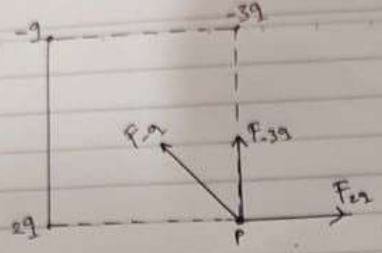
a)

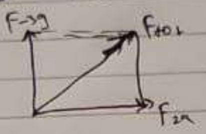


$F_{-q} = k\frac{-qP}{r^2}$

$F_{-3q} = k\frac{-3qP}{r^2}$

$F_{2q} = k\frac{2qP}{r^2}$

b) Hitung Resultan gaya F_{-3q} dan F_{2q}



$F_{tot1} = \sqrt{F_{2q}^2 + F_{3q}^2}$

$= \sqrt{\left(k\frac{2q.P}{r^2}\right)^2 + \left(k\frac{-3qP}{r^2}\right)^2}$

$= \sqrt{\frac{4k^2q^2P^2}{r^4} + \frac{9k^2q^2P^2}{r^4}}$

$= \sqrt{\frac{13k^2q^2P^2}{r^4}}$

$= \frac{kqP}{r^2}\sqrt{13}$

Hitung F_{tot} dari F_{-q} dan F_{tot1}

$F_{total} = \sqrt{F_{tot1}^2 + F_{-q}}$

$= \sqrt{\left(\frac{kqP}{r^2}\sqrt{13}\right)^2 + \left(k\frac{qP}{r^2}\right)^2}$

$= \frac{kqP}{r^2}\sqrt{14}$

c) Misal P berada pada koordinat (0,0)

$E = k\frac{Q}{r^2}\hat{r}$ Medan listrik oleh :

$E_{-q} = \sqrt{(-r-0)^2 + (r-0)^2} = r\sqrt{2}$

(2) a. Kuat medan listrik di titik P yang berjarak z di atas ujung kawat.

Asumsi:

- batang terletak sepanjang x dari 0 – L,
- P berada pada sumbu y = z.
- maka medan listrik di P, dengan mengintegralkan kontribusi
- medan listrik dari dx
  - ⇒ muatan dx ⇒ $dq = \lambda dx$
  - ⇒ jarak dari elemen dx ke titik P adalah $r = \sqrt{x^2+z^2}$
  - ⇒ medan listrik dE dari dq di titik P memiliki dEx dan dEy

$$dE_x = dE\cos\theta = \frac{k\,dq}{r^2}\cdot\frac{-x}{r} = -\frac{k\lambda\,dx\,x}{(x^2+z^2)^{3/2}}$$

$$dE_y = dE\sin\theta = \frac{k\,dq}{r^2}\cdot\frac{z}{r} = \frac{k\lambda\,dx\,z}{(x^2+z^2)^{3/2}}$$

- Integral keduanya sepanjang $x = 0 \to x = L$
  ⇒ komponen x :

$$E_x = \int_0^L -\frac{k\lambda x}{(x^2+z^2)^{3/2}}\,dx$$

⇒ substitusi $u = x^2+z^2 \Rightarrow du = 2x\,dx \Rightarrow \int_{z^2}^{L^2+z^2}$

$$E_x = -\frac{k\lambda}{2}\int_{z^2}^{L^2+z^2} u^{-3/2}\,du = -\frac{k\lambda}{2}\left[\frac{u^{-1/2}}{-1/2}\right]_{z^2}^{L^2+z^2} = k\lambda\left[\frac{1}{\sqrt{u}}\right]_{z^2}^{L^2+z^2}$$

⇒ komponen y

$$E_y = \int_0^L \frac{k\lambda z}{(x^2+z^2)^{3/2}}\,dx = k\lambda z\int_0^L (x^2+z^2)^{-3/2}\,dx$$

⇒ substitusi $x = z\tan\phi$, $dx = z\sec^2\phi\,d\phi \Rightarrow \int_0^{\arctan(L/z)}$

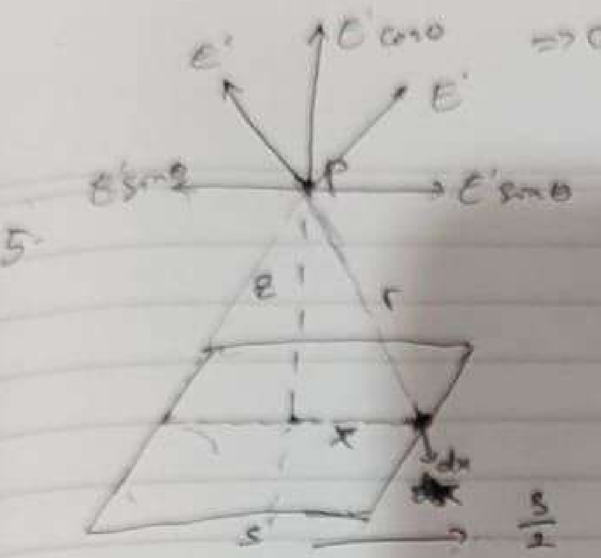
$$E_y = k\lambda z\int_0^{\mathrm{arc}(L/z)} (z^2\tan^2\phi + z^2)^{-3/2} z\sec^2\phi\,d\phi$$

$$E_y = k\lambda z\int_0^{\mathrm{arc}(L/z)} (z^2\sec^2\phi)^{-3/2}\,\phi\,d\phi$$

$$E_y = k\lambda z\int_0^{\mathrm{arc}(L/z)} \frac{1}{z^3\sec^3\phi}\,z\sec^2\phi\,d\phi = \frac{k\lambda}{z}\int_0^{\mathrm{arc}(L/z)}\cos\phi\,d\phi$$

$$E_y = \frac{k\lambda}{z}\left[\sin\phi\right]_0^{\mathrm{arc}(L/z)} = \frac{k\lambda}{z}\sin(\mathrm{arc}\,L/z) = \frac{k\lambda}{z}\frac{L}{\sqrt{L^2+z^2}} = \frac{k\lambda L}{z\sqrt{L^2+z^2}}$$

5.



$\Rightarrow \cos\theta = \frac{z}{\bullet} = \frac{z}{\sqrt{z^2 + \frac{s^2}{4}}}$

$q = \lambda \cdot s$

$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

$z = \sqrt{z^2 + \frac{s^2}{4}}$

$E = \frac{1}{4\pi\varepsilon_0}\lambda\frac{dx}{r^2}\hat{r}' \Rightarrow r = \sqrt{z^2+x^2}$

$E = \int_{-s}^{s}\frac{1}{4\pi\varepsilon_0}\frac{\lambda}{z^2+x^2}dx \quad \Rightarrow \hat{r}' = \frac{\vec{r}}{r} = \frac{zk - xi}{r}$

$E = \int_{-s}^{s}\frac{1}{4\pi\varepsilon_0}\frac{\lambda}{z^2+x^2}dx\,\frac{zk-xi}{r}$

$= \frac{1}{4\pi\varepsilon_0}\left[\int_{-s}^{s}\frac{z\lambda}{(z^2+x^2)^{\frac{3}{2}}}dx\;k - \int_{s}^{s}\frac{}{(z^2+x^2)^{\frac{3}{2}}}\,x\,dx\;i\right]$

$= \frac{1}{4\pi\varepsilon_0}\left[z\lambda\left[\frac{s}{z^2(z^2+s^2)^{\frac{1}{2}}} + \frac{s}{z^2(z^2+s^2)^{\frac{1}{2}}}\right]_{-s}^{s}k - 0\right]$

$= \frac{1}{4\pi\varepsilon_0}\frac{2\lambda s}{z\sqrt{z^2+s^2}}\,k$ ingat $\frac{s}{2}$

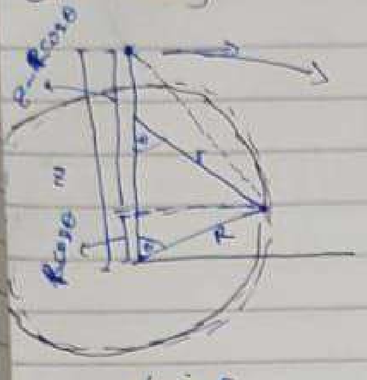
$= \frac{1}{4\pi\varepsilon_0}\frac{2\lambda\frac{s}{2}}{\sqrt{z^2+\frac{s^2}{4}}\sqrt{(z^2+\frac{s^2}{4})^2+(\frac{s}{2})^2}}$

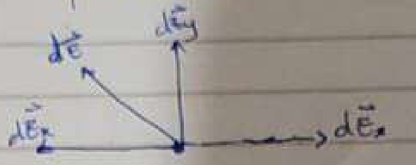
$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda s}{\sqrt{z^2+\frac{s^2}{4}}\sqrt{z^2+\frac{s^2}{4}+(\frac{s^2}{4}}}\,k$

$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda s}{\sqrt{z^2+\frac{s^2}{4}}\sqrt{z^2+\frac{s^2}{2}}}\,k.$

$= 4\cdot\frac{1}{4\pi\varepsilon_0}\frac{\lambda s}{\sqrt{z^2+\frac{s^2}{4}}\sqrt{(z^2+\frac{s^2}{2})}}\frac{z}{\sqrt{z^2+\frac{s^2}{4}}}\,k.$ Sepanjang sb x

③ Hitung medan listrik pd benda konduktor dgn rapat muatan σ dan jari² R





dari r

$r = \left[(R\sin\theta)^2 + (z - R\cos\theta)^2\right]^{1/2}$

$= \left[R^2\sin^2\theta + (z^2 + R^2\cos^2\theta - 2Rz\cos\theta)\right]^{1/2}$

$= \left[R^2(\sin^2\theta + \cos^2\theta) + z^2 - 2Rz\cos\theta\right]^{1/2}$

$= (R^2 + z^2 - 2Rz\cos\theta)^{1/2}$

$\Rightarrow \vec{E}$

$d\vec{E} = d\vec{E}x + d\vec{E}y$

$d\vec{E} = dE\cos\theta$

$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_0^{\pi}\int_0^{2\pi}\frac{\sigma\, da\cos\theta}{r^2}$

$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_0^{\pi}\int_0^{2\pi}\frac{\sigma(z - R\cos\theta)R^2\sin\theta\, d\theta\, d\phi}{(R^2+z^2-2Rz(\cos\theta))^{3/2}}$

$= \frac{\sigma R^2(2\pi)}{4\pi\varepsilon_0}\int_0^{\pi}\frac{\sin\theta\,(z - R\cos\theta)}{(R^2+z^2-2Rz(\cos\theta))^{1/2}}\,d\theta$

$= \frac{\sigma R^2}{2\varepsilon_0}\int_0^{\pi}\frac{\sin\theta\,(z-R\cos\theta)}{(R^2+z^2-2Rz\cos)^{3/2}}\,d\theta$

$\int_{-1}^{1}\frac{(z+du)(z-Ru)}{(R^2+z^2-2Rzu)^{3/2}}$ $\Rightarrow$ $u = \cos\theta$

$\frac{du}{d\theta} = -\sin\theta$

$-du = \sin\theta\, d\theta$

$R^2 + z^2 - 2Rzu = y$

$\Rightarrow \int_{(R-z)^2}^{(R+z)^2}\left(\frac{dy}{-2dz}\right)\frac{z^2-R^2+y}{(2z)\,y^{3/2}}$ $\quad -2Rz = \frac{dy}{du} \rightarrow du = \frac{dy}{2Rz}$

$= \int_{(R-z)^2}^{(R+z)^2}\left(\frac{1}{4Rz^2}\right)\left((z^2-R^2)y^{-3/2} + y^{-1/2}\right)dy$

$= \frac{1}{4Rz^2}\left[(z^2-R^2(-2y)^{-1/2} + 2y^{1/2}\right]_{(R-z)^2}^{(R+z)^2}$

$\Rightarrow \frac{1}{2Rz^2}\left[(R^2-z^2)\frac{1}{\sqrt{y}} + \sqrt{y}\right]_{(R-z)^2}^{(R+z)^2}$

$= \frac{1}{2Rz^2}\left[(R^2-z^2)\left(\frac{1}{R+z} - \frac{1}{|R-z|}\right) + (R+z) - (R-z)\right]$

$= \frac{1}{2Rz^2}\left(R-z\frac{R^2-z^2}{|R-z|} + R+z\,|R-z|\right)\, |R-z|^2 - |R-z|^2$

$= \frac{1}{2Rz^2}\left(2R - \frac{(R^2-z^2+(R-z)^2)}{|R-z|}\right)e^2 - 2Rz + z^2$

$= \frac{1}{2Rz^2}\left(2R - \frac{(2R^2-2Rz)}{|R-z|}\right) = \frac{1}{z^2}\left[1 - \frac{R-z}{|R-z|}\right]$

$\frac{\sigma R^2}{2\varepsilon_0}\,\frac{1}{z^2}\left(1 - \frac{R-z}{|R-z|}\right)$

• $z > R$

$E = \frac{\sigma R}{2\varepsilon_0}\cdot\frac{1}{z^2}\left(\frac{1-(R-z)}{z-R}\right)$

$\frac{\sigma R^2}{\varepsilon_0 z^2} = \frac{1}{4\pi\varepsilon_0}\,\frac{4\pi R^2\sigma}{z^2}$

$= \frac{1}{4\pi\varepsilon_0}\,\frac{Q}{z^2}$

• $z < R$

$E = \frac{\sigma R^2}{2\varepsilon_0}\,\frac{1}{z}$

$= \left(1 - \frac{R-z}{|R-z|}\right) = 0$

$= 1 - \frac{R-z}{R-z} = 1 - 1 = 0$

//

4. Menghitung Rapat muatan (ρ)

$$\nabla \cdot E = \frac{\rho}{\epsilon_0}$$

Koordinat bola

$$\nabla \cdot E = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 . kr^3) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta \cdot 0) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(0)$$

$$\nabla \cdot E = \frac{1}{r^2}\frac{\partial}{\partial r}(kr^5)$$

$$\nabla E = \frac{1}{r^2}(5kr^4) = 5kr^2$$

Gunakan Hk Gauss

$$\rho = \epsilon_0(\nabla \cdot E) = \epsilon_0(5kr^2) = 5k\epsilon r^2$$

$$\rho(r) = 5k\epsilon r^2$$

Muatan total

$$Q_{tot} = \int_V \rho\, dv = \int_0^R\int_0^\pi\int_0^{2\pi}(5k\epsilon_0 r^2)(r^2\sin\theta\, dr\, d\theta\, d\phi)$$

$$Q_{tot} = 5k\epsilon_0\int_0^R r^4\, dr\int_0^\pi \sin\theta\, d\theta\int_0^{2\pi} d\phi$$

$$Q_{tot} = 5k\epsilon_0\left(\frac{R^5}{5}\right)\cdot 2 \cdot 2\pi$$

$$Q_{tot} = 4\pi k\epsilon_0 R^5$$