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UTS Elektrodinamika

1. Jawaban no

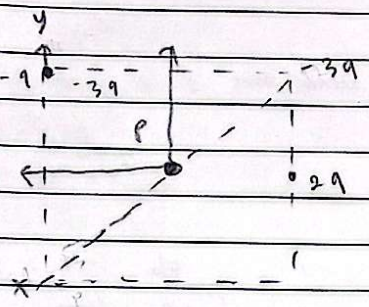
dik: $q_3 = +q$ $\quad k = \frac{1}{4\pi\varepsilon_0}$

$q_1 = -q$

$q_2 = -3q$

dit:

a.



b. $F = k\frac{q_1 q_2}{r^2}$

$F_1 = k\frac{q_1 \cdot q_3}{r_1^2} = k\frac{-q \cdot 2q}{a^2} = k\cdot\frac{2q^2}{a^2}$

* Jarak muatan $-3q$ dan $2q$

$r_2 = \sqrt{a^2 + a^2} = \sqrt{2}a$

$F = k\cdot\frac{q_2 \cdot q_3}{r_2^2} = k\frac{|-3q \cdot 2q|}{(\sqrt{2}a)^2} = k\frac{6q^2}{2a^2} = \frac{3kq^2}{a^2}$ arah 45° (kanan atas)

c. $E = k\frac{q}{r^2}$

$E_1 = k\frac{|-q|}{a^2} = \frac{kq}{a^2}$

muatan $-3q$ (atas kanan)

$r_2 = \sqrt{2}a$

$E_2 = k\frac{-3q}{(\sqrt{2}a)^2} = \frac{3kq}{2a^2}$

arah menuju muatan $-3q$ → arah 45° kekanan atas

komponen x dan y pada sudut 45°

$E_{2x} = E_2 \cos 45° = \frac{3kq}{2a^2} \cdot \frac{1}{\sqrt{2}} = \frac{3kq}{2\sqrt{2}a^2}$

$E_{2y} = E \sin 45^\circ = \frac{3kq}{2\sqrt{2}a^2}$

* total medan dititik P

$E_x = E_{2x} = \frac{3kq}{2\sqrt{2}a^2}$ $\quad E_y = E_1 + E_{2x} = \frac{kq}{a^2} + \frac{3kq}{2\sqrt{2}a^2}$

$E\ total = \left(\frac{3kq}{2\sqrt{2}a^2}\right)\hat{i} + \left(\frac{kq}{a^2} + \frac{3kq}{2\sqrt{2}a^2}\right)\hat{j}$

d. $E\ total = \sqrt{E_x^2 + E_y^2}$

$E_x^2 = E_x = \frac{3kq}{2\sqrt{2}a^2} = E_x^2 = \left(\frac{3kq}{2\sqrt{2}a^2}\right)^2 = \frac{9k^2q^2}{8a^4}$

$E_y^2 = E_y = \frac{kq}{a^2} + \frac{3kq}{2\sqrt{2}a^2}$

$E_y = \frac{kq}{a^2}\left(1 + \frac{3}{2\sqrt{2}}\right)$ $\quad E_x = \frac{kq}{a^2}\left(1 + \frac{3}{2\sqrt{2}}\right)$

$E_x^2 = \left(\frac{kq}{a^2}\ 1 + \frac{3}{2\sqrt{2}}\right)^2 = \frac{k^2q^2}{a^4}\left(1 + \frac{3}{2\sqrt{2}}\right)^2 = \frac{3}{2\sqrt{2}} = \frac{3}{2 \cdot 1414} = 1.06 = 1 + 106 = 2.06$

$E_x^2 = \frac{k^2q^2}{a^4}(2.06)^2 = \frac{k^2q^2}{a^4}\ 429$

$E\ total = \sqrt{E_x^2 + E_y^2}$

$= \sqrt{\frac{9k^2q^2}{8a^4} + \frac{9 \cdot 29k^2q^2}{a^2}}$

$= \sqrt{\frac{k^2q^2}{a^4}\left(\frac{9}{8} + 929\right)} = \sqrt{\frac{k^2q^2}{a^4} \cdot 537} = \frac{kq}{a^2} \cdot \sqrt{537} = \frac{kq}{a^2}\ 232$

P. $q_1 = -q \quad r_1 = a$

$q_2 = -3q \quad r_2 = \sqrt{2}a$

* muatan $-q$ $\qquad$ * muatan $-3q$

$V_1 = k \cdot \frac{-q}{a} = -\frac{kq}{a}$ $\qquad V_2 = k \cdot \frac{-3q}{\sqrt{2}a} = -\frac{3kq}{\sqrt{2}a}$

* total potensial dititik P

$V_{total} = V_1 + V_2 = -\frac{kq}{a} - \frac{3kq}{\sqrt{2}a} = -\frac{kq}{a}\left(1 + \frac{3}{\sqrt{2}}\right)$

$\frac{3}{\sqrt{2}} = \frac{3}{1414} = 212 \rightarrow 1 + 212 = 312$. jadi $V_{total} = \frac{kq}{a} \cdot (-3.12)$

2. dik : sebuah kawat : L

muatan linear : λ

dit : a. medan kawat

b. tegangan kawat

Jawab :

a. $x = 0$ sampai $x = L$ pada sumbu vertikal $x = 0$ sejauh z

$dq = \lambda\, dx$

$r = \sqrt{x^2 + z^2}$

$$dE = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq}{x^2+z^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\lambda\, dx}{x^2+z^2}$$

$$dE_x = dE \frac{x}{\sqrt{x^2+z^2}} = \frac{\lambda x\, dx}{4\pi\varepsilon_0 (x^2+z^2)^{3/2}}$$

$$dE_z = dE \cdot \frac{z}{\sqrt{x^2+z^2}} = \frac{\lambda x\, dx}{4\pi\varepsilon_0 (x^2+z^2)^{3/2}}$$

$$E_z = \int_{-L/2}^{L/2} dE_z = \frac{\lambda x}{4\pi\varepsilon_0} \int_{-L/2}^{L/2} \frac{dx}{(x^2+z^2)^{3/2}}$$

$$E_z = \frac{2\lambda z}{4\pi\varepsilon_0} \int_0^{L/2} \frac{dx}{(x^2+z^2)^{3/2}}$$

substitusi

misal : $x = z \tan\theta \quad dx = z \sec^2\theta\, d\theta$

$$x^2 + z^2 = z^2 \sec^2\theta \quad (x^2+z^2)^{3/2} = z^3 \sec^3\theta$$

substitusi integral

$$E_z = \frac{2\lambda z}{4\pi\varepsilon_0} \int_{\theta=0}^{\tan^{-1}(L/2z)} \frac{z \sec^2\theta\, d\theta}{z^3 \sec^3\theta} = \frac{\lambda}{2\pi\varepsilon_0 z} \int_0^{\tan^{-1}(L/2z)} \cos\theta\, d\theta$$

$$\int \cos\theta\, d\theta = \sin\theta$$

$$E_z = \frac{\lambda}{2\pi\varepsilon_0 z} \cdot \sin\left(\tan^{-1}\frac{L}{2z}\right)$$

$$\sin(\tan^{-1}(x)) = \frac{x}{\sqrt{1+x^2}}$$

dengan $x = \frac{L}{2z}$ maka

$$\sin\left(\tan^{-1}\frac{L}{2z}\right) = \frac{L}{\sqrt{L^2+4z^2}} \cdot \frac{1}{2z}$$

$$E = \frac{\lambda L}{2\pi\varepsilon_0 z\sqrt{L^2+4z^2}}$$

b. $x = -\frac{L}{2}$ hingga $x = +\frac{L}{2}$

$P = (0,0,z)$

$dq = \lambda\, dx$

$r = \sqrt{x^2+z^2}$

* elemen dq

$$dE = \frac{1}{4\pi\varepsilon_0}\cdot\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\lambda\, dx}{x^2+z^2}$$

$$dE_z = dE\cdot\cos\theta = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\lambda\, dx}{x^2+z^2}\cdot\frac{z}{\sqrt{x^2+z^2}} = \frac{\lambda z\, dx}{4\pi\varepsilon_0(x^2+z^2)^{3/2}}$$

$$E = \int_{-L/2}^{L/2} dE_z = \frac{\lambda z}{4\pi\varepsilon_0}\int_{-L/2}^{L/2}\frac{dx}{(x^2+z^2)^{3/2}}$$

$$E = \frac{2\lambda z}{4\pi\varepsilon_0}\int_{0}^{L/2}\frac{dx}{(x^2+z^2)^{3/2}}$$

* hitung integral

$$\int\frac{dx}{(x^2+z^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2+a^2}} + C$$

$$E = \frac{\lambda z}{2\pi\varepsilon_0}\left[\frac{x}{z^2\sqrt{x^2+z^2}}\right]_0^{L/2}$$

$$E = \frac{\lambda z}{2\pi\varepsilon_0}\left(\frac{L/2}{z^2\sqrt{(L/2)^2+z^2}}\right) = \frac{\lambda z}{2\pi\varepsilon_0}\cdot\frac{L}{2z^2\sqrt{z^2+L^2/4}}$$

$$E = \frac{\lambda L}{4\pi\varepsilon_0 z\sqrt{z^2+\frac{L^2}{4}}}$$

3. dik: jari-jari = R

muatan = σ

jawab:

$Q = \sigma\cdot A = \sigma\cdot 4\pi R^2$

* medan listrik luar bola ($r > R$)

$$\oint\vec{E}\cdot d\vec{A} = \frac{Q\ \text{dalam}}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{\sigma \cdot 4\pi R^2}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\sigma R^2}{r^2}$$

Jadi $r > R$ adalah $E(r) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\sigma R^2}{r^2}$ arah menjauhi permukaan jika $\sigma > 0$

* medan listrik dalam bola $(r < R)$

Q dalam $= 0 \rightarrow E = 0$

Jadi untuk $r < R$ adalah $E(r) = 0$ tidak ada listrik didalam bola konduktor

4. dik : $\vec{E} = kr^3 \hat{r}$

k konstan

Jawab :

a. rapat muatan ρ

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\vec{E} = E_r(r)\hat{r} = kr^3 \hat{r}$$

$$\nabla \cdot \vec{E} = \frac{1}{r^2} \frac{d}{dr}(r^2 E_r)$$

$$= \frac{1}{r^2} \frac{d}{dr}(r^2 \cdot kr^3) = \frac{1}{r^2} \frac{d}{dr}(kr^5) = \frac{1}{r^2} \cdot 5kr^4 = 5kr^2$$

maka :

$$\frac{\rho}{\varepsilon_0} = 5kr^2 \rightarrow \rho(r) = 5\varepsilon_0 r^2$$

b. muatan total Q dalam bola berjari - jari R

$Q = \iiint \rho(r)\, dV$ dengan $\rho(r) = 5k\varepsilon_0 r^2$

$dV = r^2 \sin\theta\, dr\, d\theta\, d\phi$

maka :

$$Q = \int_0^{2\pi}\int_0^{\pi}\int_0^{R} (5k\varepsilon_0 r^2) \cdot r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$= 5k\varepsilon_0 \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta\, d\theta \int_0^{R} r^4\, dr$$

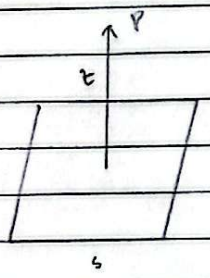
integralnya :

$\int_0^{2\pi} d\phi = 2\pi$ $\qquad$ $\int_0^{R} r^4\, dr = \frac{R^5}{5}$

$\int_0^{\pi} \sin\theta\, d\theta = 2$

Jadi $Q = 5k\varepsilon_0 \cdot 2\pi \cdot 2 \cdot \frac{R^5}{5} = 1\pi\varepsilon_0 k R^5$

5.



$x = -\frac{s}{2}$ ke $x = \frac{s}{2}$ $\quad y = \frac{s}{2}$

$\vec{r}' = (x, s/2, 0)$

$p = \vec{r} = (0, 0, z)$

$\vec{R} = \vec{r} - \vec{r}' = (-x, -s/2, z)$

$|\vec{R}| = \sqrt{x^2 + (s/2)^2 + z^2}$

$d\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\lambda\, dl\, \vec{R}}{|\vec{R}|^3}$

$dE_z = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\lambda \cdot z \cdot dl}{(x^2 + (s/2)^2 + z^2)^{3/2}}$

$E_z^{(+)} = \frac{\lambda z}{4\pi\varepsilon_0} \int_{-s/2}^{s/2} \frac{dx}{(x^2 + (s/2)^2 + z^2)^{3/2}}$

substitusikan:

$a^2 = (s/2)^2 + z^2$

$E_z^{(+)} = \frac{\lambda z}{4\pi\varepsilon_0} \int_{-s/2}^{s/2} \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{\lambda z}{4\pi\varepsilon_0} \left[\frac{x}{a^2\sqrt{x^2 + a^2}} \right]_{-s/2}^{s/2}$

$E_z^{(+)} = \frac{\lambda z}{4\pi\varepsilon_0} \left(\frac{s/2}{a^2\sqrt{(s/2)^2 + a^2}} - \left(-\frac{s/2}{a^2\sqrt{(s/2)^2 + a^2}} \right) \right)$

$= \frac{\lambda z}{4\pi\varepsilon_0} \frac{s}{a^2\sqrt{(s/2)^2 + a^2}} = \frac{\lambda z s}{4\pi\varepsilon_0 \cdot a^2\sqrt{(s/2)^2 + a^2}}$

karena $a^2 = (s/2)^2 + z^2$

maka : $a^2 = (s^2/4 + z^2)$

total medan semua sisi

$$E_z = 4 \cdot E_z^{(1)} = \frac{4\lambda z s}{4\pi\varepsilon_0 (s^2/4 + z^2) \cdot \sqrt{s^2/4 + z^2}} = \frac{2\lambda s}{\pi\varepsilon_0 (s^2/4 + z^2)^{3/2}}$$