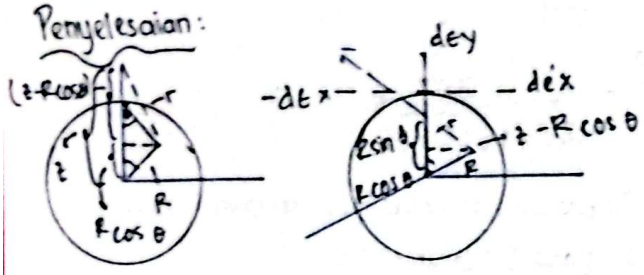
3) Hitung medan listrik pada bola konduktor dengan rapat muatan σ dan jari-jari R.

Penyelesaian:



$$r = [(R\sin\theta)^2 + (z - R\cos\theta)^2]^{1/2}$$
$$= [R^2\sin^2\theta + (z^2 + R^2\cos^2\theta - 2Rz\cos\theta)]^{1/2}$$
$$= [R^2(\sin^2\theta + \cos^2\theta) + z^2 - 2Rz\cos\theta]^{1/2}$$
$$r = (R^2 + z^2 - 2Rz\cos\theta)^{1/2}$$

Hitung $\dot{E}$

$$d\dot{E} = d\dot{E}_x + d\dot{E}_y$$
$$d\dot{E} = dE\cos\theta$$
$$\dot{E} = \frac{1}{4\pi\varepsilon_0}\int_0^{\pi}\int_0^{2\pi}\frac{\sigma\, da\cos\theta}{r^2}$$
$$= \frac{1}{4\pi\varepsilon_0}\int_0^{\pi}\int_0^{2\pi}\frac{\sigma(z - R\cos\theta)R^2\sin\theta\, d\theta\, d\phi}{(R^2 + z^2 - 2Rz\cos\theta)^{3/2}}$$
$$= \frac{\sigma R^2(2\pi)}{4\pi\varepsilon_0}\int_0^{\pi}\frac{\sin\theta(z - R\cos\theta)}{(R^2 + z^2 - 2Rz\cos\theta)^{3/2}}d\theta$$
$$= \frac{\sigma R^2}{2\varepsilon_0}\int_0^{\pi}\frac{\sin\theta(z - R\cos\theta)}{(R^2 + z^2 - 2Rz\cos\theta)^{3/2}}d\theta$$
$$-\int_{+1}^{-1}\frac{(+du)(z - Ru)}{(R^2 + z^2 - 2Rzu)^{3/2}}$$

$u = \cos\theta$

$\frac{du}{d\theta} = -\sin\theta$

$-du = \sin\theta\, d\theta$

$R^2 + z^2 - 2Rzu = y$

$-2Rz = \frac{dy}{du}$

$du = \frac{dy}{-2Rz}$

$$= \int_{(R+z)^2}^{(R-z)^2}\left(\frac{dy}{-2Rz}\right)\frac{z^2 - R^2 + y}{(2z)y^{3/2}}$$
$$= \int_{(R-z)^2}^{(R+z)^2}\left(\frac{1}{4Rz^2}\right)\left((z^2 - R^2)y^{-3/2} + y^{-1/2}\right)dy$$
$$\frac{1}{4Rz^2}\int_{(R-z)^2}^{(R+z)^2}\left((z^2 - R^2)y^{-3/2} + y^{-1/2}\right)dy$$
$$= \frac{1}{4Rz^2}\left[(z^2 - R^2)(-2)y^{-1/2} + 2y^{1/2}\right]_{(R-z)^2}^{(R+z)^2}$$
$$- \frac{1}{2Rz^2}\left((R^2 - z^2)\frac{1}{\sqrt{y}} + \sqrt{y}\right)_{(R-z)^2}^{(R+z)^2}$$
$$= \frac{1}{2Rz^2}\left((R^2 - z^2)\left(\frac{1}{R+z} - \frac{1}{R-z}\right) + (R+z) - (R-z)\right)$$
$$= \frac{1}{2Rz^2}\left(\frac{(R^2 - z^2)}{(R+z)(R-z)}\left(\frac{1}{R+z} - \frac{1}{R-z}\right) + (R+z) - (R-z)\right)$$
$$= \frac{1}{2Rz^2}\left(R - z\,\frac{-R^2 - z^2}{(R-z)} + R + z - (R-z)\right)(R-z)^2 - (R+z)^2$$
$$= \frac{1}{2Rz^2}\left(2R - \left(\frac{R^2 - z^2 + (R-z)^2}{(R-z)}\right)e^2 - 2Rz + z^2\right.$$
$$= \frac{1}{2Rz^2}\left(2R - \frac{(2R^2 - 2Rz)}{(R-z)}\right)$$
$$= \frac{1}{z^2}\left(1 - \frac{R-z}{(R-z)}\right)$$
$$\frac{\sigma R^2}{2\varepsilon_0}\frac{1}{z^2}\left(1 - \frac{(R-z)}{(R-z)}\right)$$

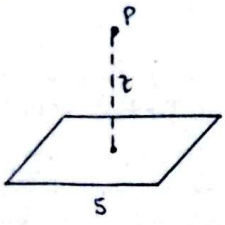
① $z > R$

$$\mathcal{E} = \frac{\sigma R^2}{2\varepsilon_0}\frac{1}{z^2}\left(1 - \frac{(R-z)}{z-R}\right)$$
$$= \frac{\sigma R^2}{\varepsilon_0 z^2}$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{4\pi R^2\sigma}{z^2}$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{z^2}$$

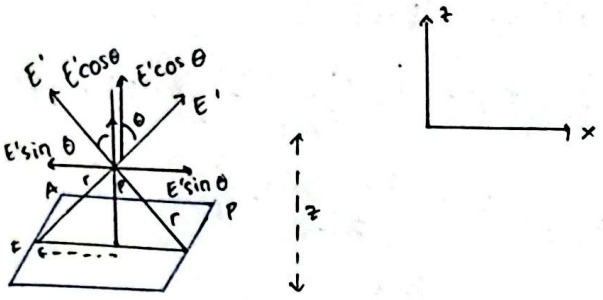
② $z > R$

$$\mathcal{E} = \frac{\sigma R^2}{2\varepsilon_0}\frac{1}{z^2}\left(1 - \frac{R-z}{R-z}\right) = 0$$
$$1 - \frac{R-z}{(R-z)} = 1 - 1 = 0$$

5.) Hitung medan listrik dititik p berjarak z di atas pusat kawat yang membentuk segi empat panjang dengan sisi s, seperti pada gambar berikut:



Penyelesaian:



$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$, dimana $\frac{1}{4\pi\varepsilon_0}$ coloumb constant

$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$ ....(1), $q = \lambda s$ ....(2)

Kerapatan perubahan garis muatan total (q) per satuan panjang (l) $\lambda = \frac{q}{T}$ → gunakan teorema phytagoras.

$FP^2 = OP^2 + FO^2$

$\Rightarrow r^2 = z^2 + \left(\frac{s}{2}\right)^2 = z^2 + \frac{s^2}{4}$

$r = \sqrt{z^2 + \frac{s^2}{4}}$ .... (3)

* Pertimbangkan skenario sisi AB

$\vec{E}' = \frac{1}{4\pi\varepsilon_0}\lambda\frac{dx}{(r)^2}r'$ .....(4)

$r' = \frac{\vec{r}}{r} = \frac{zk - xi}{r} \longrightarrow r = \sqrt{z^2+x^2}$

•) Maka:

$\vec{E}' = \int_{-s}^{s}\frac{1}{4\pi\varepsilon_0}\frac{\lambda}{z^2+x^2}dx\,\frac{zk-xi}{\sqrt{z^2+x^2}}$

$\vec{E}' = \frac{1}{4\pi\varepsilon_0}\int_{-s}^{s}\frac{\lambda}{z^2+x^2}\frac{(zk-xi)}{\sqrt{z^2+x^2}}dx$

$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\int_{-s}^{s}\frac{\lambda}{(z^2+x^2)^{3/2}}dx\,k - \int_{-s}^{s}\frac{\lambda}{(z^2+x^2)^{3/2}}xdi\right]$

$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[z\lambda\left\{\frac{x}{z^2(z^2+x^2)^{1/2}}\right\}_{-s}^{s}k - \lambda\left\{\frac{1}{(z^2+x^2)^{1/2}}\right\}_{-s}^{s}\right]$

$\vec{E}' = \frac{1}{4\pi\varepsilon_0}\left[z\lambda\left\{\frac{s}{z^2(z^2+s^2)^{1/2}} + \frac{s}{z^2(z^2+s^2)^{1/2}}\right\}_{-s}^{s}k - 0\right]$

$\vec{E}' = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda a}{z\sqrt{z^2+a^2}}k$ ... (5)

•) Sekarang, gunakan persamaan 5 untuk E' pada P karena AB

$s \to \frac{s}{2}$, $z \to \sqrt{z^2+\frac{s^2}{4}}$

Maka,

$\vec{E}' = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda\frac{s}{2}}{\sqrt{z^2+\frac{s^2}{4}}\sqrt{\left(\sqrt{z^2+\frac{s^2}{4}}\right)^2+\left(\frac{s}{2}\right)^2}}$

$\vec{E}' = \frac{1}{4\pi\varepsilon_0}\frac{\lambda s}{\sqrt{z^2+\frac{s^2}{4}}\sqrt{\left(z^2+\frac{s^2}{4}\right)+\left(\frac{s^2}{4}\right)}}k$

$\vec{E}' = \frac{1}{4\pi\varepsilon_0}\frac{\lambda s}{\sqrt{z^2+\frac{s^2}{4}}\sqrt{z^2+\frac{s^2}{4}}}k$ ... (6)

Ini adalah medan listrik di P karena satu sisi AB

•) Maka, medan listrik di P karena empat sisi:

$E = 4E'$

$E = 4E'\cos\theta$ .... (7)

$\cos\theta = \frac{z}{r} = \frac{z}{\sqrt{z^2+\frac{s^2}{4}}}$ ... (8)

Sehingga,

$E' = 4\times\frac{1}{4\pi\varepsilon_0}\frac{\lambda s}{\sqrt{z^2+\frac{s^2}{4}}\sqrt{(z^2+\frac{s^2}{2}}}\frac{z}{\sqrt{z^2+\frac{s^2}{4}}}k$

$\vec{E} = \frac{1}{\pi\varepsilon_0}\frac{\lambda s z}{\left(z^2+\frac{s^2}{4}\right)\sqrt{\left(z^2+\frac{s^2}{3}\right)}}k$