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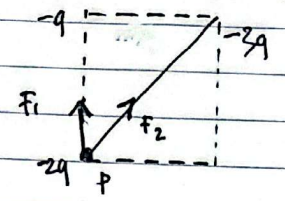
UTS GENAP 2025
ELEKTRODINAMIKA

1. Ada 3 Muatan seperti gambar berikut.



a) Gambarkan gaya-gaya pada titik P
b) hitung besar gayatsb.
gaya dari -q ke 2q (F_1)

$$F_1 = k\frac{q\,2q}{a^2} = \frac{2kq^2}{a^2}$$

Gaya dari -3q ke 2q (F_2)

$$r = \sqrt{a^2+a^2} = a\sqrt{2}$$

$$F_2 = k\frac{3q\cdot 2q}{(a\sqrt{2})^2} = \frac{6kq^2}{2a^2} = \frac{3kq^2}{a^2}$$

$$F_{2x} = F_2\cos(45) = \frac{3kq^2}{a^2}\cdot\frac{1}{\sqrt{2}} = \frac{3kq^2}{a^2\sqrt{2}}$$

$$F_{2y} = F_2\sin(45) = \frac{3kq^2}{a^2}\cdot\frac{1}{\sqrt{2}} = \frac{3kq^2}{a^2\sqrt{2}}$$

Gaya total (komponen x dan y).

$$F_x = F_{2x} = \frac{3kq^2}{a^2\sqrt{2}}$$

$$F_y = F_1 + F_{2y} = \frac{2kq^2}{a^2} + \frac{3kq^2}{a^2\sqrt{2}}$$

besar gaya total $F_{total} = \sqrt{F_x^2 + F_y^2}$

c). Hitung Medan listrik pada titik tersebut

$$E = \frac{F}{2q} = \frac{\sqrt{F_x^2+F_y^2}}{2q}$$

d). hitung Besar Medan listrik pada titik tersebut.

$$E_x = \frac{F_x}{2q} = \frac{3kq}{a^2\sqrt{2}}$$

$$E_y = \frac{F_y}{2q} = \frac{kq}{a^2} + \frac{3kq}{2a^2\sqrt{2}}$$

$$E_{total} = \sqrt{E_x^2 + E_y^2}$$

e. Hitung potensial listrik pada titik itu.

$$V = k\left(\frac{-q}{a} + \frac{-3q}{a\sqrt{2}} + \frac{0}{0}\right)$$

P berada pada muatan 2q shg.

$$V = -kq\left(\frac{1}{a} + \frac{3}{a\sqrt{2}}\right)$$

no.2 Jawab.. Kuat Medan listrik di titik yg berjarak z diatas ujung kawat

$r = \sqrt{x^2+z^2}$

$dq = \lambda dx$ Menghasilkan Medan listrik dE di P.

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{x^2+z^2}$$

$$dE_z = dE\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{x^2+z^2}\cdot\frac{z}{\sqrt{x^2+z^2}} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda z\,dx}{(x^2+z^2)^{3/2}}$$

$$dE_x = dE\sin\theta = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{x^2+z^2}\cdot\frac{x}{\sqrt{x^2+z^2}} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda x\,dx}{(x^2+z^2)^{3/2}}$$

$$E = \int_0^L dE_z = \frac{\lambda z}{4\pi\varepsilon_0}\int_0^L \frac{dx}{(x^2+z^2)^{3/2}}$$

* Komponen E_z

$$E_z = \int \frac{dx}{(x^2+z^2)^{3/2}} = \frac{x}{z^2\sqrt{x^2+z^2}} + C$$

$$E_z = \frac{\lambda z}{4\pi\varepsilon_0}\left[\frac{x}{z^2\sqrt{x^2+z^2}}\right]_0^L = \frac{\lambda}{4\pi\varepsilon_0 z}\left[\frac{L}{\sqrt{L^2+z^2}}\right]$$

* Komponen E_x

$$E_x = \int_0^L dE_x = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{x\,dx}{(x^2+z^2)^{3/2}}$$

$$E_x = \int_0^L dE_x = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{x\,dx}{(x^2+z^2)^{3/2}}$$

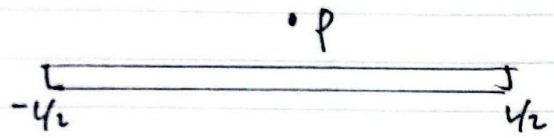
$$\int \frac{x\,dx}{(x^2+z^2)^{3/2}} = \frac{1}{\sqrt{x^2+z^2}} + C$$

Maka $E_x = \vec{E}x = \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{-1}{\sqrt{x^2+z^2}}\right]_0^L = \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{1}{z} - \frac{1}{\sqrt{L^2+z^2}}\right)$

Medan listrik total (karena P disembarang) E_z dan E_x.

$$\vec{E} = E_x\hat{x} + E_z\hat{z}$$

b). Kuat Medan listrik di titik P yang berjarak z ditengah-tengah kawat



$x = -L/2 \rightarrow x = L/2$

$$E_z = \int_{-L/2}^{L/2} dE_z = \frac{\lambda z}{4\pi\varepsilon_0} \int_{-L/2}^{L/2} \frac{dx}{(x^2+z^2)^{3/2}}$$

$$E_z = \frac{\lambda z}{4\pi\varepsilon_0} \left[\frac{x}{z^2\sqrt{x^2+z^2}} \right]_{-L/2}^{L/2} = \frac{\lambda}{4\pi\varepsilon_0 z} \left(\frac{L}{\sqrt{(L/2)^2+z^2}} \right)$$

Medan listrik total $\vec{E} = E_z \hat{z} = \frac{\lambda L}{4\pi\varepsilon_0 z\sqrt{(L/2)^2+z^2}} \hat{z}$

NO.3 jawab.

Jari2 R, rapat muatan σ
muatan total bola $Q = \sigma . 4\pi R^2$

1). Medan listrik didalam bola ($r < R$)

hk. Gauss $\oint \vec{E}\, d\vec{A} = \frac{Q_{dalam}}{\varepsilon_0}$

untuk $r < R$. $Q_{dalam} = 0$ (tdk ada muatan di dlm konduktor)

$E_{dalam} = 0$

2). Medan listrik di permukaan bola ($r = R$)

hk. Gauss $E . 4\pi R^2 = \frac{Q}{\varepsilon_0} = \frac{\sigma . 4\pi R^2}{\varepsilon_0}$.

$$E_{permukaan} = \frac{\sigma}{\varepsilon_0}$$

3). Medan listrik di luar bola ($r > R$)

hk. Gauss. $E . 4\pi r^2 = \frac{Q}{\varepsilon_0}$

$$E_{luar} = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{\sigma R^2}{\varepsilon_0 r^2}.$$

4. Jawab.

diketahui Medan listrik koordinat bola.

$\vec{E} = kr^3\hat{r}$, k = konstan.

a). Menghitung Rapat muatan (ρ).

hk. Gauss $\nabla\vec{E} = \frac{\rho}{\varepsilon_0}$

diveganti Medan listrik dlm koordinat bola.

$\vec{E} = kr^3\hat{r}$

$$\nabla\vec{E} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 E_r)$$

$$\nabla\vec{E} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 . kr^3) = \frac{1}{r^2}\frac{\partial}{\partial r}(kr^5)$$

$$\nabla\vec{E} = \frac{1}{r^2}\cdot 5kr^4 = 5kr^2$$

$$5kr^2 = \frac{\rho}{\varepsilon_0}$$

$$\rho = 5k\varepsilon_0 r^2$$

e). Hitung muatan total dalam bola berjari-jari R.

$$Q = \int_{volume} \rho\, dV \quad ; \text{ Muatan di seluruh } V$$

dlm koordinat bola $dV = r^2 \sin\theta\, dr\, d\theta\, d\phi$

substitusi ρ dan integrasi

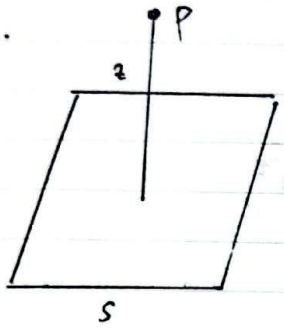
$$Q = \int_0^R \int_0^{\pi} \int_0^{2\pi} 5k\varepsilon_0 r^2 . r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$Q = 5k\varepsilon_0 \left(\int_0^R r^4\, dr\right)\left(\int_0^{\pi} \sin\theta\, d\theta\right)\left(\int_0^{2\pi} d\phi\right)$$

$$Q = 5k\varepsilon_0 \left[\frac{R^5}{4} \cdot 2 \cdot 2\pi\right]$$

$$Q = 4\pi k\varepsilon_0 R^5$$

No 2 lanjut.



$$E_z = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2\lambda L}{z\sqrt{L^2+4z^2}}$$

$x = -s/2$ ke $x = s/2$ $\quad y = -s/2$

$dq = \lambda dx$

Vektor dari elemen ke titik P

$$\vec{r} = (0-x)\hat{i} + (0+s/2)\hat{j} + (z-0)\hat{k}$$

$$= -x\hat{i} + \frac{s}{2}\hat{j} + z\hat{k}$$

$$r = \sqrt{x^2+(s/2)^2+z^2}$$

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq}{|\vec{r}|^3} \cdot \vec{r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\lambda dx}{(x^2+(s/2)^2+z^2)^{3/2}} \cdot \left(-x\hat{i} + \frac{s}{2}\hat{j} + z\hat{k}\right)$$

$$dE_z = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\lambda z\, dx}{(x^2+(\frac{s}{2})^2+z^2)^{3/2}}$$

Integralkan dari $x = -\frac{s}{2}$ ke $x = \frac{s}{2}$

$$E_z = \frac{\lambda z}{4\pi\varepsilon_0} \int_{-s/2}^{s/2} \frac{dx}{(x^2+(s/2)^2+z^2)^{3/2}}$$

$$a^2 = (s/2)^2 + z^2$$

$$E_z = \frac{\lambda z}{4\pi\varepsilon_0} \int_{-s/2}^{s/2} \frac{dx}{(x^2+a^2)^{3/2}}$$

$$\int_{-L}^{L} \frac{dx}{(x^2+a^2)^{3/2}} = \frac{2L}{a^2\sqrt{L^2+a^2}}$$

$L = s/2$

$$E_z = \frac{\lambda}{4\pi\varepsilon_0} \cdot \frac{2(s/2)}{a^2\sqrt{(s/2)^2+a^2}} = \frac{\lambda z s}{4\pi\varepsilon_0 a^2\sqrt{(s/2)^2+a^2}}$$

tapi, $a^2 = (s/2)^2 + z^2 = \frac{s^2}{4} + z^2$

Medan total dari 4 sisi

$$E_z = 4 \cdot E_z = \frac{4\lambda z s}{4\pi\varepsilon_0 \left(\frac{s^2}{4}+z^2\right)\cdot\sqrt{s^2+4z^2}}$$

$$E_z = \frac{\lambda z s}{\pi\varepsilon_0\left(\frac{s^2}{4}+z^2\right)\cdot\sqrt{s^2+4z^2}}$$

$$\vec{E}_{(P)} = \frac{\lambda z s}{\pi\varepsilon_0\left(\frac{s^2}{4}+z^2\right)\sqrt{s^2+4z^2}}\,\hat{z}$$