frac{|(3q)(q)|}{s^2} = 3k' = 3k'\hat{j}$$

• F_1 (Jarak $s\sqrt{2}$, arah $-\hat{i}, -\hat{j}$)

$$F_1 = k \frac{|(3q)(2q)|}{(s\sqrt{2})^2} = k \frac{6q^2}{2s^2} = 3k'$$

• F_3 (Jarak s , arah $-\hat{i}$)

$$F_3 = k \frac{|(3q)(-q)|}{s^2} = 3k' = 3k'\hat{j}$$

$$= 3k' \cdot \frac{1}{\sqrt{2}} (-\hat{i}, -\hat{j}) = -\frac{3k'}{\sqrt{2}} \hat{i} - \frac{3k'}{\sqrt{2}} \hat{j}$$

$$F_{\text{total}} = F_1 + F_2 + F_3$$

$$= \left(-\frac{3k'}{\sqrt{2}} - \frac{3k'}{\sqrt{2}} \right) \hat{i} + \left(3k' - \frac{3k'}{\sqrt{2}} \right) \hat{j}$$

• Besar gaya total

$$F_{\text{total}} = \sqrt{F_x^2 + F_y^2}$$

$$= 3k' \sqrt{\left(-1 - \frac{1}{\sqrt{2}} \right)^2 + \left(1 - \frac{1}{\sqrt{2}} \right)^2}$$

$$= 3k' \sqrt{1+1 + \frac{1}{2} + \frac{1}{2}} = 3k' \sqrt{3} = 3\sqrt{3} \cdot k \frac{q^2}{s^2}$$

c.) Hitung ~~Vektor~~ medan listrik

$$E_{\text{total}} = \frac{F_{\text{total}}}{3q} = \frac{3k \frac{q^2}{s^2} \left[\left(-1 - \frac{1}{\sqrt{2}} \right) \hat{i} + \left(1 - \frac{1}{\sqrt{2}} \right) \hat{j} \right]}{3q}$$

$$= k \frac{q}{s^2} \left[\left(-1 - \frac{1}{\sqrt{2}} \right) \hat{i} + \left(1 - \frac{1}{\sqrt{2}} \right) \hat{j} \right]$$

d.) Hitung Besar Medan Listrik

$$\vec{E} = \frac{F_{\text{total}}}{3q} = \frac{3\sqrt{3} \cdot k \frac{q^2}{s^2}}{3q} = \sqrt{3} \cdot k \frac{q}{s^2}$$

e.) Hitung Pula Potensial listrik pada titik itu

$$V = k \frac{q_1}{r_1} + k \frac{q_2}{r_2} + k \frac{q_3}{r_3}$$

$$\bullet V_1 = k \frac{2q}{5\sqrt{2}} = k \frac{q}{5} \sqrt{2}$$

$$\bullet V_3 = k \frac{-q}{s} = -k \frac{q}{s}$$

$$\bullet V_2 = k \frac{q}{s}$$

$$\vec{V} = V_1 + V_2 + V_3$$

$$= k \frac{q}{s} \sqrt{2} + k \frac{q}{s} - k \frac{q}{s}$$

$$= k \frac{q}{s} \sqrt{2}$$

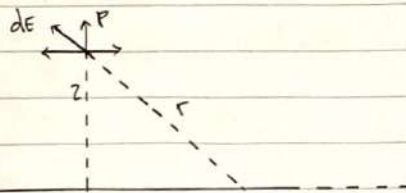
$$\vec{V} = \sqrt{2} \cdot k \frac{q}{s}$$

7.) Sebuah batang panjang dititi p yang berjarak z dari

7.) sebuah batang panjangnya L, rapat muatan per satuan panjang adalah λ .

Hitung :

a.) Kuat medan listrik di titik P yang berjarak z diatas ujung kawat.



$x = 0$ hingga $x = L$

$P = x = L + z$

$$d\vec{E} = k \frac{dq}{r^2} = k \frac{\lambda dx'}{((L+z)-x')^2}$$

$$\vec{E} = \int_0^L k\lambda \frac{dx'}{((L+z)-x')^2}$$

Gunakan $u = (L+z)-x'$, maka $du = -dx'$, batas integral berubah

• untuk $x' = 0$, $u = L+z$

• untuk $x' = L$, $u = z$

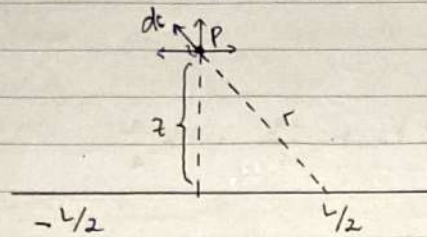
$$\vec{E} = k\lambda \int_{L+z}^z \frac{-du}{u^2} = -k\lambda \left[-\frac{1}{u} \right]_{L+z}^z = k\lambda \left[\frac{1}{u} \right]_{L+z}^z$$

$$= k\lambda \left(\frac{1}{z} - \frac{1}{L+z} \right)$$

$$= k\lambda \left(\frac{(L+z)-z}{z(L+z)} \right)$$

$$\vec{E} = k\lambda \frac{L}{z(L+z)}$$

b.) Kuat medan listrik di titik P yang berjarak z ditengah-tengah kawat



$$d\vec{E} = k \frac{\lambda dx'}{(z-x')^2}$$

$$\vec{E} = \int_{-L/2}^{L/2} k \lambda \frac{dx}{(z-x')^2}$$

Gunakan Substitusi $u = z - x'$, maka $du = -dx'$

• Untuk $x' = -L/2$, $u = z + L/2$

• Untuk $x' = L/2$, $u = z - L/2$

$$\vec{E} = k \lambda \int_{z+L/2}^{z-L/2} \frac{-du}{u^2} = -k \lambda \left[-\frac{1}{u} \right]_{z+L/2}^{z-L/2} = k \lambda \left[\frac{1}{u} \right]_{z+L/2}^{z-L/2}$$

$$= k \lambda \left(\frac{1}{z-L/2} - \frac{1}{z+L/2} \right)$$

$$= k \lambda \left(\frac{(z+L/2) - (z-L/2)}{(z-L/2)(z+L/2)} \right)$$

$$\vec{E} = k \lambda \frac{L}{z^2 - (L/2)^2}$$

8.) Hitung medan listrik pada bola konduktor dengan rapat muatan σ dan jari-jari R

Hukum Gauss $\rightarrow \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$

Total muatan pada permukaan bola $Q = \sigma \cdot A = \sigma(4\pi R^2)$

$$\oint \vec{E} \cdot d\vec{A} = \frac{0}{\epsilon_0} = E(4\pi r^2) = 0 \rightarrow \underline{\underline{E = 0}} \rightarrow \text{medan listrik dalam bola konduktor}$$

• Medan listrik dalam bola konduktor

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

$$E = \frac{\sigma R^2}{\epsilon_0 r^2} \hat{r}$$

$$E(4\pi r^2) = \frac{\sigma(4\pi R^2)}{\epsilon_0}$$

$$E = \frac{\sigma R^2}{\epsilon_0 r^2}$$

$$E = \frac{\sigma(4\pi R^2)}{4\pi \epsilon_0 r^2}$$

g.) a.) Hitung rapat muatan (ρ)

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \rightarrow \nabla \cdot \mathbf{E} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 E_r)$$

• Hitung Divergensi

$$\begin{aligned} E_r &= kr^3 \\ \nabla \cdot \mathbf{E} &= \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \cdot kr^3) \\ &= \frac{1}{r^2} \frac{\partial}{\partial r} (kr^5) \\ &= \frac{1}{r^2} (5kr^4) \\ \nabla \cdot \mathbf{E} &= 5kr^2 \end{aligned}$$

• Hitung rapat muatan (ρ)

$$\begin{aligned} \rho &= \epsilon_0 (\nabla \cdot \mathbf{E}) \\ \rho &= \epsilon_0 (5kr^2) \rightarrow \rho = \underline{\underline{5k\epsilon_0 r^2}} \end{aligned}$$

b.) Cari muatan total yang terdapat dalam suatu bola dengan jari-jari R

$$Q_{\text{total}} = \int_{\text{Volume}} \rho \, dv$$

dalam koordinat bola, volume $dv = r^2 \sin \theta \, dr \, d\theta \, d\phi$

$$\begin{aligned} Q_{\text{total}} &= \int_0^{2\pi} \int_0^\pi \int_0^R (5k\epsilon_0 r^2) (r^2 \sin \theta \, dr \, d\theta \, d\phi) \\ &= 5k\epsilon_0 \left(\int_0^R r^4 \, dr \right) \left(\int_0^\pi \sin \theta \, d\theta \right) \left(\int_0^{2\pi} d\phi \right) \\ &= \int_0^\pi \sin \theta \, d\theta = [-\cos \theta]_0^\pi = -(\cos \pi - \cos 0) = -(-1 - 1) = 2 \\ \int_0^{2\pi} d\phi &= 2\pi \\ \int_0^R r^4 \, dr &= \left[\frac{1}{5} r^5 \right]_0^R = \frac{1}{5} R^5 \end{aligned}$$

→ muatan total

$$\begin{aligned} Q_{\text{total}} &= 5k\epsilon_0 \left(\frac{1}{5} R^5 \right) (2) (2\pi) \\ &= \underline{\underline{4\pi k\epsilon_0 R^5}} \end{aligned}$$