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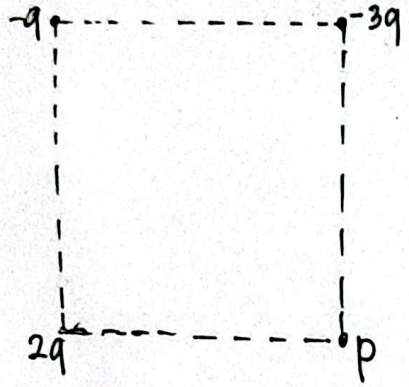
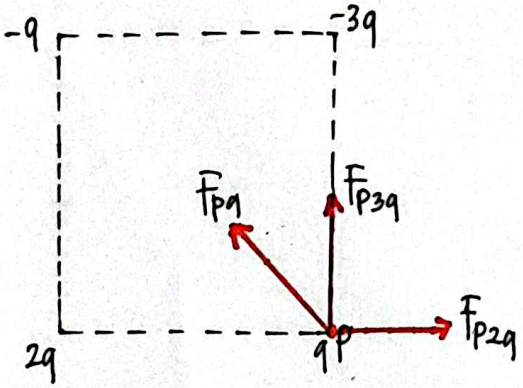
UTS Elektrodinamika Semester Genap 2025
Sabtu, 19 April 2025.

1) Ada tiga muatan seperti gambar berikut ini :



a) Gambar gaya-gaya pada titik p



b). Besar gaya-gaya tersebut (F_{pq}, F_{p2q}, F_{p3q})!

$$F_{pq} = k\frac{q_p(q)}{r_{pq}^2} = k.\frac{(q_p)(-q)}{r_{pq}^2}(\hat{i}+\hat{j})$$

$$F_{p,2q} = \frac{k.(q_p)(2q)}{(r_{p,2q})^2}(i)$$

$$F_{p,-3q} = \frac{k(q_p)(-3q)}{(r_{p,-3q})^2}(j)$$

$$F_p = F_{pq} + F_{p,2q} + F_{p,-3q}$$

$$= k\frac{(q_p)(-q)}{(r_{pq})^2}(i+j) + k\frac{(q_p)(2q)}{(r_{p,2q})^2}(i) + k\frac{(q_p)(-3q)}{(r_{p,-3q})^2}(j)$$

$$= kq_p\left(\frac{-q}{(r_{pq})^2}(i+j) + \frac{(2q)}{(r_{p,2q})^2}(i) + \frac{(-3q)}{(r_{p,3q})^2}(j)\right)$$

$$F_p = kq_p\left[\left(\frac{-q}{(r_{pq})^2}+\frac{2q}{(r_{p,2q})^2}\right)i + \left(\frac{-q}{(r_{p,q})^2}+\frac{(-3q)}{(r_{p,3q})^2}\right)j\right]$$

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c) Medan listrik pada titik tersebut

$$E_q = k.\frac{q}{(r_{pq})^2}(i+j) = k\frac{(q)}{(r_{pq})^2}(i) + k\frac{q}{(r_{pq})^2}j$$

$$E_{(-3q)} = k.\frac{(-3q)}{(r_{p,3q})^2}(\hat{j})$$

$$E_{(2q)} = k\frac{(2q)}{(r_{p,2q})^2}(\hat{i})$$

d) $E_{total} = E_q + E_{(-3q)} + E_{(2q)}$

$$= k\frac{(-q)}{(r_{pq})^2}(i+j) + k\frac{(-3q)}{(r_{p,3q})^2}(\hat{j}) + k\frac{(2q)}{(r_{p,2q})}(i)$$

$$E_{tot} = \left[\frac{k(q)}{(r_{pq})^2} + \frac{k(2q)}{(r_{p2q})^2}\right]i + \left[k\frac{(q)}{(r_{pq})^2} + \frac{k(-3q)}{(r_{p,3q})^2}\right]j$$

$$\boxed{E_{tot} = \left[\frac{k(-q)}{(r_{pq})^2} + \frac{k(2q)}{(r_{p,2q})^2}\right]i + \left[\frac{k(-q)}{(r_{pq})^2} + \frac{k(-3q)}{(r_{p,3q})^2}\right]j}$$

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e) Potensial listrik pada titik p

$$V_{(-q)} = k\frac{(-q)}{(r_{pq})^2}(i+j) = k\frac{(-q)}{(r_{pq})}(\hat{i}) + k\frac{(-q)}{r_{pq}}(j)$$

$$V_{(2q)} = k\frac{(2q)}{(r_{p,2q})}(\hat{i})$$

$$V_{(-3q)} = k\frac{(-3q)}{(r_{p,3q})}(j)$$

$$V_{total} = V_{(-q)} + V_{(2q)} + V_{(-3q)}$$

$$= \left(k\frac{(-q)}{(r_{pq})}(i) + k\frac{(-q)}{(r_{pq})}(j)\right) + k\frac{(2q)}{(r_{p2q})}(i) + k\frac{(-3q)}{(r_{p3q})}(j)$$

$$\boxed{V_{total} = \left[k\frac{(-q)}{(r_{pq})} + k\frac{(2q)}{(r_{p2q})}\right]\hat{i} + \left[k\frac{(-q)}{r_{pq}} + k\frac{(-3q)}{(r_{p,3q})}\right]\hat{j}}$$

$$\boxed{V_{total} = k\left[\left(\frac{-q}{r_{pq}} + \frac{2q}{r_{p2q}}\right)i + \left(\frac{-q}{r_{pq}} + \frac{(-3q)}{r_{p,3q}}\right)j\right]}$$

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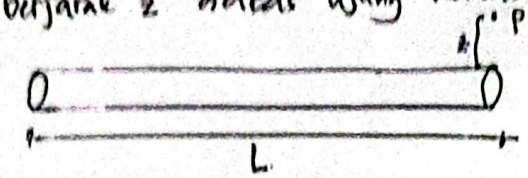
2) Sebuah batang panjang L, rapat muatan per satuan panjang adalah λ.

a) Kuat Medan Listrik di titik P yang berjarak z diatas ujung kawat



Penyelesaian:

$dq = \lambda\, dx$

$dE = k\frac{dq}{r^2} \Rightarrow r^2 = x^2 + z^2$

$r = \sqrt{x^2+z^2}$

$dE = k.\frac{\lambda}{(x^2+z^2)}\, dx$

$dE = k\frac{\lambda.z}{(x^2+z^2)^{3/2}}.dx$

maka:

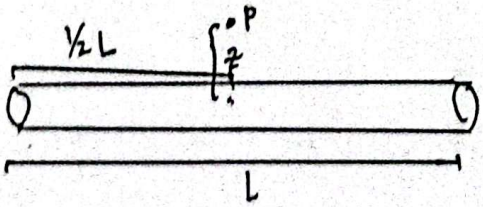
$$E = \int_0^L k\frac{\lambda.z}{(x^2+z^2)^{3/2}}.dx$$

$$= k\lambda z \int_0^L \frac{1}{(x^2+z^2)^{3/2}}\,dx$$

$$= \frac{k\lambda.z}{z^2}\cdot\frac{L}{\sqrt{L^2+z^2}}$$

$$\boxed{E = k.\frac{\lambda.L}{z\sqrt{L^2+z^2}}}$$

b) Kuat Medan listrik di titik P yang berjarak z di tengah-tengah kawat



Penyelesaian:

$$E = \int_{-\frac{1}{2}L}^{\frac{1}{2}L} \frac{k\lambda z}{(x^2+z^2)^{3/2}}.dx$$

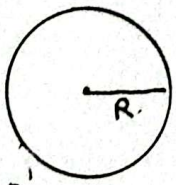
$$= \int_{-\frac{1}{2}L}^{\frac{1}{2}L} k\,\frac{\lambda.z}{(x^2+z^2)^{3/2}}\,dx$$

$$= k\lambda \int_{-\frac{1}{2}L}^{\frac{1}{2}L} \frac{z}{(x^2+z^2)}\,dx$$

$$= k.\lambda.z.\frac{L}{2z\sqrt{\left(\frac{L}{2}\right)^2+z^2}}$$

$$\boxed{E = k.\frac{\lambda L}{2\sqrt{\left(\frac{L}{2}\right)^2+z^2}}}$$

(3) Hitung medan listrik pada bola konduktor dengan rapat muatan σ dan jari-jari R



A = Luas permukaan Bola

Penyelesaian:

$A = 4\pi R^2$

$Q = \sigma.A = \sigma.4\pi R^2$

Menggunakan thk. Gauss.

$$\oint E.dA = \frac{Q}{\varepsilon_0}$$

$$E.4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$E.4\pi r^2 = \frac{\sigma.4\pi R^2}{\varepsilon_0}$$

$$\boxed{E = \frac{\sigma R^2}{\varepsilon_0 r^2}}$$

untuk: $r \geq R$ maka $E_{luar} = \frac{\sigma R^2}{\varepsilon_0 r^2}$

$r \leq R$ maka $E_{dalam} = 0$

4. Bila kuat medan listrik $E = kr^3$ dalam koordinat bola, dengan k konstan

a) Hitung rapat muatannya.

Penyelesaian:

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_{total}}{r^2} \quad \text{(Hk Gauss)}$$

Menggunakan pers Maxwell

$$E = \frac{\rho}{\varepsilon_0}$$

$$E = kr^3$$

$$\frac{\rho}{\varepsilon_0} = kr^3 \Rightarrow \boxed{\rho = \varepsilon_0 . kr^3}$$

b) Cari muatan total yang terdapat dalam suatu bola dgn jari-jari R berpusat di 0

Penyelesaian:

Dalam koordinat bola,

$$dV = r^2 \sin\theta \, dr \, d\theta \, d\phi$$

maka:

$$Q = \int \rho \, dV$$

$$= \int_0^{2\pi}\int_0^{\pi}\int_0^{R} \rho \, r^2 \sin\theta \, dr \, d\phi \, d\theta$$

$$Q = \int_0^{2\pi}\int_0^{\pi}\int_0^{R} \varepsilon_0 k r^3 \, r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$Q = \int_0^{2\pi}\int_0^{\pi}\int_0^{R} \varepsilon_0 k r^3 . \, r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= \varepsilon_0 k \int_0^{R} r^{n+2} dr . \int_0^{\pi} \sin\theta \, d\theta . \int_0^{2\pi} d\phi$$

$$Q = \varepsilon_0 . k \int_0^{R} r^{n+2} dr \cdot 2 \cdot 2\pi$$

$$= \varepsilon_0 . k . \frac{R^{n+3}}{n+3} . 2 . 2\pi$$

$$= \frac{4\pi\varepsilon_0 k R^{n+3}}{n+3.}$$

$$\boxed{Q = \frac{4\pi\varepsilon_0 k R^{n+3}}{n+3}}$$

5) Hitung medan listrik titik P berjarak z di atas pusat

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

Dengan theorema Pythagoras

$$FP^2 = OF^2 + FO^2$$

$$r^2 = z^2 + \left(\frac{q}{2}\right)^2 = z^2 + \frac{q^2}{4}$$

$$r = \sqrt{z^2 + \frac{1}{4}q^2}$$

Medan listrik di titik P

$$\vec{E'} = \frac{1}{4\pi\varepsilon_0} \lambda \frac{dx}{(r')^2} r' \Rightarrow r' = \frac{\vec{r}}{r} = \frac{zk - xi}{r}$$

$$E' = \int_{-a}^{a} \frac{1}{4\pi\varepsilon_0} \frac{\lambda}{z^2+x^2} dx \frac{zk - xi}{\sqrt{z^2+x^2}}$$

$$= \frac{1}{4\pi\varepsilon_0} \int_{-a}^{a} \frac{\lambda}{z^2+x^2} \left(\frac{zk - xi}{\sqrt{z^2+x^2}}\right) dx$$

$$= \frac{1}{4\pi\varepsilon_0} \left[z\lambda \left\{ \frac{x}{z^2(z^2+x^2)^{1/2}} \right\} \cdot k - \lambda \left(\frac{1}{(z^2+x)^{1/2}} \right)^{a} \right.$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[z\lambda \left[\frac{a}{z(z^2+a^2)^{1/2}} + \frac{a}{z^2(z^2+a^2)^{1/2}} \right] k - 0 \right]$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{2\lambda a}{z\sqrt{z^2+a^2}} \cdot k.$$

$a \to \frac{a}{2}$ dan $z = \sqrt{z^2 + \left(\frac{a^2}{4}\right)}$

$$\vec{E}' = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2\pi \cdot a}{2} \Big/ \sqrt{z^2+\frac{a^2}{4}} \sqrt{\left(\sqrt{z^2+\left(\frac{a^2}{4}\right)^2}\right)^2 + \left(\frac{a}{2}\right)^2}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \left(\frac{\lambda a}{\sqrt{z^2+\frac{a^2}{4}}\sqrt{z^2\left(\frac{a^2}{4}\right)+\left(\frac{a^2}{4}\right)}} \right)$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{\lambda \cdot a}{\sqrt{z^2+\frac{a^2}{4}} \cdot \sqrt{\left(z^2+\frac{a^2}{4}\right)}}$$

maka Medan Listrik di titik P pd Loop persegi

$$E = 4E' \cos\theta$$

$$\cos\theta = \frac{z}{r} = \frac{z}{\sqrt{z^2+\frac{a^2}{4}}} \text{ / maka}$$

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{\lambda \cdot az}{\left(z^2+\frac{a^2}{4}\right)\left(\sqrt{\left(z^2+\frac{a^2}{2}\right)}\right)}}$$

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