

Hukum Gauss

$$\oint \vec{E} \cdot d\vec{s} = \iiint \nabla \cdot \vec{E} \, dV$$

$$\oint \vec{E} \cdot d\vec{s} = \int_V \nabla \cdot \vec{E} \, dV$$

$$\oint \vec{E} \cdot d\vec{s} = \frac{1}{4\pi\epsilon_0} \iiint \rho \sin \theta \, d\theta \, d\phi$$

$$= \frac{1}{4\pi\epsilon_0} (2\pi) \int_0^\pi \rho \sin \theta \, d\theta$$

$$= \frac{Q (2\pi)}{4\pi \epsilon_0} \left[-\cos \theta \Big|_0^\pi \right]$$

$$= \frac{4\pi Q}{4\pi \epsilon_0} = \frac{Q}{\epsilon_0}$$

$$\oint \vec{E} \cdot d\vec{s} = \frac{Q}{\epsilon_0}$$

$$\iiint \nabla \cdot \vec{E} \, dV \rightarrow \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \rightarrow Q = \rho \, dV$$

$$\nabla \cdot \vec{E} = \frac{1}{4\pi\epsilon_0} \iiint \nabla \cdot \left(\frac{\hat{r}}{r^2} \right) \rho \, dV$$

$$Q = \rho \, dV$$

$$= \frac{1}{4\pi\epsilon_0} \rho \int_V \nabla \cdot \left(\frac{\hat{r}}{r^2} \right) dV \rightarrow \int_V \delta \, dV = 1$$

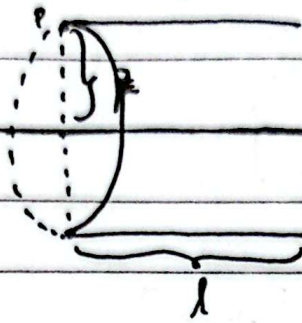
$$= \frac{\rho}{4\pi\epsilon_0} \left[4\pi \int \delta \, dV \right]$$



$$= \frac{4\pi\rho}{4\pi\epsilon_0} \longleftrightarrow \nabla \vec{E} = \frac{\rho}{\epsilon_0} \quad \text{operator}$$

Contoh :

1. Hitung medan listrik pada kawat lurus panjang sepanjang z



$$E \oint d\vec{s} = \frac{Q}{\epsilon_0}$$

$$\oint E \cdot d\vec{s} = \frac{Q}{\epsilon_0}$$

$$E \cdot 2\pi r l = \frac{\lambda l}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

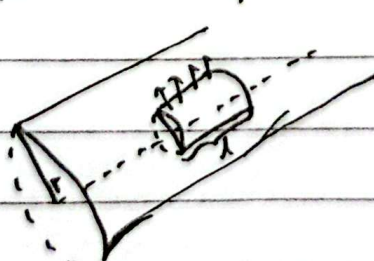
$Q = \lambda l$
$Q = \rho A$
$Q = \rho V$

$$E \cdot 2\pi r l = \frac{\lambda l}{\epsilon_0}$$

$$E \cdot 2\pi r l = \frac{\lambda l}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

2. Sebuah silinder pejal panjang sekali, dengan jari-jari silinder R , rapat muatan ρ . hitung medan listrik didalam silinder



$$\oint E \cdot d\vec{s} = \frac{Q}{\epsilon_0}$$

$$= \frac{1}{\epsilon_0} \iiint \rho d\vec{v}$$

lanjutan di belakng

$$= \frac{1}{\epsilon_0} \int_0^L \int_0^{2\pi} \int_0^r r d\phi dr dz$$

$$= \frac{2\pi \lambda}{\epsilon_0} \int_0^L \int_0^r r dr dz$$

$$= \frac{2\pi \rho z}{\epsilon_0} \int_0^L r dr$$

$$= \frac{2\pi \rho \lambda}{\epsilon_0} \left[\frac{1}{2} r^2 \right]_0^r$$

$$E \cdot S =$$

$$E (2\pi r \lambda) = \frac{2\pi \rho \lambda}{2\epsilon_0} [r^2 - 0]$$

$$E (2\pi r \lambda) = \frac{\pi \rho \lambda r^2}{\epsilon_0}$$

$$E = \frac{\rho r^2}{2\epsilon_0} = \frac{\rho r}{2\epsilon_0}$$

$$E = \frac{\rho r}{2\epsilon_0}$$

jadi, medan listrik didalam silinder dengan jari-jari r bergantung pada rapat dan tempatnya.

