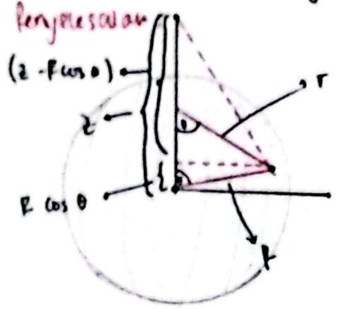
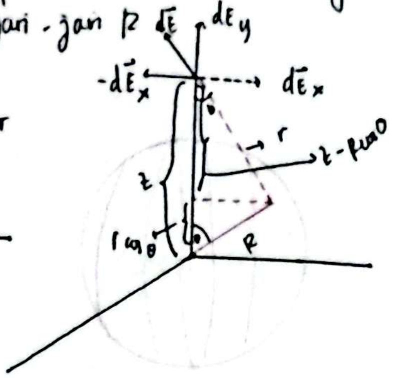
3] Hitung medan listrik pada bola konduktor dengan rapat muatan σ dan jari - jari R

Penyelesaian





Dari r

$$r = \left[(R\sin\theta)^2 + (z - R\cos\theta)^2\right]^{1/2}$$
$$= \left[R^2\sin^2\theta + (z^2 + R^2\cos^2\theta - 2Rz\cos\theta)\right]^{1/2}$$
$$= \left[R^2(\sin^2\theta + \cos^2\theta) + z^2 - 2Rz\cos\theta\right]^{1/2}$$
$$= (R^2 + z^2 - 2Rz\cos\theta)^{1/2}$$

Sekarang hitung $\bar{E}$

$$d\vec{E} = d\vec{E}_x + d\vec{E}_y$$
$$d\vec{E} = dE\cos\theta$$
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_0^{\pi}\int_0^{2\pi}\frac{\sigma\, da\cos\theta}{r^2}$$
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_0^{\pi}\int_0^{2\pi}\frac{\sigma(z - R\cos\theta)R^2\sin\theta\, d\theta\, d\phi}{(R^2 + z^2 - 2Rz\cos\theta)^{3/2}}$$
$$= \frac{\sigma R^2(2\pi)}{4\pi\varepsilon_0}\int_0^{\pi}\frac{\sin\theta\,(z - R\cos\theta)}{(R^2 + z^2 - 2Rz\cos\theta)^{3/2}}\, d\theta$$
$$= \frac{\sigma R^2}{2\varepsilon_0}\int_0^{\pi}\frac{\sin\theta\,(z - R\cos\theta)}{(R^2 + z^2 - 2Rz\cos)^{3/2}}\, d\theta$$
$$\int_{-1}^{1}\frac{(+du)(z - Ru)}{(R^2 + z^2 - 2Rzu)^{3/2}}$$

$u = \cos\theta$

$\frac{du}{d\theta} = -\sin\theta$

$-du = \sin\theta\, d\theta$

$R^2 + z^2 - 2Rzu = y$

$-2Rz = \frac{dy}{du} \rightarrow du = \frac{dy}{-2Rz}$

$$= \int_{(R-z)^2}^{(R+z)}\left(\frac{dy}{-2dz}\right)\frac{z^2 - R^2 + y}{(2z)\, y^{3/2}}$$
$$= \int_{(R-z)}^{(R+z)^2}\left(\frac{1}{4Rz^2}\right)\left((z^2 - R^2)\, y^{-3/2} + y^{-1/2}\right) dy$$
$$= \frac{1}{4Rz^2}\left[(z^2 - R^2(-2y)^{-1/2} + 2y^{1/2}\right]_{(R-z)^2}^{(R+z)^2}$$
$$= \frac{1}{2Rz^2}\left((R^2 - z^2)\frac{1}{\sqrt{y}} + \sqrt{y}\right)_{(R-z)^2}^{(R+z)^2}$$
$$= \frac{1}{2Rz^2}\left((R^2 - z^2)\left(\frac{1}{R+z} - \frac{1}{|R-z|}\right) + (R+z) - (R-z\right.$$
$$= \frac{1}{2Rz^2}\left(R - z\frac{R^2 - z^2}{|R-z|} + R + z - |R-z|\right) \quad |R-z|^2 = (R-z)^2$$
$$= \frac{1}{2Rz^2}\left(2R - \frac{(R^2 - z^2 + (R-z)^2}{(R-z}\right) \quad e^2 - 2Rz + z^2$$
$$= \frac{1}{2Rz^2}\left(2R - \frac{(2R^2 - 2Rz)}{|R-z|} = \frac{1}{z^2}\right|1 - \frac{R-z}{|R-z|}\Big)$$
$$\frac{\sigma R^2}{2\varepsilon_0}\frac{1}{z^2}\left(1 - \frac{R-z}{|R-z|}\right)$$

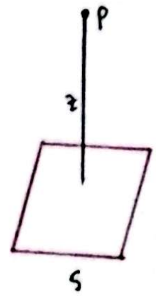
① $z > R$

$$E = \frac{\sigma R}{2\varepsilon_0}\frac{1}{z^2}\left(\frac{1 - (R-z)}{z - R}\right)$$
$$= \frac{\sigma R^2}{\varepsilon_0 z^2}$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{4\pi R^2\sigma}{z^2}$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{z^2}$$

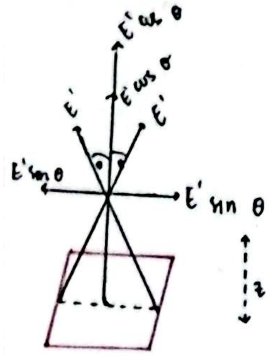
② $z < R$

$$E = \frac{\sigma R^2}{2\varepsilon_0}\frac{1}{z}$$
$$= \left(1 - \frac{R-z}{|R-z|}\right) = 0$$
$$= 1 - \frac{R-z}{R-z} = 1 - 1 = 0 \quad //$$

5) Hitung medan listrik P berjarak z diatas pusat kawat yang membentuk segi empat panjang dengan sisi s, seperti pada gambar berikut :



Pengelesaian :



$$F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$$, dimana

$\frac{1}{4\pi\varepsilon_0}$ coulumb konstan

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \dots (1) \qquad q = \lambda \cdot s \dots (2)$$

$\lambda = \frac{q}{t}$ → gunakan teorema phytagoras

$$FP^2 = OP^2 + FO^2 \rightarrow r^2 = z^2 + \left(\frac{s}{2}\right)^2 = z^2 + \frac{s^2}{4}$$

$$r^2 = \sqrt{z^2 + \frac{s^2}{4}} \dots (3)$$

∴ danano sisi AB

$$\vec{E}' = \frac{1}{4\pi\varepsilon_0} \lambda \frac{dx}{(r)^2} r' \dots (4)$$

$$r' = \frac{\vec{r}}{r} = \frac{zk - xi}{r} \rightarrow r = \sqrt{z^2 + x^2}$$

$$\vec{E}' = \int_{-s}^{s} \frac{1}{4\pi\varepsilon_0} \frac{\lambda}{z^2 + x^2} dx \frac{zk - xi}{\sqrt{z^2 + x^2}}$$

$$\vec{E}' = \frac{1}{4\pi\varepsilon_0} \int_{-s}^{s} \frac{\lambda}{z^2 + x^2} \frac{(zk - xi)}{\sqrt{z^2 + x^2}} dx$$

$$\vec{E}' = \frac{1}{4\pi\varepsilon_0} \left[\int_{-s}^{s} \frac{\lambda}{(z^2 + x^2)^{3/2}} dx\, k - \int_{-s}^{s} \frac{\lambda}{(z^2 + x^2)^{3/2}} x\, dx\, i \right]$$

$$\vec{E}' = \frac{1}{4\pi\varepsilon_0} \left[z\lambda \left[\frac{s}{z^2 (z^2 + s^2)^{1/2}} + \frac{s}{z^2 (z^2 + s^2)^{1/2}} \right]_{-s}^{s} k - 0 \right]$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{2\lambda s}{z\sqrt{z^2 + s^2}} k \dots (5)$$

$$s \rightarrow \frac{s}{2} \qquad z \rightarrow \sqrt{z^2 + \frac{s^2}{4}}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{2\lambda \frac{s}{2}}{\sqrt{z^2 + \frac{s^2}{4}} \sqrt{\left(z^2 + \frac{s^2}{4}\right)^2 + \left(\frac{s}{2}\right)^2}}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{\lambda s}{\sqrt{z^2 + \frac{s^2}{4}} \sqrt{\left(z^2 + \frac{s^2}{4}\right) + \left(\frac{s^2}{4}\right)}} k$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{\lambda s}{\sqrt{z^2 + \frac{s^2}{4}} \sqrt{\left(z^2 + \frac{s^2}{2}\right)}} k \dots (6)$$

Dari gambar. $\cos\theta = \frac{z}{r} = \frac{z}{\sqrt{z^2 + \frac{s^2}{4}}} \dots (8)$

$$\vec{E} = 4 \times \frac{1}{4\pi\varepsilon_0} \frac{\lambda s}{\sqrt{z^2 + \frac{s^2}{4}} \sqrt{\left(z^2 + \frac{s^2}{2}\right)}} \frac{z}{\sqrt{z^2 + \frac{s^2}{4}}} k$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{\lambda s z}{\left(z^2 + \frac{s^2}{4}\right)\left(\sqrt{z^2 + \frac{s^2}{2}}\right)} k$$ sepanjang arah z