Hukum Gauss

Merupakan hukum yang menghubungkan fluks total pada permukaan tertutup dengan jumlah muatan yang dikandung oleh permukaan tersebut. Fluks total yang melewati permukaan tertutup sebanding dengan muatan total yang dilingkupi permukaan tertutup tersebut.

$$\Phi_E = \oint \vec{E}\cdot d\vec{A} = \frac{Q_{dalam}}{\varepsilon_0}$$

Non diskrit

$$\oiint \vec{E}\, ds = \iiint \nabla \vec{E}\, d\tau_0 \rightarrow E = \frac{q}{4\pi\varepsilon_0} r^{-2}$$

$$\oint_s \vec{E}\, d\vec{s} = \int_v \nabla\cdot\vec{E}\, d\tau_0 \rightarrow r$$

$$\oint \vec{E}\, ds = \frac{1}{4\pi\varepsilon_0}\int_0^{2\pi}\int_0^{\pi}\frac{Q}{r^2} r^2 \sin\theta\, d\theta\, d\phi$$

$$= \frac{1}{4\pi\varepsilon_0}(2\pi)\int_0^\pi Q\sin\theta\, d\theta = \frac{Q(2\pi)}{4\pi\varepsilon_0}\left[-\cos\theta\Big|_0^\pi\right]$$

$$= \frac{4\pi Q}{4\pi\varepsilon_0} = \frac{Q}{\varepsilon_0}$$

$$\boxed{\oint_s E\, ds = \frac{Q}{\varepsilon_0}}$$

$$\iiint \nabla \vec{E}\, d\tau_0 \rightarrow \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

$$\nabla\cdot\vec{E} \rightarrow \frac{1}{4\pi\varepsilon_0}\iiint \nabla\left(\frac{\hat{r}}{r^2}\right)\rho\, d\tau_0$$

$$Q = \rho\, d\tau_0$$

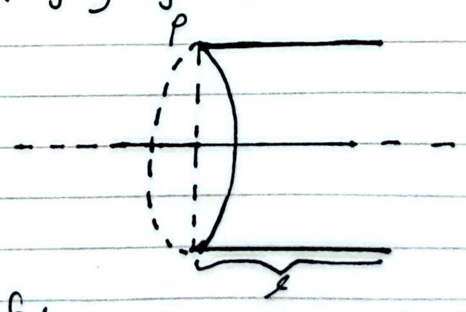
$$= \frac{1}{4\pi\varepsilon_0}\int_v \nabla\left(\frac{\hat{r}}{r^2}\right) d\tau_0$$

$$= \frac{\rho}{4\pi\varepsilon_0}\left[\int 4\pi\delta\, d\tau_0\right]$$

$$= \frac{4\pi\rho}{4\pi\varepsilon_0} \rightarrow \underbrace{\nabla}_{\text{operator}}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

Contoh:

1. Hitung medan listrik pada kawat lurus panjang sejauh z ...



Satuan dimensi = λl
Luas = δA
Volume = ρV

$$E\oint ds = E(s)$$

$$\oint \vec{E}\, ds = \frac{Q}{\varepsilon_0}$$

$$E_s = \frac{\lambda l}{\varepsilon_0}$$

$$E\cdot 2\pi z \cancel{l} = \frac{\lambda \cancel{l}}{\varepsilon_0}$$

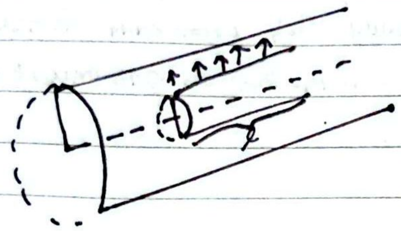
$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 z}}$$

⇒

No.

Date

2. Pada silinder dengan jari-jari r, hitung medan magnet di dalam dengan keadaan sangat rapat



$$\oint_s E\,ds = \frac{Q}{\varepsilon_0} \rightarrow \int dE$$

$$= \frac{1}{\varepsilon_0} \iiint \rho\, d\tau_0$$

$$= \frac{\rho}{\varepsilon_0} \int_0^{\ell} \int_0^{r} \int_0^{2\pi} r\, \theta\, \phi\, dr\, dz$$

$$= \frac{2\pi\rho}{\varepsilon_0} \int_0^{\ell} \int_0^{r} r\, dr\, dz$$

$$= \frac{2\pi\rho z}{\varepsilon_0} \int_0^{r} r\, dr = \frac{2\pi\rho\ell}{\varepsilon_0} \left[\frac{1}{2} r^2 \Big|_0^{r} \right]$$

$\oint E \cdot S$

$$E(2\pi r\ell) = \frac{2\pi\rho\ell}{2\varepsilon_0}\left[r^2 - 0 \right]$$

$$E(\cancel{2\pi r\ell}) = \frac{\cancel{\pi} \cdot \rho \cancel{\ell} r^2}{\varepsilon_0}$$

$$\vec{E} = \frac{\rho r^2}{2r\varepsilon_0} = \frac{\rho r}{2\varepsilon_0}$$

$$\boxed{\vec{E} = \frac{\rho r}{2\varepsilon_0}} \rightarrow$$ yang mempengaruhi ukuran, rapat massa, letaknya.