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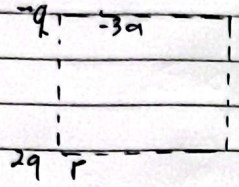
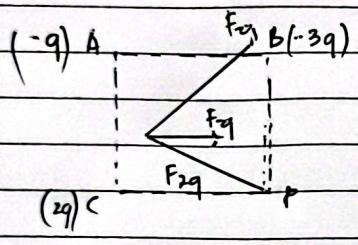
" UTS ELEKTRODINAMIKA "

1. Ada 3 muatan seperti gambar berikut



a. Gambar gaya pada titik P



Muatan A : -q
Muatan B : -3q
Muatan C : 2q

b. Hitung besar gaya

Hukum Coulomb :

$$F = k\frac{q_1 q_2}{r^2}$$

Muatan (-q)
Jarak A ke P : $\sqrt{2}a$ maka

$$F_A = k\cdot\frac{q_0 q}{(\sqrt{2}a)^2}$$

$$F_{-3q} = k\frac{2q\,3q}{a^2} = \frac{6kq^2}{a^2}$$

Gaya $-q$ (jarak $a\sqrt{2}$) :

$$F_{-q} = k\cdot\frac{2q\cdot q}{(a\sqrt{2})^2}$$
$$= \frac{2kq^2}{2a^2}$$
$$= \frac{kq^2}{a^2}$$

c. $\vec{E} = \frac{\vec{F}}{q}$

Medan $-3q$ (jarak a) arah kiri

$$E_{-3q} = k\cdot\frac{3q}{a^2}$$

medan dari $-q$ (jarak $a\sqrt{2}$ arah menuju muatan $-q$, kiri atas

$$E_{-q} = k\cdot\frac{q}{(a\sqrt{2})^2}$$
$$= \frac{kq}{2a^2}$$

Untuk medan $2q$ (muatan itu sendiri) : tidak memberikan medan

d. $E_{-qx} = \frac{kq}{2a^2}\cos 45^\circ = \frac{kq}{2a^2}\cdot\frac{\sqrt{2}}{2}\cdot\frac{kq\sqrt{2}}{4a^2}$

$E_{-qy} = E_{-qx} = \frac{kq\sqrt{2}}{4a^2}$

$$E_x = -\left(\frac{3k\,q}{a^2} + \frac{k\,q\sqrt{2}}{4a^2}\right)$$

$$E_y = -\frac{kq\sqrt{2}}{4a^2}$$

$$|\vec{E}| = \sqrt{E_x^2 + E_y^2}$$

e. $$V = \Sigma k \frac{q_i}{r}$$

$$= -\frac{3kq}{a} - \frac{kq}{\sqrt{2}a}$$

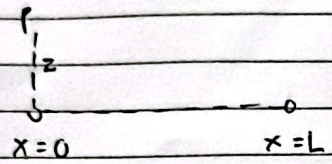
2. Sebuah batang panjangnya L, rapat muatan per satuan panjang adalah λ. Hitung.

a. Kuat medan listrik di titik P yg berjarak z di atas ujung kawat.

Batang berada dari x = 0 sampai x = L, titik P ujung kiri (x = 0) pada (0, 0, z)

$$dE = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq}{r^2}$$

karena $dq = \lambda dx$, $r = \sqrt{x^2+z^2}$ maka



$$dE = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\lambda z\, dx}{(x^2+z^2)^{3/2}}$$

$$E = \frac{1}{4\pi\varepsilon_0} \lambda z \int_0^L \frac{dx}{(x^2+z^2)^{3/2}}$$

substitusikan

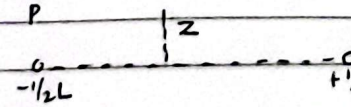
$$\int \frac{dx}{(x^2+z^2)^{3/2}} = \frac{x}{z^2\sqrt{x^2+z^2}}$$

sehingga

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \lambda z \left[\frac{x}{z^2\sqrt{x^2+z^2}}\right]_0^L = \frac{\lambda}{4\pi\varepsilon_0} \cdot \left(\frac{L}{z\sqrt{L^2+z^2}}\right)$$

$$E = \frac{\lambda}{4\pi\varepsilon_0} \cdot \frac{L}{z\sqrt{L^2+z^2}}$$

b. Kuat medan listrik di titik P yg berjarak z di tengah kawat.



$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\lambda z\, dx}{(x^2+z^2)^{3/2}}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \lambda z \int_{-L/2}^{L/2} \frac{dx}{(x^2+z^2)^{3/2}}$$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \lambda z \left[\frac{x}{z^2\sqrt{x^2+z^2}} \right]_{-L/2}^{L/2} = \frac{\lambda}{4\pi\varepsilon_0} \cdot \frac{L}{z\sqrt{(L/2)^2+z^2}}$$

Maka

$$E = \frac{\lambda}{4\pi\varepsilon_0} \cdot \frac{L}{z\sqrt{(L^2/4)+z^2}}$$

$$= \frac{\lambda L}{4\pi\varepsilon_0 z\sqrt{(L^2/4)+z^2}}$$

3. Hitung medan listrik pada bola konduktor dg rapat muatan σ dan jari-jari R.

↳ Hukum Gauss:

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$$, untuk $r > R$ maka

dengan $Q = \sigma \cdot 4\pi R^2$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0} \Rightarrow E \cdot 4\pi r^2 = \frac{\sigma \cdot 4\pi R^2}{\varepsilon_0} = \frac{\sigma R^2}{\varepsilon_0 r^2}$$

atau

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\sigma 4\pi R^2}{r^2}$$

$$= \frac{\sigma R^2}{\varepsilon_0 r^2}$$

Untuk medan listrik dalam konduktor

$E = 0$ karena konduktor dalam distribusi muatan nol

maka:

$E = 0$ untuk $r < R$

$E = \frac{\sigma R^2}{\varepsilon_0 r^2}$ untuk $r > R$

4. Bila kuat listrik $E = kr^3 r_0$ dlm koordinat bola
k = konstan

a. rapat muatan.
hukum gauss bentuk diferensial

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

dlm koordinat bola $\vec{E} = kr^3 \hat{r}$

$$\nabla \vec{E} = \frac{1}{r^2} \cdot \frac{d}{dr}(r^2 \cdot kr^3)$$

$$= \frac{1}{r^2} \cdot \frac{d}{dr}(kr^5)$$

$$= \frac{1}{r^2} \cdot 5kr^4$$

$$= 5kr^2$$

$$\rho = \varepsilon_0 \cdot 5kr^2$$

$$\rho(r) = 5k\varepsilon_0 r^2$$

b. total muatan dlm bola berjari-jari R

$$Q = \int \rho dv$$

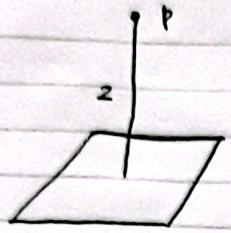
$$= \int_0^R 5k\varepsilon_0 r^2 \cdot 4\pi r^2 dr$$

$$= 20\pi k\varepsilon_0 \int_0^R r^4 dr$$

$$= 20\pi k\varepsilon_0 \cdot \frac{R^5}{5}$$

$$Q = 4\pi k\varepsilon_0 R^5$$

5. medan listrik di titik p berjarak z di atas pusat kawat yg berbentuk segi empat



$$E_z = \frac{\sigma}{4\pi\varepsilon_0} \cdot 4\sin^{-1}\left(\frac{s^2}{\sqrt{(s^2+4z^2)^2}}\right)$$

komponen z dari medan listrik hkp masing

$$E_z = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \cos\theta$$

$$\cos\theta = \frac{z}{r} \Rightarrow E_z = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qz}{r^3}$$

$$E_{total,z} = 4 \cdot \left(\frac{1}{4\pi\varepsilon_0} \cdot \frac{q_z}{r^3}\right)$$

$$= \frac{q_z}{\pi\varepsilon_0\left((s^2/2)+z^2\right)^{3/2}}$$

$$\bar{E}_p = \frac{q_z}{\pi\varepsilon_0\left((s^2/2)+z^2\right)^{3/2}} \hat{z}$$

KKY