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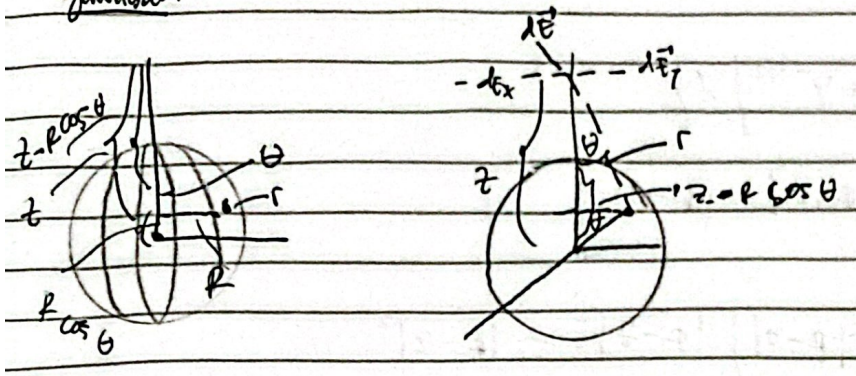
3) Hitung medan listrik pada bola konduktor dengan rapat muatan σ dan jari-jari R

Jawaban



dari r

$$r = \left[(R\sin\theta)^2 + (z - R\cos\theta)^2 \right]^{1/2}$$
$$= \left[R^2\sin^2\theta + (z^2 + R^2\cos^2\theta - 2Rz\cos\theta) \right]^{1/2}$$
$$= \left[R^2(\sin^2\theta + (\cos^2\theta) + z^2 - 2Rz\cos\theta \right]^{1/2}$$
$$= \left[R^2(\sin^2\theta + \cos^2\theta) + z^2 - 2Rz\cos\cos\theta \right]^{1/2}$$
$$r = \left[R^2 + z^2 - 2Rz\cos\theta \right]^{1/2}$$

Menghitung $\vec{E}$

$$d\vec{E} = d\vec{E}_x + d\vec{E}_y$$
$$d\vec{E} = dE\cos\theta$$
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_0^{\pi}\int_0^{2\pi} \frac{\sigma\, da\cos\theta}{r^2}$$
$$= \frac{1}{4\pi\varepsilon_0}\int_0^{\pi}\int_0^{2\pi} \frac{\sigma(z - R\cos\theta)R^2\sin\theta\, d\theta\, d\varphi}{(R^2 + z^2 - 2Rz\cos\theta)^{3/2}}$$
$$= \frac{\sigma R^2(2\pi)}{4\pi\varepsilon_0}\int_0^{\pi} \frac{\sin\theta(z - R\cos\theta)}{(R^2 + z^2 - 2Rz\cos\theta)^{3/2}}\, d\theta$$
$$= \frac{\sigma R^2}{2\varepsilon_0}\int_0^{\pi} \frac{\sin\theta(z - R\cos\theta)}{(R^2 + z^2 - 2Rz(\cos\theta)^{3/2}}\, d\theta$$
$$= \int_{-1}^{1} \frac{(+du)(z - Ru)}{(R^2 + z^2 - 2Rzu)^{3/2}}$$

$u = \cos\theta$

$\frac{du}{d\theta} = \sin\theta$

$-du = \sin d\theta$

$R^2 + z^2 - 2Rzu = Y$

$-2Rz = \frac{dY}{du}$, $du = \frac{dY}{-2Rz}$

$$= \int_{(R+z)^2}^{(R-z)} \left(\frac{dY}{-2Rz}\right) \frac{z^2 - R^2 + Y}{(2z)Y^{3/2}}$$

langsung 3

$$= \int_{(R-z)^2}^{(R+z)^2} \left(\frac{1}{4Rz^2}\right)\left[(z^2-R^2)y^{-3/2} + y^{-1/2}\right] dy$$

$$= \frac{1}{4\pi\varepsilon_0} \int_{(R-z)^2}^{(R+z)^2} \left[(z^2-R^2)y^{-3/2} + y^{-1/2}\right] dy$$

$$= \frac{1}{2Rz^2}\left[(R^2-z^2)\left(\frac{1}{R+z} - \frac{1}{(R-z)}\right) + (R+z) - |R-z|\right]$$

$$= \frac{1}{2Rz^2}\left(R-z - \frac{R^2-z^2}{|R-z|} + R+z - |R-z|\right) \quad |R-z|^2 = |R-z|^2$$

$$= \frac{1}{2Rz^2}\left[2R - \left[\frac{R^2-z^2+(R-z)^2}{(R-z)}\right]\right] \quad e^2 - 2Rz + z^2$$

$$= \frac{1}{2Rz^2}\left(2R - \frac{(2R^2-2Rz)}{|R-z|}\right)$$

$$= \frac{1}{z^2}\left(1 - \frac{R-z}{|R-z|}\right)$$

$$= \frac{\sigma R^2}{2\varepsilon_0}\frac{1}{z^2}\left(1 - \frac{R-z}{|R-z|}\right)$$

$z > R$

$$E = \frac{\sigma R^2}{2\varepsilon_0}\frac{1}{z^2}\left(1 - \frac{|R-z|}{|z-R|}\right) = \frac{\sigma R^2}{\varepsilon_0 z^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{4\pi R^2\sigma}{z^2}$$

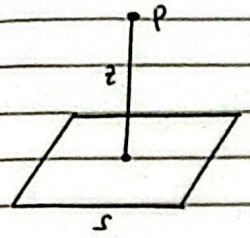
$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{z^2}$$

$z < R$

$$E = \frac{\sigma R^2}{2\varepsilon_0}\frac{1}{z^2}\left(1 - \frac{R-z}{|R-z|}\right) = 0$$

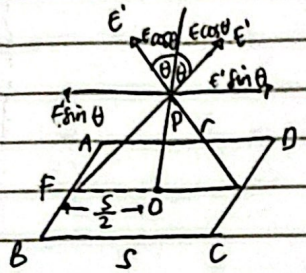
$$1 - \frac{R-z}{|R-z|} = 1-1 = 0$$

5) # Jawaban



Dik. Medan listrik di titik P
Berjarak z
Panjang kiri s

Penyelesaian :



Gunakan Hukum coulumb

$$E = \frac{1}{4\pi\varepsilon_0}\frac{s}{r^2} \quad (1)$$

Muatan total AB

$$q = \lambda s \quad (2)$$

Gunakan Pitagoras untuk r dan AB

$$FP^2 = DP^2 + FO^2$$
$$r^2 = z^2 + \left(\frac{s}{2}\right)^2$$
$$= z^2 + \left(\frac{s}{4}\right)^2$$

$$r = \sqrt{z^2 + \frac{s^2}{4}} \quad (3)$$

Medan P

$$\vec{E'} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{(r)^2} r' \quad (4)$$

$$E' = \int_{-s}^{s} \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{z^2+x^2}\frac{dx}{\sqrt{z^2+x^2}}\frac{zk - x_i}{\sqrt{z^2+x^2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\int_{-s}^{s}\frac{\lambda}{z^2+x^2}\frac{(zk - x_i)}{\sqrt{z^2+x^2}}\,dx$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\int_{-s}^{s}\frac{z\,\lambda}{(z^2+x^2)^{3/2}}\,dx\,k - \int_{-s}^{s}\frac{\lambda}{(z^2+x^2)^{3/2}}\,x\,dx_i\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[z\lambda\left(\frac{x}{z^2(z+x^2)^{1/2}}\right)_{-s}^{s}k - \lambda\left(\frac{-1}{(z^2+x^2)^{1/2}}\right.\right.$$

$$= \frac{1}{4\pi\varepsilon_0}\left[z\lambda\left(\frac{s}{z^2(z^2+s^2)^{1/2}} + \frac{s}{z^2(z^2+s^2)^{1/2}}\right)_{-s}^{s}k - 0\right.$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{2\lambda s}{z\sqrt{z^2+s^2}}k \quad \ldots \quad (5)$$

ganti $s \to \frac{s}{2}$ dan $z \to \sqrt{z^2 + \frac{s^2}{4}}$

$$\vec{E'} = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda\frac{s}{2}}{\sqrt{z^2+\frac{s^2}{4}}\sqrt{\left(z^2+\frac{s^2}{4}\right)^2+\left(\frac{s}{2}\right)^2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda s}{\sqrt{z^2+\frac{s^2}{4}}\sqrt{\left(z^2+\frac{s^2}{4}\right)+\left(\frac{s}{2}\right)^2}}k$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda s}{\sqrt{z^2+\frac{s^2}{4}}\sqrt{\left(z^2+\frac{s^2}{2}\right)}}$$

Untuk Medan P

$$E = 4E'$$

Lanjutan 5

Medan listrik 1 loop di z

$$E = 4\varepsilon' \cos\theta \qquad (7)$$

$$\cos\theta = \frac{z}{r} = \frac{z}{\sqrt{z^2 + \frac{s^2}{4}}} \qquad (8)$$

Substitusi (6) dan (7)

$$\vec{E} = 4 \times \frac{1}{4\pi\varepsilon_0} \frac{\lambda s}{\sqrt{z^2+\frac{s^2}{4}}\sqrt{\left(z^2+\frac{s^2}{2}\right)}} \frac{z}{\sqrt{z^2+\frac{s^2}{4}}} k$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{\lambda s z}{\left(z^2+\frac{s^2}{4}\right)\sqrt{\left(z^2+\frac{s^2}{2}\right)}} k \;//$$