# ELEKTRODINAMIKA

Dr. I wayan Distrik, M.Si
Dr. Kartini Herlina, M.Si

No Sabtu
Date 15 feb 2025

## Kelistrikan

1. Statis : Gaya, medan, potensial, energi usaha, Polarisasi,
2. Dinamis

## Kemagnetan

1. Medan magnet
2.
3.
4. Elektromagnetik

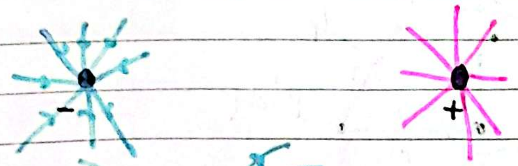
## Muatan Listrik

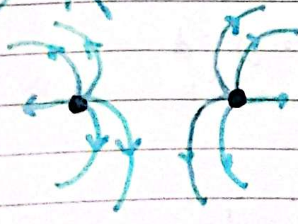
a/ Partikel yang menyebabkan mereka mengalami interaksi elektromagnetik

$q = \pm ne$

(+) Proton $= +1,602 \times 10^{-19}$ Coulumb
(-) Electron : $-1,602 \times 10^{-19}$ Coulumb

Muatan positif  Muatan Negatif





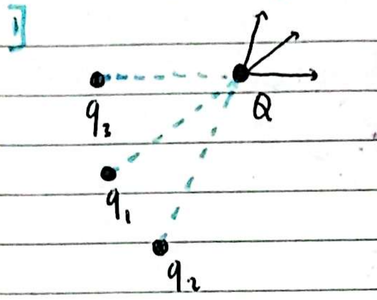
$\hat{r} = \frac{\vec{r}}{|\vec{r}|}$

$\vec{F} = K \frac{q\,q}{r^2}$

$K = \frac{1}{4\pi\varepsilon_0}$

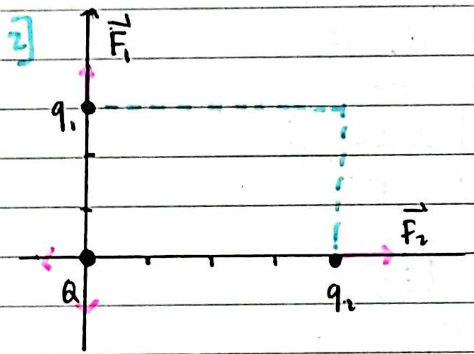
## Contoh Penerapan :

1)




$\vec{F_Q} = \vec{F_1} + \vec{F_2} + \vec{F_3}$

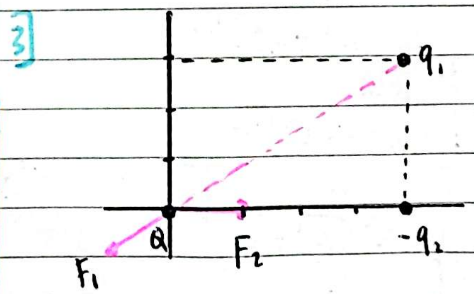
2)




$F_Q = kQ\left[\frac{q_1}{3^2}\hat{j} + \frac{q_2}{4^2}-\hat{i}\right]$

$= -kq\left[\frac{q_1}{9}\hat{j} + \frac{q_2}{16}\hat{i}\right]$

3)




$\vec{F} = kQ\left[\frac{q_1}{5^2}\frac{(-4\hat{i} - 3\hat{j})}{5} + \frac{q_2}{4^2}(\hat{i})\right]$

r

## Kuat Medan Listrik

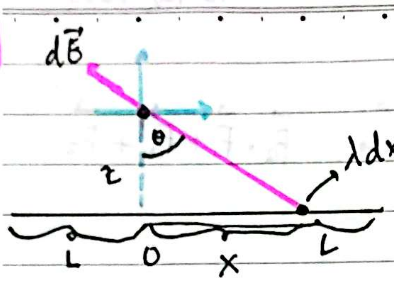
$\vec{E} = E_1 +$

# ELEKTRODINAMIKA

Anna Hafsah (2423022009)

No Sabtu
Date 8 maret 2025

1]




$\vec{E} = \frac{1}{4\pi\varepsilon_0} \sum \frac{q}{r_i^2} \hat{r}_0$

$\vec{E} = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r^2} \hat{r}_0$

$= \frac{1}{4\pi\varepsilon_0} \int \frac{\lambda\, dl}{r^2} \hat{r}_0$

$d\vec{E} = d\vec{E}_x + d\vec{E}_y$

$= d\vec{E} \cos\theta$

$= \frac{1}{4\pi\varepsilon_0} \int_{-L}^{L} \frac{\lambda\, dx}{r^2} \cos\theta$

$= \frac{1}{4\pi\varepsilon_0} \int_0^L \frac{2\lambda\, dx}{r^2} \cos\theta$

$= \frac{1}{4\pi\varepsilon_0} \int_0^L \frac{z\, 2\lambda\, dx}{(x^2+z^2)^{3/2}}$

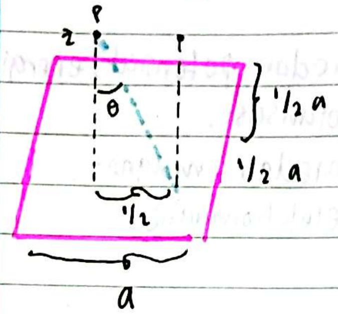
$= \frac{2z\lambda}{4\pi\varepsilon_0} \int_0^L \frac{dx}{(x^2+z^2)^{3/2}}$

$= \frac{2z\lambda}{4\pi\varepsilon_0} \left[ \frac{x}{z^2\sqrt{x^2+z^2}} \right]_0^L$

$= \frac{\lambda}{2\pi\varepsilon_0 z} \left[ \frac{L}{\sqrt{L^2+z^2}} \right]$

$\vec{E} = \frac{\lambda L}{2\pi\varepsilon_0 z\sqrt{L^2+z^2}}$

bersebrangan (cos)




$\vec{E}_z = E'_{z_1} \cos\theta$

$= 4\vec{E}_z \cdot \cos\theta$

$= 4\left[\frac{\lambda L}{2\pi\varepsilon_0 z\sqrt{L^2+z^2}}\right]\cos\theta$

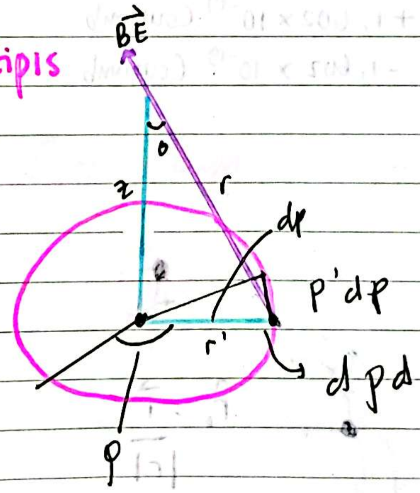
$z \to L$, $L \to \frac{1}{2}a$

$\cos\theta = \frac{z}{z}$

$z = z' \cos\theta$

$z' = \sqrt{z^2 + (\frac{1}{2}a)^2}$

tipis




$E = \frac{1}{4\pi\varepsilon} \int_0^R \int_0^{2\pi} \frac{\sigma\, da}{r^2} \cdot \cos\theta\, \hat{j}$

$r = (z^2 + r'^2)^{1/2}$

$= \frac{\sigma}{4\pi\varepsilon_0} 2\pi z \int_0^R \frac{r'\, dr'}{(z^2+r'^2)^{3/2}}$

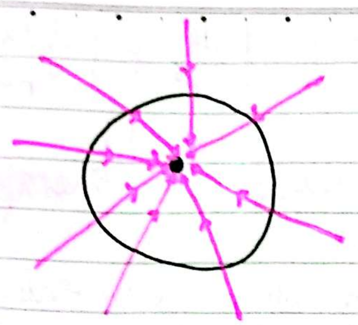
$= \frac{2\pi z\sigma}{4\pi\varepsilon_0} \cdot \frac{1}{2} \int_0^R \frac{d(r'^2+z^2)}{(z^2+r^2)^{3/2}}$

$= \frac{2\pi z\sigma}{4\pi\varepsilon_0} \frac{1}{2}$

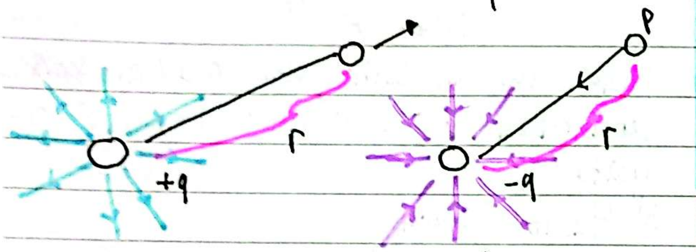
# MEDAN LISTRIK Part 2

No

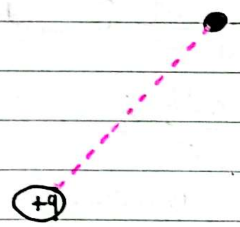
Date



$\vec{g} = G\frac{M}{r^2}\hat{r}$ dengan $\vec{g}$ = kuat medan gravitasi pada suatu titik




garis-garis yang ditimbulkan oleh muatan listrik




$\vec{E} = k\frac{Q}{r^2}\hat{r}$ $\vec{E}$ = kuat medan listrik

$$\vec{g} = G\frac{M_1}{r_1^2}\hat{r}_1 + G\frac{M_2}{r_2^2}\hat{r}_2 + \dots = \sum_{i=1}^{N} G\frac{M_i}{r_i^2}\hat{r}_i$$

$$\vec{E} = k\frac{q_1}{r_1^2}\hat{r}_1 + k\frac{q_2}{r_2^2}\hat{r}_2 + \dots = \sum_{i=1}^{N} k\frac{Q_i}{r_i^2}\hat{r}_i$$

**Contoh Soal :**

Tiga partikel bermuatan terdistribusi sebagai berikut : $q_1$ (4.0) = 2 C, $q_2$ (0, 3) = 5 C. $q_3$ (0,0) = 10 C. Tentukan : Medan listrik pada posisi (4, 3). dimana jarak dalam meter

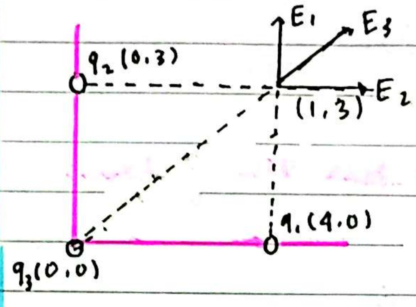
**Penyelesaian :**

$q_1 = +2\ C \rightarrow (4,0)\ m$

$q_2 = +5\ C \rightarrow (0,3)\ m$

$q_3 = +10\ C \rightarrow (0,0)\ m$

$k = 9 \cdot 10^9\ N \cdot m^2/C^2$




$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$ Dengan, $\hat{r} = \frac{\vec{r}}{|r|}$

medan oleh $q_1$

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{2}{16}(\hat{i})$$

$$\vec{E}_3 = \frac{1}{4\pi\varepsilon_0}\frac{10}{25}\left(\frac{4\hat{i}+3\hat{j}}{5}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{40}{125}\hat{i} + \frac{30}{125}\hat{j}\right)$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\left(\frac{2}{16}+\frac{40}{125}\right)(\hat{i}) + \left(\frac{5}{9}+\frac{30}{125}\right)\hat{j}\right]$$

## Distribusi Muatan

$dm = \rho\, dv$ $\qquad dq = \sigma\, dA$

$dq = \rho\, dV$ $\qquad dq = \lambda\, dl$

Hk. Coulumb untuk muatan kontinu

$dq = \lambda\, dl,\ \sigma\, da,\ \rho\, d\tau$

$$\sum_{i=1}^{n}(\ )q_i \sim \int_{line}(\ )\lambda\, dl \sim (\ )\sigma\, da$$

JOYKO 36 Lines, 6 mm

No ......
Date

Besarnya medan listrik oleh muatan garis

$$\vec{E} = k \int \frac{\hat{r}_0}{r^2}\, dq$$

$$\vec{E} = k \int \frac{\hat{r}_0}{r^2}\, \lambda dl$$

Muatan kontinu pada bidang atau permukaan

$$\vec{E} = k \int_p \int_\ell \left( \frac{\hat{r}_0}{r^2} \right) dq$$

$$\vec{E} = k \int_p \int_\ell \left( \frac{\hat{r}_0}{r^2} \right) \sigma ds$$

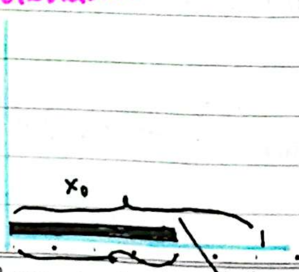
Muatan kontinu pada volume dapat dianalogikan bigi di dalam buah jambu bigi

$$\vec{E} = k \iiint \left( \frac{\hat{r}_0}{r^2} \right) \rho d\tau$$

Contoh Penerapan

1] Muatan terdistribusi pada garis dengan panjang tertentu (L), Muatan seragam Q terletak di sepanjang garis yang panjangnya L, terletak pada sepanjang sumbu x dari x = 0 sampai x = L, densitas muatannya adalah $Q = \lambda L$. Hitung medan listrik pada titik P yang terletak pada $x_0 > L$!

Penyelesaian:




maka, $E_x = k \int \frac{dq}{r^2} = k \int \frac{\lambda dx}{r^2}$

Part 3

# Hk. Gauss & Penerapannya

1] Medan listrik oleh muatan terdistribusi

Analog dari medan listrik oleh muatan titik.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}_0$$

maka medan listrik oleh muatan kontinue terdistribusi dapat dihitung dengan 2 cara, yaitu:

a] Hukum Coulumb

Hukum Coulumb dalam penerapannya menggunakan integral dalam sistem koordinat kartesian, bola dan silinder.

1) untuk integral garis

$$\vec{E} = \int_{line} \frac{1}{4\pi\varepsilon_0} \frac{dq}{r^2} \hat{r}_0 \rightarrow dq = \lambda dl$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \int \frac{\lambda dl}{r^2} \hat{r}_0$$

2) untuk luas permukaan

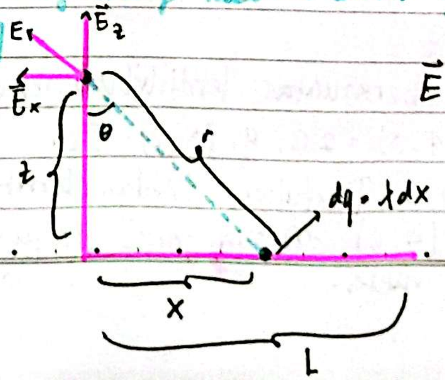
$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \iint \frac{\sigma\, ds}{r^2} \hat{r}_0$$

3) Untuk volume

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \iiint \frac{\rho\, d\tau}{r^2} \hat{r}_0$$

Contoh Penerapan Hk. Coulumb untuk menghitung medan listrik!

1]




$$\vec{E} = \vec{E}_z + \vec{E}_x$$

JOYKO® 36 Lines, 6 mm

Ambil $\vec{E}_z = \frac{1}{4\pi\varepsilon_0}\int_0^L \frac{dq}{r^2} = \frac{1}{\square}\int_0^L \frac{\lambda\, dx}{r^2}\cos\theta$

$\Rightarrow \cos\theta = \frac{z}{r}$

$$\vec{E}_z = \frac{1}{\square}\int_0^L \frac{z\lambda\, dx}{r^3} = \frac{1}{\square}\int_0^L \frac{z\lambda\, dx}{(z^2+x^2)^{3/2}}$$

$$= \frac{z\lambda}{4\pi\varepsilon_0}\int_0^L \frac{dx}{(z^2+x^2)^{3/2}}$$

Lihat di daftar integral

$$\int_0^L \frac{dx}{(z^2+x^2)^{3/2}} = \frac{1}{z^2}\frac{x}{\sqrt{z^2+x^2}}\Bigg|_0^L$$

$$= \frac{1}{4\pi\varepsilon_0}\lambda z\left[\frac{1}{z^2}\frac{x}{\sqrt{z^2+x^2}}\right|_0^L$$

$$\vec{E}_z = \frac{\lambda z}{4\pi\varepsilon_0}\lambda z\frac{L}{z^2\sqrt{z^2+x^2}} = \frac{\lambda}{4\pi\varepsilon_0}\frac{L}{z\sqrt{z^2+x^2}}$$

$$\vec{E}_x = -\frac{1}{4\pi\varepsilon_0}\int_0^L \frac{\lambda\, dx}{r^2}\sin\theta$$

$$= -\frac{\lambda}{4\pi\varepsilon_0}\cdot\frac{1}{2}\int_0^L \frac{dx^2}{(z^2+x^2)^{3/2}}$$

$$= -\frac{\lambda}{4\pi\varepsilon_0}\cdot\frac{1}{2}\left[-2\frac{1}{(z^2+x^2)^{1/2}}\right]_0^L$$

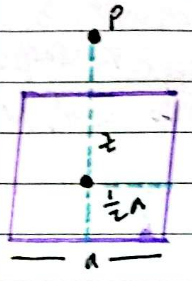
$$= -\frac{\lambda}{4\pi\varepsilon_0}\left[-\left(\frac{1}{\sqrt{z^2+x^2}} - \frac{1}{z}\right)\right]$$

$$\vec{E}_x = -\frac{\lambda}{4\pi\varepsilon_0}\left[\frac{1}{z} - \frac{1}{\sqrt{z^2+x^2}}\right]$$

$$\vec{E} = \vec{E}_z + \vec{E}_x$$

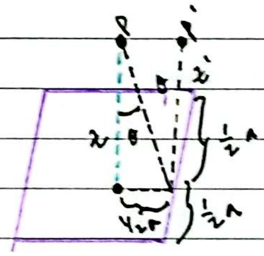
$$= \frac{\lambda}{4\pi\varepsilon_0}\frac{L}{z\sqrt{z^2+x^2}} - \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{1}{z} - \frac{1}{\sqrt{z^2+x^2}}\right] \quad //$$

Ulasan Tugas No. 2 Hal 53




Solusi :
Untuk menyelesaikan soal diatas gunakan contoh no. 3, yaitu kawatnya 2L, dugaan $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda L}{z'^2}$

$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda L}{z'^2}$ Hasil contoh no. 3




$P = P'\cos\theta$; karena ada 4 sisi, maka
$P = 4P'\cos\theta$; sehingga

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda L}{z'^2} = 4\cdot\frac{1}{4\pi\varepsilon_0}\frac{2\lambda L}{z'^2}\cos\theta$$

karena $L = \frac{1}{2}a$, dan $z' = r = \sqrt{z^2+(\frac{1}{2}a)^2}$

$$\vec{E} = 4\cdot\frac{1}{4\pi\varepsilon_0}\frac{2\lambda(\frac{1}{2}a)\cdot z}{(z^2+(\frac{1}{2}a)^2)\sqrt{z^2+(\frac{1}{2}a)^2}}$$

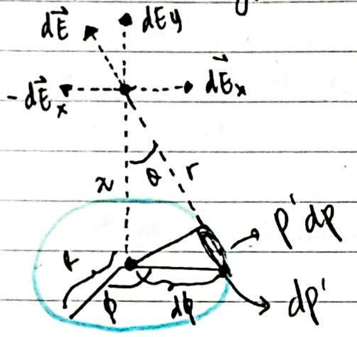
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{4\lambda a z}{\sqrt{z^2+\frac{a^2}{4}}}\hat{z}$$

No

Date

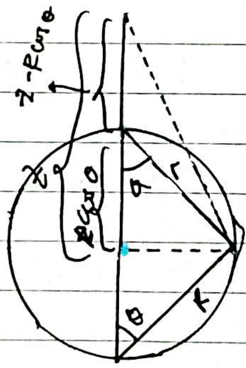
Tugas no. 4 Hal 53

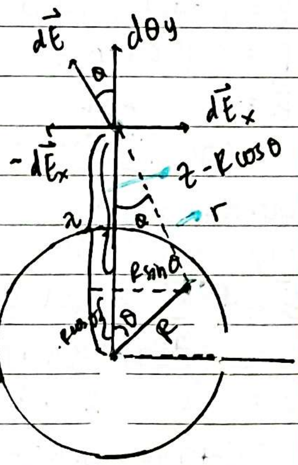
Tentukan medan listrik berjarak z diatas pusat permukaan terbentuk lingkaran dengan jari-jari R yang membawa permukaan seragam.




Penyelesaian ada di lembar sebelumnya

Problems 2.7 Buku Griffith




Pandang r

$$r = [(R\sin\theta)^2 + (z - R\cos\theta)^2]^{1/2}$$
$$= [R^2\sin^2\theta + (z^2 + R^2\cos^2\theta - 2Rz\cos\theta)]^{1/2}$$
$$= [R^2(\sin^2\theta + \cos^2\theta) + z^2 - 2Rz\cos\theta]^{1/2}$$
$$r = (R^2 + z^2 - 2Rz\cos\theta)^{1/2}$$

Sekarang hitung $\vec{E}$

$$d\vec{E} = d\vec{E}_x + d\vec{E}_y$$
$$d\vec{E} = dE\cos\theta$$
$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int_0^{\pi}\int_0^{2\pi}\frac{\sigma\, da\cos\theta}{r^2}$$
$$= \frac{1}{4\pi\epsilon_0}\int_0^{\pi}\int_0^{2\pi}\frac{\sigma\,(z - R\cos\theta)\,R^2\sin\theta\, d\theta\, d\phi}{(R^2 + z^2 - 2Rz\cos\theta)^{3/2}}$$

# Muatan Kontinu

Part 1

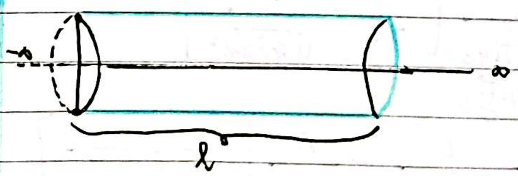
1. Medan Listrik
2. Potensial Listrik
3. Energi Listrik

Pembahasan : Medan listrik

Medan listrik pada benda ≈ tinggi tinggi dihitung dengan "Hukum Gauss"

1. $\nabla\vec{E} = \dfrac{\rho}{\epsilon_0}$
2. $\oint \vec{E}\, d\hat{a} = \dfrac{Q}{\epsilon_0}$

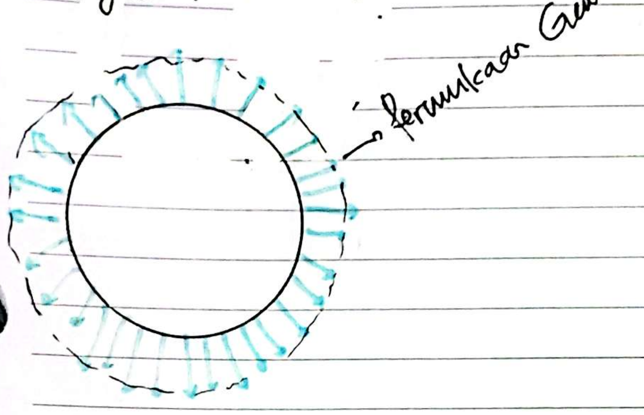
a) Pada kawat panjang sekali




Permukaan Gauss

$$\oint \vec{E}\cdot d\vec{a} = \frac{Q}{\epsilon_0}, \quad Q = \lambda \cdot l$$
$$\vec{E}\cdot a = \frac{\lambda \cdot l}{\epsilon_0}$$
$$\vec{E}\,(2\pi r l) = \frac{\lambda\, l}{\epsilon_0}$$
$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r}\,\hat{r}$$

b) bola pejal (bola penuh muatan) jari-jarinya a mempunyai muatan seragam q. Hitung medan listrik di luas bola sejauh r $(r > a)$

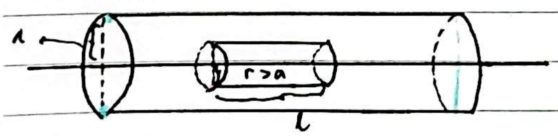



$$\oint \vec{E}\cdot d\vec{a} = \frac{Q}{\epsilon_0}$$

$$\vec{E}\,(4\pi r^2) = \frac{q}{\epsilon_0}$$

$$\vec{E} = \frac{q}{4\pi\epsilon_0 r^2}\,\hat{E}$$

c) silinder panjang membawa rapat muatan volume $\rho = kr$. Hitung medan listrik di dalam silinder dengan di luas silinder, jika jari-jari silinder a




$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{inc}}{\epsilon_0} \rightarrow \rho v = q$$

$$\vec{E}\,(2\pi r \cdot l) = \frac{1}{\epsilon_0}\int_v \rho \, dG$$

$$= \frac{1}{\epsilon_0}\int_0^r\int_0^{2\pi}\int_0^l (kr)\, r\, dr\, d\phi\, dz$$

$$= \frac{2\pi l k}{\epsilon_0}\int_0^r r^2\, dr$$

$$\vec{E}\,(2\pi r l) = \frac{2}{3}\,\frac{\pi l k}{\epsilon_0}\, r^3$$

$$\vec{E} = \frac{kr^2}{3\epsilon_0}\,\hat{r} \quad , r < a$$

$$\vec{E}\,(2\pi r l) = \frac{1}{\epsilon_0}\int_0^a\int_0^{2\pi}\int_0^l (k\rho)\, r\, d\rho\, d\phi\, dz$$

$$= \frac{2\pi k l}{\epsilon_0}\int_0^a r^2\, dr = \frac{2}{3}\,\frac{\pi k l}{\epsilon_0}\, a^3$$

$$\boxed{\vec{E} = \frac{k a^3}{3\epsilon_0 r}}$$

# Potensial Listrik

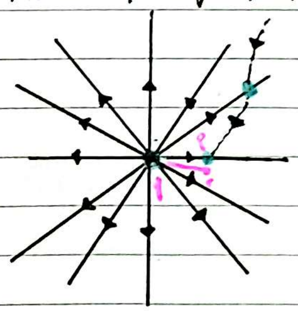
part 5

1 Pengenalan potensial melalui analogi

$$V = \frac{W}{m} = \frac{G\frac{M\cdot m}{r}}{m} = G\frac{M}{r}$$

$$V = G\frac{m_1}{r_1} + G\frac{m_2}{r_2} + \dots G\frac{m_n}{r_n} = \sum_{i=1}^{n} G\frac{m_i}{r_i}$$

muatan q digerakkan oleh suatu gaya ke titik P mendekati muatan



$$dV = \frac{dU}{q_0} = -\vec{E}\cdot d\vec{l}$$

No
Date

Beda potensial listrik dari titik a ke titik b adalah

$$\int_a^b dV = \int_{r_a}^{r_b} -\vec{E}\cdot d\vec{l}$$

$$V(b) - V(a) = -\int_0^b E\cdot dl + \int_0^a E\cdot dl$$

$$= -\int^b E\cdot dl + \int_a^0 E\cdot dl$$

$$= -\int_0^{b} E\cdot dl$$

$$V(b) - V(a) = \int_a^b (\nabla V)\, dl$$

$$\int_a^b (\nabla V)\cdot dl = -\int_a^b E\cdot dl$$

$$E = -\nabla V$$

$$V_b - V_a = kQ\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

$$V_b - V_a = kQ\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

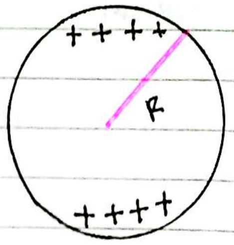
maka besarnya potensial secara umum dapat ditulis:

$$V_p = k\frac{Q}{r}$$

masing-masing pada jarak $r_1, r_2, r_3 \ldots r_n$

$$V_p = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \ldots \frac{q_n}{r_n}\right]$$

## Potensial listrik oleh bola konduktor



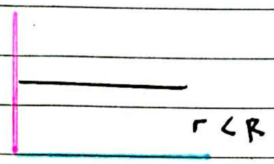

Medan listrik di dalam bola adalah nol, tetapi potensial listrik di dalam bola konduktor tersebut tidak sama dengan nol.

$$V_b - V_a = -\oint E\,dl$$

$V_b - V_a = 0$, dimana $V_a = V_b = k\frac{q}{R}$

$$V = k\frac{q}{R}$$

Hubungan antar potensial listrik (V) dan jarak (r) di dalam bola konduktor




Pert 6

# Metoda Penentuan Potensial

Dalam elektrostatis adalah menentukan medan listrik $\vec{E}$ yang tidak tergantung kepada waktu (stasioner)

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\iiint \frac{\rho\, d\tau}{r^2}\hat{r}$$

$$\nabla\vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \times \vec{E} = 0$$

$$\vec{E} = -\nabla V$$

Jika dalam bentuk integral:

$$V(r) = -\int_{acuan}^{r} \vec{E}\cdot d\vec{l}$$

$$V = \frac{1}{4\pi\varepsilon_0}\iiint \frac{\rho\cdot d\tau}{r}$$

$$= \frac{1}{4\pi\varepsilon_0}\iiint \left(\frac{1}{r}\right)\rho\, d\tau$$

No
Date

## Persamaan Poisson

$\nabla\cdot\vec{E} = -\nabla\cdot\nabla V = \frac{\rho}{\varepsilon_0}$

$\nabla^2 V = -\frac{\rho}{\varepsilon_0}$ → persamaan Poisson

$\nabla^2 V = 0$ → persamaan Laplace

$V = V(x, y, z)$

$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$ → kartesius $V = V(x, y, z)$

### koordinat bola

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

### koordinat silinder

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} + \frac{\partial^2 V}{\partial z^2}$$

## Persamaan Laplace

$\nabla^2 V = -\frac{\rho}{\varepsilon_0}$ → $\rho = 0$

$\nabla^2 V = 0$

misalkan potensial V hanya bergantung pada suatu variabel :

$\frac{d^2 V}{dx^2} = \frac{d}{dx}\left(\frac{dV}{dx}\right) = 0$

$\frac{dV}{dx} = m$

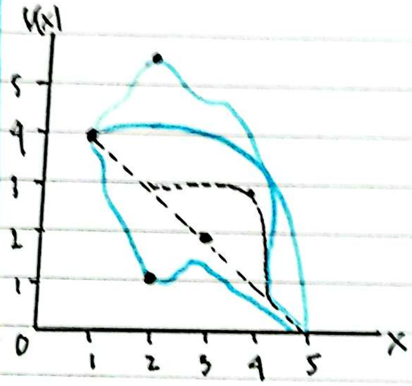
$V = mx + b$ → $V = 4$ di $x = 1$
$V = 0$ di $x = 5$

Dari syarat batas tersebut :

$4 = m + b$
$0 = 5m + b$ −
$4 = -4m + 0$

$m = -1$ maka $b = -5$

Sehingga : $V(x) = -x + 5$




### Persamaan Laplace dua dimensi

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

V bergantung dari dua variabel koordinat untuk sistem koordinat kartesius.

## Syarat Batas dan Teori Keunikan

$\nabla^2 V_1 = 0$ dan $\nabla^2 V_2 = 0$

$V_1 \neq V_2$

$V_3 = V_1 - V_2$ → $\nabla^2 V_3 = \nabla^2 V_1 - \nabla^2 V_2 = 0$

$\nabla^2 V = -\frac{\rho}{\varepsilon_0}$ , $V_3 = V_1 - V_2$ , $V_1 = V_2$

$\nabla^2 V_3 = \nabla^2 V_1 - \nabla^2 V_2 = -\frac{\rho}{\varepsilon_0} + \frac{\rho}{\varepsilon_0} = 0$