

hukum Gauss.

$$\oint \vec{E} \cdot d\vec{s} = \iiint \nabla \cdot \vec{E} \, dV$$

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

$$\oint_s \vec{E} \cdot d\vec{s} = \int_V \nabla \cdot \vec{E} \, dV \xrightarrow{r}$$

$$\oint \vec{E} \cdot d\vec{s} = \frac{1}{4\pi\epsilon_0} \int_0^{2\pi} \int_0^\pi \frac{Q}{r^2} r^2 \sin\theta \, d\theta \, d\phi$$

$$= \frac{1}{4\pi\epsilon_0} 2\pi \int_0^\pi Q \sin\theta \, d\theta$$

$$= \frac{Q(2\pi)}{4\pi\epsilon_0} \left[-\cos\theta \Big|_0^\pi \right]$$

$$= \frac{4\pi Q}{4\pi\epsilon_0} = \frac{Q}{\epsilon_0}$$

$$\oint \vec{E} \cdot d\vec{s} = \frac{Q}{\epsilon_0}$$

$$\iiint \nabla \cdot \vec{E} \, dV \xrightarrow{\left[\frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \right]} Q = \rho \, dV.$$

$$= \frac{1}{4\pi\epsilon_0} \iiint \nabla \cdot \left(\frac{\vec{r}}{r^2} \right) \rho \, dV$$

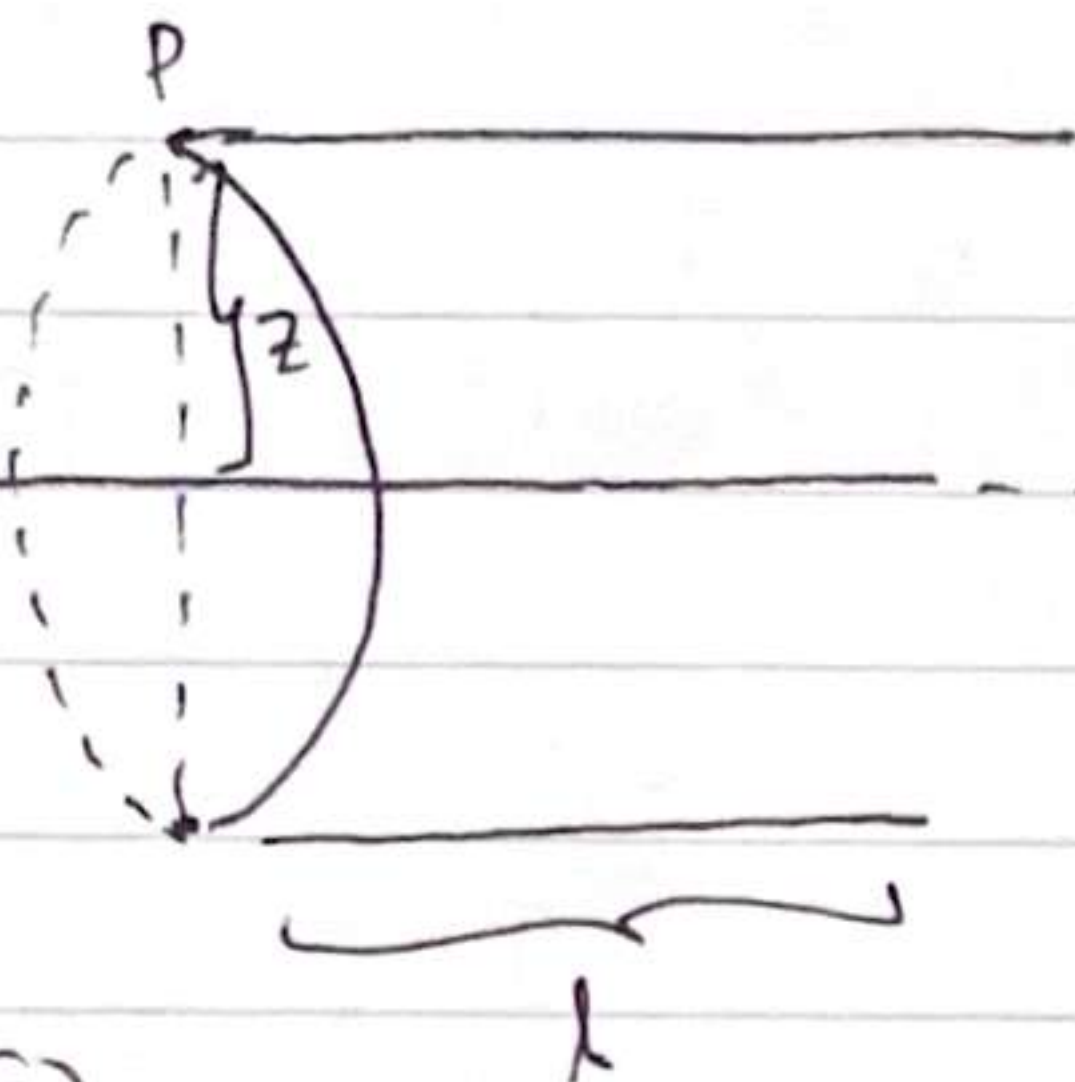
$$= \frac{1}{4\pi\epsilon_0} \int_V \left(\frac{\vec{r}}{r^2} \right) dV \xrightarrow{\int_V \delta \, dV = 1}$$

$$= \frac{\rho}{4\pi\epsilon_0} \left[4\pi (\delta \, dV) \right]$$

$$= \frac{4\pi \rho}{4\pi\epsilon_0} \xrightarrow{\text{operator}} \nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

Contoh :

1) Hitung medan listrik pada kawat lurus sepanjang z .



$$E \oint ds = E(S)$$

$$E = \frac{\lambda}{2\pi \epsilon_0 z}$$

$$Q = \lambda l.$$

$$Q = qA$$

$$Q = \rho V.$$

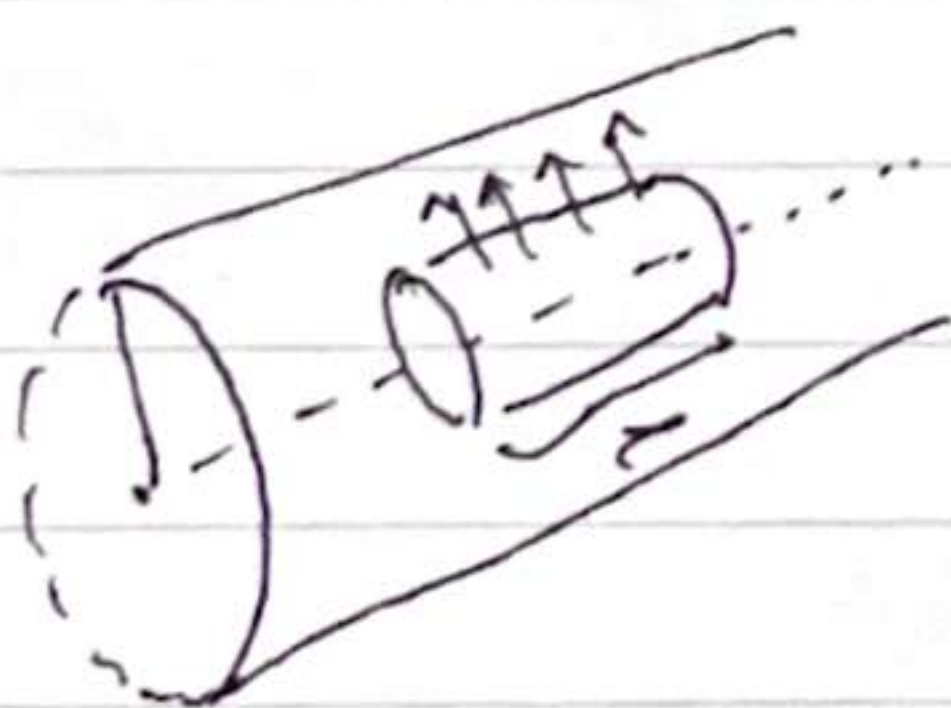
$$\oint \vec{E} \cdot d\vec{s} = \frac{Q}{\epsilon_0}.$$

$$E \cdot S = \frac{\lambda l}{\epsilon_0}.$$

$$E \cdot 2\pi z K = \frac{\lambda K}{\epsilon_0}.$$

$$E = \frac{\lambda}{2\pi \epsilon_0 z}.$$

2) Sebuah silinder pejal panjang setali, dengan jari-jari silinder R , rapat muatan ρ , hitung medan listrik di dalam silinder!



$$\oint \vec{E} \cdot d\vec{s} = \frac{Q}{\epsilon_0} \rightarrow \rho d\tau.$$

$$= \frac{1}{\epsilon_0} \iiint \rho d\tau$$

$$= \frac{\rho}{\epsilon_0} \int_0^l \int_0^{2\pi} \int_0^r r d\phi dr dz$$

$$= \frac{2\pi \rho l}{\epsilon_0} \int_0^l \int_0^r r dr dz$$

Lanjutan no 2.

$$= \frac{2\pi \rho l}{\epsilon_0} \int_0^r r dr$$

$$= \frac{2\pi \rho l}{\epsilon_0} \left[\frac{1}{2} r^2 \right]_0^r$$

E . S .

$$E (2\pi r l) = \frac{2\pi \rho l}{\epsilon_0} [r^2 - 0]$$

$$E (2\pi r l) = \frac{2\pi \rho l r^2}{\epsilon_0}$$

$$E = \frac{\rho r^2}{2r\epsilon_0} = \frac{\rho r}{2\epsilon_0}$$

jadi, medan listrik di dalam silinder dengan jari-jari r bergantung pada rapat dan tempatnya.